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MATHEMATICS AND ITS APPLICATIONS IN THE DEVELOPMENT OF SCIENCES AND TECHNOLOGY

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PREFACE

It is an honor and great pleasure for the Department of Mathematics – Universitas Gadjah Mada, Yogyakarta – INDONESIA, to be entrusted by the Southeast Asian Mathematical Society (SEAMS) to organize an international conference every four years. Appreciation goes to those who have developed and established this tradition of the successful series of conferences. The SEAMS - Gadjah Mada University (SEAMS-GMU) 2011 International Conference on Mathematics and Its Applications took place in the Faculty of Mathematics and Natural Sciences of Universitas Gadjah Mada on July 12th – 15th, 2011. The conference was the follow up of the successful series of events which have been held in 1989, 1995, 1999, 2003 and 2007.

The conference has achieved its main purposes of promoting the exchange of ideas and presentation of recent development, particularly in the areas of pure, applied, and computational mathematics which are represented in Southeast Asian Countries. The conference has also provided a forum of researchers, developers, and practitioners to exchange ideas and to discuss future direction of research. Moreover, it has enhanced collaboration between researchers from countries in the region and those from outside.

More than 250 participants from over the world attended the conference. They come from USA, Austria, The Netherlands, Australia, Russia, South Africa, Taiwan, Iran, Singapore, The Philippines, Thailand, Malaysia, India, Pakistan, Mongolia, Saudi Arabia, Nigeria, Mexico and Indonesia. During the four days conference, there were 16 plenary lectures and 217 contributed short communication papers. The plenary lectures were delivered by Halina France-Jackson (South Africa), Jawad Y. Abuihlail (Saudi Arabia), Andreas Rauber (Austria), Svetlana Borovkova (The Netherlands), Murk J. Bottema (Australia), Ang Keng Cheng (Singapore), Peter Filzmoser (Austria), Sergey Kryzhevich (Russia), Intan Muchtadi-Alamsyah (Indonesia), Reza Pulungan (Indonesia), Salmah (Indonesia), Yudi Soeharyadi (Indonesia), Subanar (Indonesia) Supama (Indonesia), Asep K. Supriatna (Indonesia) and Indah Emilia Wijayanti (Indonesia). Most of the contributed papers were delivered by mathematicians from Asia.

We would like to sincerely thank all plenary and invited speakers who warmly accepted our invitation to come to the Conference and the paper contributors for their overwhelming response to our call for short presentations. Moreover, we are very grateful for the financial assistance and support that we received from Universitas Gadjah Mada, the Faculty of Mathematics and Natural Sciences, the Department of Mathematics, the Southeast Asian Mathematical Society, and UNESCO.

We would like also to extend our appreciation and deepest gratitude to all invited speakers, all participants, and referees for the wonderful cooperation, the great coordination, and the fascinating efforts. Appreciation and special thanks are addressed to our colleagues and staffs who help in editing process. Finally, we acknowledge and express our thanks to all friends, colleagues, and staffs of the Department of Mathematics UGM for their help and support in the preparation during the conference.

The Editors
October, 2012

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MATHEMATICAL ANALYSIS OF FLUID FLOW THROUGH RECTANGULAR MICROCHANNEL WITH SLIP BOUNDARY UNDER CONSTANT PRESSURE GRADIENT

SUHARSONO , Y. H. WU, B. WIWATANAPATAPHE

Abstract. In this paper, we derive analytical solutions for transient flow of Newtonian fluid through rectangular microchannel with Navier slip boundary under constant pressure gradient. The derivation of the solutions is based on Fourier series expansion in space. We then investigate the influence of the slip parameters on the phenomena. The effects of the slip parameter on transient pressure field and velocity as well as flow rate will be presented in this paper.

Keywords and Phrases : Rectangular, Slip, transient flow, pressure gradient

I. INTRODUCTION

Recently, one of the important scientific research focuses worldwide has been on the study of the behaviour of materials at micro and nanoscales. Advances from the research community in this area led to the development of many biological and engineering devices and systems. Most of these devices and systems involve fluid flow through microchannels, referred to as microflows. Some models involve transient or steady flows. Many methods have been used to solve the models. One of the methods has been applied to solve the model of transient flow of Newtonian fluids with slip boundary [1, 2].

The governing field equations for the flow of incompressible Newtonian fluids are the incompressible continuity equation and the Navier–Stokes equations. In addition, a boundary condition has to be imposed on the field equations. A number of evidence of slip flow of a fluid on a solid surface has been reported. More recently, Y H Wu et al studied pressure gradient driven transient flows of incompressible Newtonian liquid in micro-annuals under a Navier slip boundary condition. They use Fourier series in time and Bessel functions in space to find out exact solution [1]. Some steady state and transient slip solutions for the flows through a pipe, a channel and an annulus have been obtained [1, 2, 3]. In this paper, we will derive a new exact solution for the transient flow of Newtonian fluids in rectangular microtubes with a slip boundary condition under a constant pressure gradient.

2. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

We consider the flow of an incompressible Newtonian fluid through a rectangular micro tube with the z-axis being in the axial direction as shown in Figure 1. The differential equations governing the flow include the continuity equation and the Navier–Stokes equations as follows

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$$\frac{\partial U_j}{\partial x_j} = 0 \,, \tag{1}$$

$$\rho\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j \partial x_j} + \rho g_i \,, (i = 1,2,3;\ j = 1,2,3) \tag{2}$$

where p and U_j are respectively the fluid pressure and velocity vector, g_i is the gravitational acceleration , ρ and μ are respectively the fluid density and viscosity and x_i denotes coordinates.

As the flow is axially symmetric, the velocity components in the x and y directions vanish, namely $U_1 = U_x = 0$ and $U_2 = U_y = 0$. Thus the continuity equation (1) becomes

$$\frac{\partial U_3}{\partial x_3} = \frac{\partial U_z}{\partial z} = 0 \,,$$

which gives rise to $U_3 = v = v(x, y, t)$.

As the flow is horizontal, $g_3 = g_z = 0$, and hence Eq. (2) becomes

$$\rho\left(\frac{\partial v}{\partial t}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial z}$$

In this work, we consider the fluid flow driven by the pressure field with a pressure gradient *q(t)* which can be expressed by a Fourier series, namely

$$\frac{\partial p}{\partial z} = q(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right] \tag{3}$$

We use complex number to express it by exponential functions, namely

$$\frac{\partial p}{\partial z} = \mathrm{Re}\left(\sum_{n=0}^{\infty} c_n e^{in\omega t}\right)$$

where

$$c_n = a_n - b_n i \,;\ e^{in\omega t} = \cos(n\omega t) + \sin(n\omega t) \,.$$

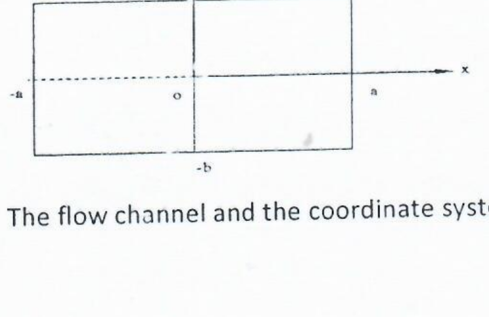

Figure 1. The flow channel and the coordinate system used

As the problem is axially symmetric, we only need to consider a quadrant of the cross-section in the computation.
By applying the Navier slip conditions in the first quadrant of the rectangular cross section, as in the paper by Wu et al [1] and Duan & Murychka [3], for every time t, we have

$$\frac{\partial v}{\partial y}(x,0)=0; \quad 0\le x\le a$$
$$\frac{\partial v}{\partial x}(0,y)=0; \quad 0\le y\le b$$
$$v(x,b)+l\frac{\partial v}{\partial y}(x,b)=0; \quad 0\le x\le a$$
$$v(a,y)+l\frac{\partial v}{\partial x}(a,y)=0; \quad 0\le y\le b \tag{4}$$

3. EXACT SOLUTION FOR THE TRANSIENT VELOCITY FIELD

Consider the unsteady Navier Stokes equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{\rho}{\mu}\frac{\partial u}{\partial t}=\frac{1}{\mu}\frac{\partial p}{\partial z} \tag{5}$$

If u_n is the solution of (5) for $\frac{\partial p}{\partial z}=c_n e^{in\omega t}$, then the complete solution of (5) for

$\frac{\partial p}{\partial z}=\mathrm{Re}\sum_{n=1} c_n e^{in\omega t}=\sum_{n=1} a_n\cos(n\omega t)+b_n\sin(n\omega t)$ is $u=\sum_{n=1}\mathrm{Re}(u_n)$.

Therefore, the equation (5) becomes

$$\frac{\mu}{\rho}\left(\frac{\partial^2 u_n}{\partial x^2}+\frac{\partial^2 u_n}{\partial y^2}\right)-\frac{\partial u_n}{\partial t}=\frac{c_n}{\rho}e^{in\omega t}.$$

To solve this equation, we let,

$$u_n(x,y,t)=f_n(x,y)e^{in\omega t},$$

so that

$$e^{in\omega t}\left(\frac{\partial^2 f_n}{\partial x^2}+\frac{\partial^2 f_n}{\partial y^2}\right)-\frac{in\omega\rho}{\mu}f_n\, e^{in\omega t}=\frac{c_n}{\mu}e^{in\omega t}$$

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which is equivalent to

$$\left(\frac{\partial^2 f_n}{\partial x^2}+\frac{\partial^2 f_n}{\partial y^2}\right)-\frac{in\omega\rho}{\mu}f_n=\frac{c_n}{\mu}. \qquad (6)$$

In case of $\frac{\partial p}{\partial z}=a_0\in\mathcal{R}$, it means $n=0$ so that the equation (6) becomes

$$\frac{\partial^2 f_0}{\partial x^2}+\frac{\partial^2 f_0}{\partial y^2}=\frac{a_0}{\mu}. \qquad (7)$$

Now we let $u_0=f_0(x,y)$ is the solution of the equation (7), then

$$\left(\frac{\partial^2 u_0}{\partial x^2}+\frac{\partial^2 u_0}{\partial y^2}\right)=\frac{a_0}{\mu}. \qquad (8)$$

We write

$$u_0(x,y)=U_0(x,y)+V_0(x,y)+C(x^2+y^2).$$

By substituting it into (8) we have $\frac{\partial^2 U_0}{\partial x^2}+\frac{\partial^2 U_0}{\partial y^2}=0$ and $\frac{\partial^2 V_0}{\partial x^2}+\frac{\partial^2 V_0}{\partial y^2}=0$. This implies $C=\frac{a_0}{4\mu}$, so that

$$u_0(x,y)=U_0(x,y)+V_0(x,y)+\frac{a_0}{4\mu}(x^2+y^2). \qquad (9)$$

Base on boundary condition 4_4, this equation becomes

$$U_0(a,y)+V_0(a,y)+\frac{a_0}{4\mu}(a^2+y^2)+l\left[\frac{\partial U_0}{\partial x}(a,y)+\frac{\partial V_0}{\partial x}(a,y)+\frac{a_0}{4\mu}(2a)\right]=0$$

which yields

$$U_0(a,y)+l\frac{\partial U_0}{\partial x}(a,y)=-\frac{a_0}{4\mu}(a^2+y^2+2al) \text{ and}$$

$$V_0(a,y)+l\frac{\partial V_0}{\partial x}(a,y)=0.$$

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Similarly, boundary condition 4_3 gives

$$U_0(x,b)+V_0(x,b)+\frac{a_0}{4\mu}\left(x^2+b^2\right)+l\left[\frac{\partial U_0}{\partial x}(x,b)+\frac{\partial V_0}{\partial x}(x,b)+\frac{a_0}{4\mu}(2b)\right]=0$$

which yields

$$U_0(x,b)+l\frac{\partial U_0}{\partial x}(x,b)=0 \text{ and}$$

$$V_0(x,b)+l\frac{\partial V_0}{\partial x}(x,b)=-\frac{a_0}{4\mu}\left(x^2+b^2+2bl\right).$$

Hence, boundary condition 4 can be split into

$$\text{BVP1}\begin{cases}\dfrac{\partial^2 U_0}{\partial x^2}+\dfrac{\partial^2 U_0}{\partial y^2}=0, \quad \dfrac{\partial U_0}{\partial x}(0,y)=0, \quad \dfrac{\partial U_0}{\partial y}(x,0)=0\\ U_0(a,y)+l\dfrac{\partial U_0}{\partial x}(a,y)=-\dfrac{a_0}{4\mu}\left(a^2+y^2+2al\right)\\ U_0(x,b)+l\dfrac{\partial U_0}{\partial x}(x,b)=0\end{cases}$$

$$\text{BVP2}\begin{cases}\dfrac{\partial^2 V_0}{\partial x^2}+\dfrac{\partial^2 V_0}{\partial y^2}=0, \quad \dfrac{\partial V_0}{\partial x}(0,y)=0, \quad \dfrac{\partial V_0}{\partial y}(x,0)=0\\ V_0(a,y)+l\dfrac{\partial V_0}{\partial x}(a,y)=0\\ V_0(x,b)+l\dfrac{\partial V_0}{\partial x}(x,b)=-\dfrac{a_0}{4\mu}\left(x^2+b^2+2bl\right).\end{cases}$$

Thus, the problem becoming simple and remaining work for finding f_n is to solve the two BVPs.

We first solve (BVP1) to obtain $U_0(x,y)$ by the separation of variables. For this purpose, let

$$U_0 = X(x)\,Y(y) \tag{10}$$

Then from (10) and the homogeneous boundary conditions in BVP1, we have

$$Y''+\lambda Y=0, \quad Y'(0)=0, \quad Y(b)+l\,Y'(b)=0 \tag{11}$$

$$X''-\lambda X=0, \quad X'(0)=0 \tag{12}$$

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It can be proved that non trivial solutions exist only for $\lambda = \upsilon^2 > 0$. From the ordinary differential equations, we have

$$Y = C_1 \cos\left(\sqrt{\lambda}\, y\right) + C_2 \sin\left(\sqrt{\lambda}\, y\right), \tag{13}$$

$$X = D_1 \cosh\left(\sqrt{\lambda}\, x\right) + D_2 \sinh\left(\sqrt{\lambda}\, x\right) \tag{14}$$

The boundary conditions $(11)_2$ and $(12)_2$ require that $C_2 = D_2 = 0$; while the boundary condition $(11)_3$ implies

$$\cot\left(\sqrt{\lambda}\, b\right) = l\sqrt{\lambda} \quad \text{or} \quad \cot\left(b\upsilon\right) = l\upsilon \tag{15}$$

This equation has infinite number of solutions $\upsilon_1, \upsilon_2, \upsilon_3, \ldots$ which being the value of the intersections of the graphs $y = l\upsilon$ and $y = \cot\left(b\upsilon\right)$. Consequently there exist an infinite number of corresponding eigenvalues and eigenfunctions as follows

$$\lambda_m = \upsilon_m^2, \quad \Phi_m = \cos\left(\sqrt{\lambda_m}\, y\right), \quad m = 1, 2, 3 \cdots \tag{16}$$

Thus, the solution of (BVP1) can be written as

$$U_0 = \sum_{m=1}^{\infty} A_m \cosh\left(\upsilon_m x\right)\cos\left(\upsilon_m y\right) \tag{17}$$

To meet the nonhomogeneous boundary condition BVP1 for U_0, it requires

$$\sum_{m=1}^{\infty}\left[\cosh\left(\upsilon_m a\right) + l\,\upsilon_m \sinh\left(\upsilon_m a\right)\right] A_m \cos\left(\upsilon_m y\right) = -\frac{\alpha_0}{4\mu}\left(a^2 + y^2 + 2al\right). \tag{18}$$

It can be proved that the eigen function $\Phi_m = \cos\left(\upsilon_m y\right), \quad (m = 1, 2, 3 \cdots)$ are orthogonal on $[0.b]$ with

$$\int_0^b \Phi_m \Phi_n \, dy = 0 \quad for \quad n \neq m \quad \text{and}$$

$$M_{m,m} = \int_0^b \Phi_m \Phi_m \, dy = \int_0^b \cos^2(\upsilon_m y)\, dy = \frac{2b\upsilon_m + \sin(2b\upsilon_m)}{4\upsilon_m} \tag{19}$$

Thus, the coefficients of A_m can be determined by

$$A_m = \frac{-a_0}{4\mu M_{mm}\left[\cosh(a\upsilon_m) + l\upsilon_m \sinh(a\upsilon_m)\right]} \int_0^b \left(a^2 + y^2 + 2al\right)\cos(\upsilon_m y)\, dy$$

$$= \frac{-a_0\left[\left(a^2 + 2al\right)\sin(b\upsilon_m) + b^2\sin(b\upsilon_m) + \frac{2}{\upsilon_m}\left(b\cos(b\upsilon_m) - \frac{\sin(b\upsilon_m)}{\upsilon_m}\right)\right]}{\mu\left[2b\upsilon_m + \sin(2b\upsilon_m)\right]\left[\cosh(a\upsilon_m) + l\upsilon_m \sinh(a\upsilon_m)\right]} \tag{20}$$

Similarly, the solution V_0 of the (BVP2) is

$$V_0 = \sum_{m=1}^{\infty} B_{nm} \cosh(\bar{\upsilon}_m y)\cos(\bar{\upsilon}_m x) \tag{21}$$

where $\bar{\upsilon}_m$ is the root of the equation

$$\cot(a\bar{\upsilon}) = l\bar{\upsilon}, \tag{22}$$

and

$$\bar{M}_{mm} = \int_0^a \cos^2(\bar{\upsilon}_m x)\, dx = \frac{2a\bar{\upsilon}_m + \sin(2a\bar{\upsilon}_m)}{4\bar{\upsilon}_m} \tag{23}$$

Similarly, the coefficients of B_m can be determined by

$$B_m = \frac{-a_0}{4\mu \bar{M}_{mm}\left[\cosh\left(b\bar{\upsilon}_m\right)+l\bar{\upsilon}_m \sinh\left(b\bar{\upsilon}_m\right)\right]}\int_0^a \left(x^2+b^2+2bl\right)\cos\left(\bar{\upsilon}_m x\right)dx$$

$$= \frac{-a_0\left[\left(b^2+2bl\right)\sin\left(a\bar{\upsilon}_m\right)+a^2\sin\left(a\bar{\upsilon}_m\right)+\frac{2}{\bar{\upsilon}_m}\left(a\cos\left(a\bar{\upsilon}_m\right)-\frac{\sin\left(a\bar{\upsilon}_m\right)}{\bar{\upsilon}_m}\right)\right]}{\mu\left[2a\bar{\upsilon}_m+\sin\left(2a\bar{\upsilon}_m\right)\right]\left[\cosh\left(b\bar{\upsilon}_m\right)+l\bar{\upsilon}_m\sinh\left(b\bar{\upsilon}_m\right)\right]} \tag{24}$$

Substituting (17) and (21) into (9) yields the solution

$$u_0(x,y,t) = \frac{a_0}{4\mu}\left(x^2+y^2\right)+\sum_{m=1}^{\infty}\left[A_m\cosh\left(\upsilon_m x\right)\cos\left(\upsilon_m y\right) + B_m\cosh\left(\bar{\upsilon}_m y\right)\cos\left(\bar{\upsilon}_m x\right)\right] \tag{25}$$

From the axial velocity solution (25), the flow rate can be determined by

$$Q(t) = 4\int_0^b\int_0^a u(x,y,t)\,dx\,dy = Q_0 + \sum_{n=1}^{\infty} Q_n \tag{26}$$

where Q_0 and Q_n are respectively, the flow rate corresponding to the constant component and the nth harmonic component of the pressure gradient and

$$Q_0 = \frac{a_0}{3\mu}\left[a^3b+ab^3\right]+4\,\mathrm{Re}\sum_{m=1}^{\infty}\left[\frac{A_m}{\upsilon_m^2}\sinh\left(a\upsilon_m\right)\sin\left(b\upsilon_m\right) + \frac{B_m}{\bar{\upsilon}_m^2}\sinh\left(b\bar{\upsilon}_m\right)\sin\left(a\bar{\upsilon}_m\right)\right] \tag{27}$$

We let that

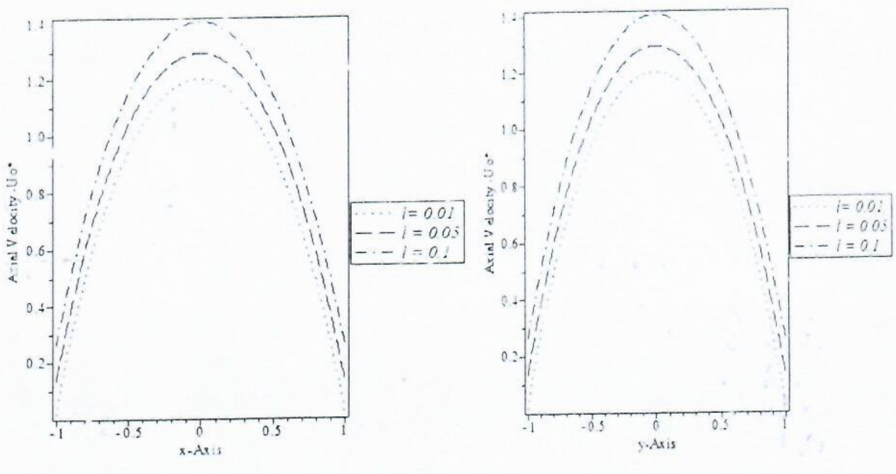
$$x^* = \frac{x}{a};\ y^* = \frac{y}{b},\ t^* = \frac{\omega t}{2\pi},\ \varepsilon = \frac{b}{a}. \tag{28}$$

From (25), (27), and (28), we obtain the following normalized velocity and normalized flow rate

$$u_0^*\left(x^*,y^*\right)=\frac{4\mu}{a_0a^2}u_0=x^{*2}+\left(\varepsilon y^*\right)^2+\frac{4\mu}{a_0a^2}\operatorname{Re}\sum_{m=1}^{\infty}\Big[A_m\cosh\left(a\upsilon_m x^*\right)\cos\left(b\upsilon_m y^*\right)$$
$$+B_m\cosh\left(b\bar{\upsilon}_m y^*\right)\cos\left(a\bar{\upsilon}_m x^*\right)\Big] \tag{29}$$

$$Q_0^*=\frac{3\mu}{a_0\varepsilon a^4}Q_0=1+\varepsilon^2+\frac{12\mu}{a_0\varepsilon a^4}\operatorname{Re}\sum_{m=1}^{\infty}\left[\frac{A_m}{\upsilon_m^2}\sinh\left(a\upsilon_m\right)\sin\left(b\upsilon_m\right)\right.$$
$$\left.+\frac{B_m}{\bar{\upsilon}_m^2}\sinh\left(b\bar{\upsilon}_m\right)\sin\left(a\bar{\upsilon}_m\right)\right] \tag{30}$$

To demonstrate the influence of the slip length in the flow behavior, we analyze the solutions graphically. Figures 2 shows 2D velocity profiles on the cross-section of the channel for various different values of l. Figure 3 shows the influence of the slip length l on the flow rate Q^*.



(a) (b)

Figure 2. 2D graphs showing the axial velocity profiles along the x- axis and y- axis for different l values (a) along the x – axis; (b) along the y - axis.

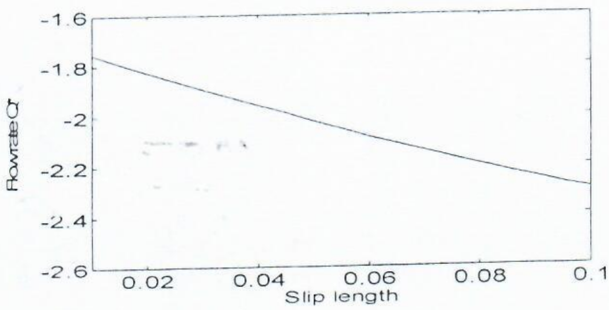


Figure 3. Variation of flow rate with slip length l.

4. CONCLUDING REMARK

In this paper we present an exact solution for transient flow of incompressible Newtonian fluid in rectangular microtubes with a Navier slip condition on the boundary under constant pressure gradient. We also show the velocity profile and variation of flow rate with slip length l in the cross section for the case of $\frac{\partial p}{\partial z} = -2$.

5. Reference

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