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Modeling and Forecasting Time Series Data by EGARCH Model

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Abstract: In analysis of time series data, the class of GARCH Models in many studies has proved very valuable in modeling time series with time varying volatility, especially in financial time series data. The behavior of financial data sometimes are not only have a high volatility and heterogeneous variances but also have an asymmetric effect or leverage effect due to the price down (bad news) and the price increase (good news). One of the models that can cope with the asymmetric effect is Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) Model. The aims of this study is to find the best EGARCH Model for forecasting data share of PT. Tambang Batu Bara Bukit Asam Tbk from January 2009-February 2016. The results shown that the best model are ARIMA (1, 1, 0) Model and EGARCH (1.1) Models. The forecasting results also sound good and within the 95% confidence interval.

Key words: Heteroscedasticity asymmetric effect, ARIMA, EGARCH Model, Tambang, forecasting, sound

INTRODUCTION

The commonly used methods of analysis time series data are Autoregressive (AR) or Moving Average (MA) models or the combination of both of them, i.e., Autoregressive Moving Average (ARMA) model which assume that the variance is Homoscedasticity (homogeneous variance). But for some cases of financial data, the fluctuations are very fast from time to time such that the variances and error change over time (heterogeneous). One of the models that can cope with the problem of heterogeneous variance is Autoregressive Conditional Heteroscedasticity (ARCH) Model which was introduced by Engle (1982). The model was generalized by Bollerslev (1986) to overcome the high order on the ARCH Model and it is well known as Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model (Tsay, 2005). The Model ARCH or GARCH assume that the errors either positive or negative will give the same effect on volatilities. But the fact is that this assumption sometimes is violated because generally, the time series data shows asymmetric phenomenon between the positive and negative error to the volatility (Tsay, 2005). A method of analysis that can be used to deal with the asymmetric effect is Exponential GARCH (EGARCH) Model which was introduced by Nelson (1991). This model will be used to analyze data share of PT Tambang Batu Bara Bukit Asam Tbk in the period from January 2009-February 2016. In this study, the application of EGARCH Model, estimation of its parameters to find the best model and then use the model for forecasting data.

MATERIALS AND METHODS

Time series modeling: Time series is an ordered sequence of observation. It is usually through time, particularly in terms of some equally space time interval (Wei, 2006). Prior to conduct an analysis of time series data, the assumption of stationary should be fulfilled by the data. Stationary is one of the fundamental concepts in time series analysis (Sampson, 2001). A stationary time series is one whose statistical properties such as mean, variance, autocorrelation, etc. are all constant over time. In time series modeling tools that commonly used to identify model from the data are ACF and PACF (Pankratz, 1991; Wei, 2006). A stationary time series data satisfied the properties that the mean and variance are constant. To test the nonstationary, the Augmented Dickey-Fuller test can be used.

Augmented Dickey-Fuller (ADF) test: Some time series data tend to be nonstationary, for example, a price series

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data, the nonstationary due to the fact that there is no fixed level for price. Such a nonstationary series is called unit-root nonstationary time series (Tsay, 2005). A unit root is a feature of some stochastic processes that can cause problems in time series modeling. Let the regression equation of AR (p) is as follows:

$$x_t = \delta + \varphi_1 x_{t-1} + \sum_{i=1}^{p-1} \varphi_i \Delta x_{t-i} + e_t \quad (1)$$

$\Delta x_j = x_j - x_{j-1}$ is the difference sequence of x_t. One of the unit root tests is Augmented Dickey Fuller (ADF) test. The ADF test is conducted through the calculation of the value of τ (tau) statistic as follows:

$$\tau = \frac{\hat{\phi}_1}{SE(\hat{\phi}_1)} \quad (2)$$

The hypothesis is as follows: H_0: $\phi<1$ (x_t is nonstationary) against H_1: $\phi<1$ (x_t is stationary). If τ statistic is $<\tau$ table, then H_0 is not rejected and the data are nonstationary (Tsay, 2005; Maddala and In-Moo Kim, 2004; Ogaki, 1993).

White noise process: The white noise process is used to diagnostic check a model to test the ARIMA Model and Exponential GARCH (EGARCH) Model. A process ϵ_t is called a white noise process if the random variable are uncorrelated and normally distributed with constant mean $E(\epsilon_t) = 0$ and constant variance $Var(\epsilon_t) = \sigma^2$ and $\gamma = Cov(\epsilon_t \epsilon_{t+k}) = 0$ for $k \neq 0$. Thus, a stationary white noise process satisfied. Autocovariance function:

$$\gamma_k = \begin{cases} \sigma^2 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$$

Autocorrelation function:

$$\rho_k = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$$

Partial autocorrelation function:

$$\phi_{kk} = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$$

The white noise process can be check by using ljung box-pierce test with the hypotheses is as follows: H_0: $\rho_1 = \rho_2 = \rho_3 = ,..., = 0$ (There is no autocorrelation) H_1: $\exists \rho_k \neq 0$, $k = 1, 2, \ldots, K$ (there is an autocorrelation). The level of significance $\alpha = 5\%$, the Ljung Box-Pierce test statistic is:

$$Q_k = T\,(T+2) \sum_{k=1}^{K} \frac{\hat{\rho}_k^2}{T-k} \quad (3)$$

Where:

- T = Total observation
- K = Number of lag tested
- ρ_k = The estimation of autocorrelation residuals of k period

The distribution of Q_k is asymptotically chi-squares distribution with df K minus number of parameters. So, H_0 is rejected if $Q_k > x^2_{(\alpha,\, df)}$ table with the df K minus number of parameters (Wei, 2006).

Jarque-bera test: To check the normality of residuals, the Jarque Bera test can be used. The statistic that is introduced by Bera and Jarque (1982) provides a formal assessment of how much the skewness and kurtosis deviate from the normality assumption. The hypothesis to be tested is as follows. H_o: The residuals are normal distribution and H_1: The residuals are non-normal distribution. The test is as follows:

$$JB = \left[\left(\frac{T}{6}\right) S^2 + \left(\frac{T}{24}\right)(K-3)^2\right] \quad (4)$$

Where:

- T = Total number of observation
- S = Skewness
- K = Kurtosis

The JB asymptotically has chi-squares distribution with 2° of freedom. Reject H_0 if JB is $>x^2_{(2)}$.

Volatility: In statistical time series data, the conditional variance given the past is:

$$Var\,(x_t | F_{t-1})$$

where, F_{t-1} = The available set of information at time t-1 is not constant over time and the stochastic process of $\{x_t\}$ is conditionally heteroscedastic. The volatility is defined as:

$$\sigma_t = \left[Var\,(x_t | F_{t-1})\right]^{1/2}$$

change over time (Straumann, 2005). Tsay (2005) states that volatility means the conditional standard deviation. The volatility as a standard deviation has lead a time series modeling approach based on the properties of the variance, namely, variance constant and variance not constant (heteroscedastic). The Autoregressive (AR), Moving-Average (MA) and Autoregressive Moving

Average (ARMA) Models are based on the assumption that the variances are constant while ARCH and GARCH Models are based on the assumption that the variances are not constant.

Autoregressive (AR) and Moving Average (MA) Models: General form of AR (p) Model is as follows:

$$x_t = \delta+\phi_1 x_{t-1}+\phi_2 x_{t-2}+, ..., +\phi_p x_{t-p}+\varepsilon_t \quad (5)$$

Where ϵ_t white noise. The Eq. 5 can be written as:

$$\Phi (B) x_t = \delta+\varepsilon_t$$

Where:

$$\Phi (B) = 1-\phi_1 B-\phi_2 B^2-...-\phi_p B^P \text{ and } B^k x_t = x_{t-k}$$

The AR (p) time series $\{x_t\}$ is stationary if the roots of the polyomial:

$$S^p-\phi_1 S^{p-1}-\phi_2 S^{p-2}, ..., \phi_p = 0 \quad (7)$$

are less than one in absolute value. It has been shown (Montgomery *et al*., 2008) that for stationary AR (p) the mean, covariance and variance are as folows (Montgomery *et al*., 2008):

$$E (x_t) = \mu = \frac{\delta}{1-\phi_1-\phi_2, ..., \phi_p} \quad (8)$$

$$\gamma (k) = Cov (x_t, x_{t-k}) \quad (9)$$

$$= \sum_{i=1}^{p} \varphi_i \gamma (k-i)+\begin{cases} s^2 & \text{if } k=0 \\ 0 & \text{if } k>0 \end{cases} \quad (10)$$

$$\gamma (0) = \sigma^2$$

Moving Average (MA) Model with order q is defined by MA (q) and can be written as follows:

$$x_t = \mu+\varepsilon_t-\theta_1 \varepsilon_{t-1}-\theta_2 \varepsilon_{t-1}-,...,-\theta_3 \varepsilon_{t-3}-, ...,-\theta_q \varepsilon_{t-q} \quad (11)$$
$$\varepsilon_t \sim N (0, \sigma^2)$$

Where:

x_t = Variable at time t
ϵ_t = An error at time t
θ_i = Regression coefficient
I = 1, 2, 3, ..., q
q = The order of MA

Equation 11 can be written by using Backshift (B) operator as follows:

$$\begin{aligned} x_t &= \mu+(1-\theta_1 B-\theta_2 B^2-...-\theta_q B^q) \varepsilon_t \\ &= \mu+(1-\textstyle\sum_{i=1}^{q}\theta_i B^i) \varepsilon_t \\ &= \mu+\Theta (B) \varepsilon_t \end{aligned} \quad (12)$$

where, Θ (B) = $1-\Sigma^q_{i-1}$. Since ϵ_t is white noise, the expected value, variance, autocovariance and autocorrelation are as follows:

$$\begin{aligned} E (X_t) &= E (\mu+\varepsilon_t-\theta_1 \varepsilon_{t-1}-\theta_2 \varepsilon_{t-2}-\theta_3 \varepsilon_{t-3}, ..., \theta_q \varepsilon_{t-q}) \\ &= \sigma^2 (1+\theta_1^2+, ...,+\theta_q^2) \end{aligned} \quad (13)$$

$$\begin{aligned} Var (x_t) &= \gamma_x (0) \\ &= Var (\mu+\varepsilon_t-\theta_1 \varepsilon_{t-1}-\theta_2 \varepsilon_{t-2}-\theta_3 \varepsilon_{t-3}, ..., \theta_q \varepsilon_{t-q}) \\ &= \sigma^2 (1+\theta_1^2+\theta_2^2+...+ \theta_q^2) \end{aligned} \quad (14)$$

And autocovariance at lag k is:

$$\gamma_y (k) = Cov (x_t, x_{t+k}) = \begin{cases} \sigma^2 \left(-\theta_k+\theta_1 \theta_{k+1}+, ..., +\theta_{q-k} \theta_q\right) & K = 1, 2, ..., q \\ 0 & k>q \end{cases} \quad (15)$$

So, the value of autocorrelation at lag k is:

$$\rho_y (k) = \frac{\gamma_y (k)}{\gamma_y (0)} = \begin{cases} \dfrac{(-\theta_k+\theta_1 \theta_{k+1}+, ..., +\theta_{q-k} \theta_q)}{1+\theta_1^2+, ...,+\theta_q^2}, & k = 1, 2, 3, ..., q \\ 0 & k>q \end{cases} \quad (16)$$

The values of ACF can be used to identify the MA model and order cut off after lag q (Montgomery *et al*., 2008).

Autoregressive Integrated Moving Average (ARIMA) Model: Let d be a nonnegative integer then $\{x_t\}$ is said to be an ARIMA process if Y_t: = $(1-B)^d x_t$ is generated from ARMA process. So, $\{x_t\}$ satisfied the equation:

$$\phi^* (B) x_t \equiv \Phi (B) (1-B)^d x_t = T (B) \varepsilon_t, \{\varepsilon_t\} \sim WN (0, \sigma)$$

In a compact notation it can be written as:

$$\Phi (B) \nabla^d_{xt} x_t = T (B) \varepsilon_t \quad (17)$$

Where Φ (B), Θ (B) are polynomial with the degree p and q, respectively and ∇^d $(1-B)^d$ (Brockwell and Davis, 2002; Pankratz, 1983). In selecting a best model, the Akaike Information Criterion (AIC) is used.

Autoregressive Conditional Heteroscedastic (ARCH) Model: The basic idea of the least squares model assumes that the expected value of all of the squares error terms is the same at any given point. This assumption is called homoscedasticity (Engle, 2001). The ARCH/GARCH models are built based on the assumption that the variances are not constant overv time. This assumption is called heteroscedasticity. The ARCH and GARCH Models treat heteroscedasticity as a variance to be modeled (Engle, 2001; Bollerslev,1986). Engle (1982) introduced a Model time varying conditional variance with Autoregressive Conditional Heteroscedasticity (ARCH) model using lagged disturbances. ARCH is a function of autoregression which assume that the variance is not constant over time and also affected by the past data. The idea behind this model is to see the relationship between the random variable and the previous random variable. The basic idea of ARCH Model is the time series data x_t is serially uncorrelated but dependent, the dependence of x_t can be written as a simple quadratic function (Tsay, 2001). The ARCH Model is built as follows, Let $x_1, x_2, ..., x_T$ be the sequence of random data and F_T be the set of random data up to time t, then ARCH Model with degree q with respect to x_T is: $x_T|F_{t-1}\sim N(0, \sigma_t^2)$ where F_{t-1} is the information set available at time t-1. Conditional variance of the residual ϵ_t which is σ_t^2, can be written as:

$$\sigma_t^2 = \omega+\lambda\ \varepsilon_{t-1}^2+\lambda_2\ \varepsilon_{t-2}^2+...+\lambda_q\ \varepsilon_{t-q}^2$$

where the variance residual depend on the-q squares of residual and is called Autoregressive Conditional Heteroscedasticity (ARCH). The ARCH Model can be written as:

$$\begin{aligned} x_t &= \delta+\sum_{i=1}^{p}\phi_i\ x_{t-i}-\sum_{i=1}^{q}\theta_i\ \varepsilon_{t-i}+\varepsilon_t \\ \varepsilon_t &\sim N(0, \sigma^2) \\ \sigma_t^2 &= \omega+\lambda_1\ \varepsilon_{t-1}^2+\lambda_2\ \varepsilon_{t-2}+...+\lambda_q\ \varepsilon_{t-q} \end{aligned} \tag{18}$$

x_t is the equation of conditional mean (Brooks, 2014: Tsay, 2005).

Lagrange Multiplier (LM) test: Engle (1982) stated that the time series data beside has a problem with autocorrelation also has a problem with heteroscedasticity. Weiss (1984) has shown the importance of detecting the present of ARCH effect in time series data. He showed that ignoring the presence of heteroscedasticity not only cause the estimation of parameters to be inefficient but it also could result in an over parameterized ARMA Model. The test that can be used to detect the heteroscedasticity or ARCH effect is ARCH-Lagrange Multiplier (ARCH-LM) (Engle, 1982; Tsay, 2005). The steps are as follows. Define the linear regression as follows $x_t = \mu+\lambda_1\ x_{t1}+\lambda_2\ x_{t2}+, ..., +\lambda_p\ x_{tp}+\epsilon_t$ Squares the residual and regress on the variance t to test the order of q ARCH $\sigma_t^2 = \lambda_0+\lambda_1\ \epsilon_{t-1}^2+\lambda_2\ \epsilon_{t-2}^2+, ..., +\lambda_q\ \epsilon_{t-q}^2$ where is residual. Find the R^2 from this residual. The test Statistic is:

$$LM = TR^2 \tag{19}$$

$$R^2 = \frac{\sum_{i=1}^{n}(\hat{x}_i-\bar{x})^2}{\sum_{i=1}^{n}(\hat{x}_i-\bar{x})^2} \tag{20}$$

Where:

T = Total number of observation
R^2 = R-square
x^2 (q) = Distribution

The null and alternative hypothesis is: $H_0 = \lambda_1 = \lambda_2 = ,...,= \lambda_q = 0$ H_1: $\lambda \neq 0$ or $\lambda_2 \neq 0$ or ... or $\lambda_q \neq 0$ (Brooks, 2014). Although, the Lagrange multiplier is helpful in detecting ARCH effect but it is still difficult in practice to determine the order of the process. One method to determine the order of the model is to fit several competing models and then compare the AIC (Akaike Information Criterion) values for these competing models.

Generalized ARCH (GARCH) Model: GARCH Model (Generalized Autoregresive Conditional Heteroscedastic) Model is a generalized of ARCH. This model is built to avoid the order of ARCH Model which is to high. Terasvita (2009) and Lindner (2009) have pointed out that the GARCH Model is a special case of ARCH (∞). GARCH Model is not only to see the relationship among some residual but also depend on some past residuals. GARCH was introduced by Bollerslev (1986). GARCH Model with order p and q is defined:

$$x_t|F_{t-1}\sim N(0, \sigma_t^2) \tag{21}$$

GARCH Model was developed by Bollerslev (1986). GARCH Model allows the conditional variance depend on the conditional variance of the previous lag. So that, the equation of conditional variance become

$$\sigma_t^2 = \omega+\sum_{i=1}^{p}\lambda_i\ \varepsilon_{t-1}^2+\sum_{i=1}^{q}\beta_j\ \sigma_{t-j}^2 \tag{22}$$

Where the present values of the conditional variance was parameterized and depend on the-q lag from the squares residual and the-p lag of the conditional variance is written as GARCH (p,q). So, GARCH Model if its time varying conditional variance is heteroscedastic with both autoregression and moving average (Peijie, 2009). GARCH Model can be written as:

$$\begin{aligned} x_t &= \delta+\sum_{i=1}^{p}\phi_i\ x_{t-i}-\sum_{i=1}^{q}\theta_i\ \varepsilon_{t-i}+\varepsilon_t \\ \varepsilon_t &\sim N(0, \sigma^2) \\ \sigma_t^2 &= \omega+\sum_{i=1}^{q}\lambda_i\ \varepsilon_{t-i}^2+\sum_{j=1}^{p}\beta_j\ \sigma_{t-j}^2 \end{aligned} \tag{23}$$

Where x_t is the equation of conditional mean (Bollerslev, 1986).

Asymmetric effects: The condition error which is <0 or the asset price decrease is called bad news and the condition of error which is larger than zero or the asset price increase is called good news. Whenever good news and bad news have an impact which is non-symmetrical toward volatility or volatility tends to respond asymmetrically to positive and negative asset return, this condition is called leverage effect (Chen *et al*., 2005; Straumann, 2005). To use EGARCH Model it is assume that the residuals have to have asymmetric effects. In order to examine the presence of asymmetric effects (volatility-return correlation) in the data share of PT. Tambang Batu Bara Bukit Asam Tbk, we first analyze the dynamics of stock returns volatility. To this end, we apply the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model. In the volatility modeling process using GARCH Models, the first moment (mean) and second moment (variance) of the series are estimated simultaneously. A GARCH Model for stock returns assuming that the distribution of the return series for period t, conditional on all previous returns. Engle and Ng (1993) proposed three diagnostic tests for checking the asymmetric effects, the sign bias test, the negative size bias test and the positive size bias test. These tests are used to evaluate whether we need asymmetric model or GARCH Model is fitted for the data. To detect asymmetric volatility in the stock return series, Engle and Ng (1993) proposed a set of tests, known as sign and bias tests. These tests should be used to investigate whether an asymmetric GARCH Model is required for stock return series. In volatility modeling of the stock return series, the diagnostic Engle-Ng tests are used to detect misspecifications related to asymmetric effects based on the residuals of a GARCH Model fitted to the stock return series. The underlying idea is that if the volatility process is correctly specified then the squared standardized residuals should not be predictable on the basis of observed variables. These tests can be individually computed from the following equations. Sign bias t-test:

$$\hat{e}_t^2 = j_0 + j_1 \bar{S}_{t-1}^{-} + e_t$$

Positive size bias t-test:

$$\hat{e}_t^2 = j_0 + j_1 S_{t-1}^{-} e_{t-1} + e_t$$

Negative size bias t-test:

$$\hat{e}_t^2 = j_0 + j_1 \ (1 - S_{t-1}^{-}) e_{t-1} + e_t$$

Joint test for the three effects:

$$\hat{e}_t^2 = j_0 + j_1 S_{t-1}^{-} + j_2 S_{t-1}^{-} e_{t-1} + j_3 \ (1 - S_{t-1}^{-}) \ e_{t-1} + e_t$$

Where ϵ_t error term under the null hypothesis is S_{t-1} is dummy variable which takes the value of 1 if ϵ_{t-1} and 0 otherwise. φ_1 is a parameter of sign bias (effect positive or negative), φ_2 is parameter of size bias (the largest of negative effects), φ_3 is parameter of size bias (the largest positive effects). The null hypothesis of no asymmetric effect is as follows, H_0: $\varphi_1 = \varphi_2 = \varphi_3 = 0$ and is tested by using Lagrange Multiplier Test (LM Test). The LM test is asymptotically follow a x^2 distribution with 3° of freedom (df = 3).

Exponential GARCH (EGARCH) Model: EGARCH Model was introduced by Nelson (1991). EGARCH Model is defined as follows:

$$\text{In}\ (\sigma_t^2) = \omega + \beta\ \text{In}\ (\alpha_{t-1}^2) + \zeta\ \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \lambda \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \quad (24)$$

Where ω, β, ς are λ parameters to be estimated, (σ_t^2) is Exponential GARCH Model, ω is parameter from ARCH Model, β is the effect of positive issues with respect to the current variances. ς is the effect of volatility of the past period which has effect on the current variance and λ is parameter from GARCH Model. In the Eq. 24 conditional variance used natural logarithm. So, the conditional variance is nonnegative (Brooks, 2014).

The estimation of parameter of EGARCH Model: Given $\epsilon_t \sim N\ (0,\ \sigma_t^2)$ and $\epsilon_1, \epsilon_2, ..., \epsilon_3$ are n random sample of Identically Independent Distribution (IID) from f (ϵ_t; θ) with θ = 0, σ_t^2. By using the density function above, we can define the likelihood function:

$$L\ (\theta) = f\ (\varepsilon_1; 0)\ f\ (\varepsilon_2; \theta) ... f\ (\varepsilon_n; \theta) \quad (25)$$

$$L\ (\theta) = \prod_{i=1}^{T} f\ (\varepsilon_t; 0)$$

$$L\ (\theta) = \prod_{i=1}^{T} \frac{1}{\sqrt{2\pi\ \sigma_t^2}}\ e^{\frac{-\varepsilon_t^2}{2\ \sigma_t^2}} \quad (26)$$

$$L\ (\theta) = (2\pi\ \sigma_t^2)^{-\frac{n}{2}}\ \exp \left[-\frac{1}{2\ \sigma_t^2} \sum_{t=1}^{n} \varepsilon_t^2 \right]$$

By applying log to both sides, we have:

$$\text{In}\ L\ (0) = \text{In}\ (2\pi\sigma_t^2)^{-\frac{n}{2}}\ \exp \left[-\frac{1}{2\ \sigma_t^2} \sum_{t=1}^{n} \varepsilon_t^2 - \right.$$
$$= -\frac{n}{2}\ \text{In}\ 2\pi - \frac{n}{2}\ \text{In}\ \sigma_t^2 - \sum_{t=1}^{n} \frac{\varepsilon_t^2}{2\ \sigma_t^2} \quad (27)$$

According to Bollerslev (1986) the iteration method Berndt-Hall-Hall-Hausman (BHHH) can be used to estimate the parameters of EGARCH (p, q) Model. The iteration Berndt-Hall-Hall-Hausman (BHHH) method.

Bernt-Hall-Hall-Hausman (BHHH): This method used exploited algorithm iteration of scoring. Part of the exploited is the P_n from the method of scoring namely:

$$P_n = -\left[E\left(\frac{\partial^2 L}{\partial\theta\,\partial\theta}|\theta_n\right)\right]^{-1} \quad (28)$$

Become:

$$P_n = -\left[E\left(\frac{a^2\,(L_1+L_2+,...,+L_n)}{\partial\theta\,\partial\theta'}|\theta_n\right)\right]^{-1}$$

$$= -\left[E\left(\frac{\partial^2 \sum_{t=1}^{N} L_t}{\partial\theta\,\partial\theta'}|\,\theta_n\right)\right]^{-1} = -\left[E\left(\sum_{t=1}^{N}\frac{\partial^2 L_t}{\partial\theta\,\partial\theta'}|\,\theta_n\right)\right]^{-1}$$

$$= -\left[\sum_{t=1}^{N} E\left(\frac{\partial^2 L_t}{\partial\theta\,\partial\theta'}|\theta_n\right)\right]^{-1} = -\left[NE\left(\frac{\partial^2 L_t}{\partial\theta\,\partial\theta'}|\theta_n\right)\right]^{-1}$$

$$= \left[-N\frac{1}{N}\left(\sum_{t=1}^{N}\frac{\partial^2 L_t}{\partial\theta\,\partial\theta'}|\theta_n\right)\right]^{-1}$$

Finally, we have:

$$P_n = \left[-\left(\sum_{t=1}^{N}\frac{\partial^2 L_t}{\partial\theta\,\partial\theta'}|\theta_n\right)\right]^{-1} = \left[\sigma-\left(\sum_{t=1}^{N}\frac{\partial^2 L_t}{\partial\theta\,\partial\theta'}|\,\theta_n\right)\right]^{-1} \quad (29)$$

The general form of BHHH iteration by using algorithm iteration is as follows:

$$\theta_{n+1} = \theta_n + \left[-\left(\sum_{t=1}^{N}\frac{\partial L_t}{\partial\theta}\frac{\partial L_t}{\partial\theta}|\theta_n\right)\right]^{-1}\left[\frac{\partial L_t}{\partial\theta}|\theta_n\right] \quad (30)$$

RESULTS AND DISCUSSION

Identification: Before we analyze data, first of all we will check the stationary data. The data are stationary if there is no drastic change in the data. Test for stationary data are conducted by looking at the plot of the data weekly share price of PT. Tambang Batu Bara Bukit Asam in the periods of January 2009-February 2016 and given in Fig. 1 and the result of Augmented Dickey Fuller test given in Table 1 from the data. Figure 1 shows that the movement of the graph of share price data PT Tambang Batu Bara Bukit Asam Tbk is not stationary. The trend shows that the data increase and decrease sharply such that the mean of the data are not stationary. Test Augmented Dickey Fuller (ADF), the null and alternative hypotheses:

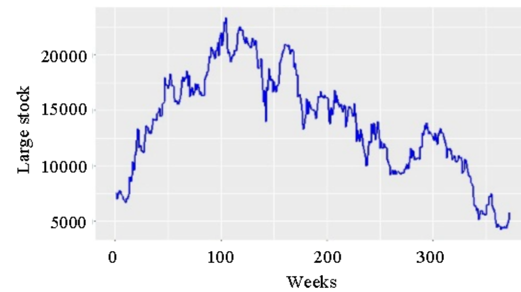

Fig. 1: Plot data share price of PTBA in the period January 2009-February 2016

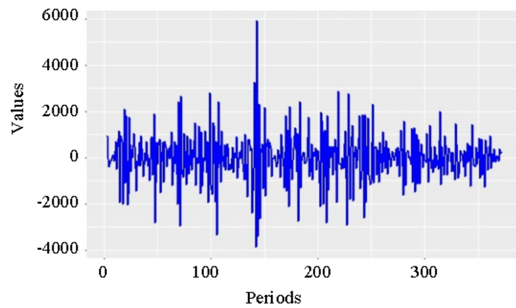

Fig. 2: The graph share price of PTBA in the period January 2009-February 2016 after differencing

Table 1: The output of ADF test for the data share price of PTBA Augmented Dickey-Fuller test

Variables	Values
Test results parameter:	
Lag order	0
Statistic:	
Dickey-Fuller	-0.5444
p-value	0.4428

H_0: $\phi = 0$ (Thereis a unit root or data nonstationary)

H_0: $\phi<0$ (no unit root or data stationary) Reject H_0 if p-value <0.05

From Table 1 it found that the p-value (0.4428)>0.05), so that, there is not enough evidence to reject Ho. This means that the data share price of PTBA has a unit root which implies that the data are nonstationary. So, we need to conduct a difference or transform the data to make them stationary. The method of transformation that we will use is Difference Stationary Processes (DSP). By using this DSP, the data are transformed by differencing the-k lag by th-(k-1) lag. We got the graph as follows. The result of ADF test after differencing is given below. Table 2 shows that the p-value (0.01<0.05) and we reject Ho. This means that the share price data of PTBA has no unit root which implies that the data are stationary (Fig. 2).

ARIMA Model estimation: After the transformation and the data are stationary then we can identify the order and

Table 2: The result of ADF test after differencing augmented Dickey-Fuller test

Variables	Values
Test results parameter:	
Lag Order	0
Statistic:	
Dickey-Fuller	-22.3248
p-value	0.01

Table 3: Result of model estimation and the best model share price of PTBA based on the value of AIC

Variables	Values
ARIMA (2,1,2) with drift	5952.546
ARIMA (0,1,0) with drift	5954.504
ARIMA (1,1,0) with drift	5948.575
ARIMA (0,1,1) with drift	5949.222
ARIMA (0,1,0)	5952.519
ARIMA (2,1,0) with drift	5949.941
ARIMA (1,1,1) with drift	5949.677
ARIMA (2,1,1) with drift	5951.604
ARIMA (1,1,0)	5946.587
ARIMA (2,1,0)	5947.955
ARIMA (1,1,1)	5947.689
ARIMA (2,1,1)	5949.619

Best model: ARIMA (1,1,0)

Table 4: The estimation of parameter ARIMA (1.1,0) Model

Coefficient	Estimation	t-value	p-value
AR (1)	-0.1495	-2.9086	0.0038

estimate ARIMA Model. Below are the results of the best model estimation by using software R 3.2.3 ,Table 3 show that the best model based on the smallest value of AIC is ARIMA (1, 1, 0) where the AIC value is 5946.587. The mean model presented in the following Table 4. The mean Model can be written as follows:

$$x_t = -0.149\, x_{t-1} + \varepsilon_t$$

Where:

x_{t-k} = Differencing data share price of PTBA at the-(t-k)
ϵ_t = the-t residual
k = 0, 1, 2, ..., n

The Evaluation ARIMA Model: At the stage of model evaluation, we will see whether the residuals of the ARIMA Model are white noise and normally distributed.

White noise test: To see whether the residuals are white noise, we will conduct a residual test by using Ljung-Box test. The results of the Ljung-Box test are given below. Figure 3, shows that the graph of p-value for Ljung-Box statistic are above 0.05 this means that the residuals has no correlation and we can say that the residuals of ARIMA (1,1,0) Model are white noise.

Normality test: Figure 4 shows that the distribution of the residuals is not on the straight line, so the residual of the ARIMA (1, 1, 0) Model not normally distributed. Besides using Normal QQ Plot, we can use normality test, Jarque-Bera test as given in Eq. 4 and the result of the test is given in Table 5.

Table 5: The results of Jacque-bera test for ARIMA (1,1,0)

Variables	Values
Jarque-bera normality test; test results	
Statistic X-squared	47.5729
p-value	4.674e-11

Table 6: Test ARCH lagrange multiplier for ARIMA (1, 1, 0)

Variables	Values
Chi-squared	35.729
df	12
p-value	0.0003581

ARCH LM-test (Null hypothesis: no ARCH effects) data: diff-t

Table 7: The estimation results GARCH (1,1) model

Parameters	Coefficient	SE	p-value
δ	-18.398891	32.81.000	0.574955
ϕ_1	-0.119539	0.053655	0.025886
a_0	562.632783	6987.3.0000	0.935822
λ_1	0.031798	0.021548	0.140019
β_1	0.966044	0.030264	0.000000

Since, the p-value ($4.674e^{-11}$ or $0.0000780637<0.05$), H_0 is rejected, so that, the residuals not normally distributed.

Lagrange Multiplier (LM) test: Data time series besides have a problems with autocorrelation, also has problem with heteroscedasticity. A test that can detect of ARCH effects is the ARCH Lagrange Multiplier test, the null and alternative hypothesis is as follows:

H_0: $\lambda_0 = \lambda_1 = ,..., = \lambda_p = 0$ (No ARCH effects)
H_1: $\lambda_0 \neq \lambda_1 \neq ,..., \neq \lambda_p \neq 0$ (has ARCH effects)

Reject H_0 if the p-value<0.05. Table 6 shows that the p-value (0.0003581<0.05), so, we can conclude that the model ARIMA (1, 1, 0) has ARCH effects. Thus, the modeling by using model ARCH/GARCH is recommended.

Model GARCH: A model that we chose from the estimation of mean function and variance function simultaneously with the model GARCH is AR(1)- GARCH (1,1). The estimation results can be seen in the Table 1. Based on the results in Table 7, GARCH (1,1) Model can be written as follows:

$$x_t = -18.398891 - 0.119539\, x_{t-1} + \varepsilon_t$$

And:

$$\sigma_t^2 = 562.632783 + 0.031798\, x_{t-1} + 0.966044\, \sigma_{t-1}^2$$

where x_t is equation of conditional mean.

The test for asymmetric effects: The condition where an error is <0 or the asset price go down is called bad news and the condition of the error is greater than zero or the asset price is go up is called good news. When a good news and bad news give asymmetric effects to volatilities, this situation is called leverage effect. Below is the results

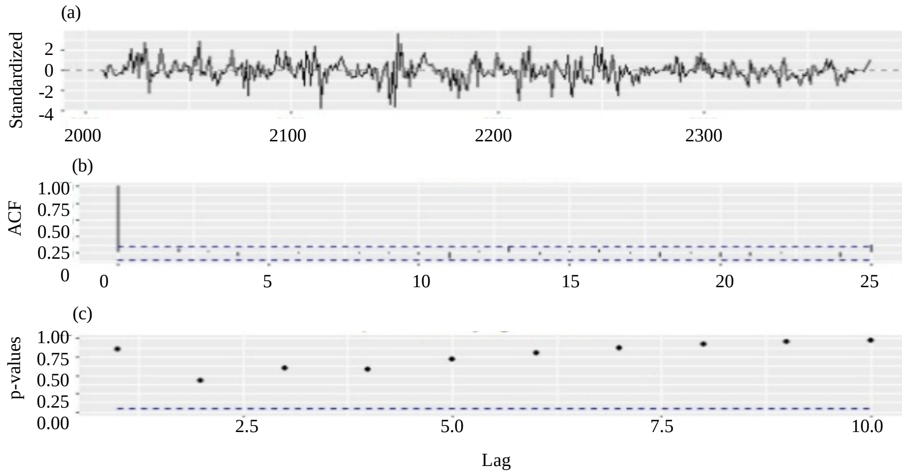

Fig. 3: The results of Ljung-Box test for the Residuals ARIMA (1, 1, 0) Model; a) Standardized residuals; b) ACF of rasidualsa and c) p-values for Ljung-box statistic

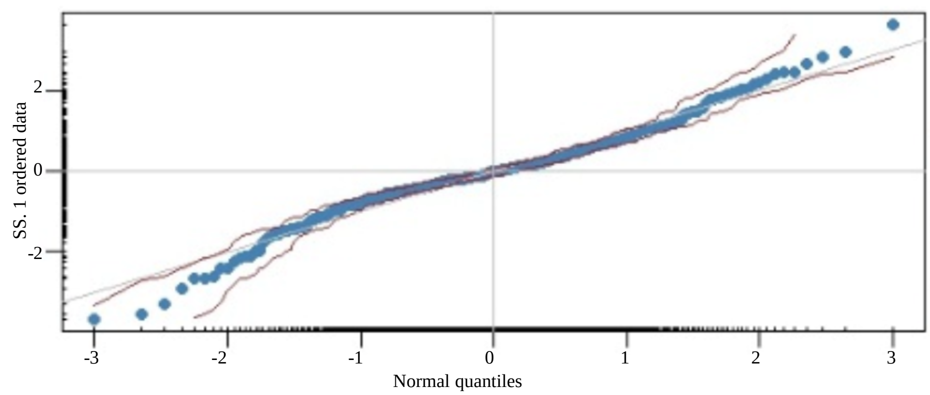

Fig. 4: Normal quantil-quantil plot with confidence interval 95% for ARIMA (1, 1, 0) Model

of the value of sign bias test data weekly share price PTBA. The values of sign bias test and negative sign bias test are <0.05 (α) this shows that the test is significant at α = 5% , while the values of joint effect is not significant at α = 5% but significant at α = 10% or 0,1. From the above results we can conclude that the data weekly share price of PTBA have asymmetric effects. To overcome the asymmetric effects we can use EGARCH Model. The asymmetric of data can be seen from the plot of news impact curve below (Table 8 and 9).

EGARCH Model: To estimate the parameters of the EGARCH Model we use maximum likelihood. Below are the results of the estimation of parameters EGARCH (1,1) Model.

From Table 10, the mean and variance models can be written as follows:

$$x_t = -11.97936-0.116093\ x_{t-1}+\epsilon_t$$

And:

Table 8: The results of sign bias test

Variables	Values
t-value	prob sig
Sign bias	2.624 0.009065***
Negative sign bias	2.017 0.044443**
Positive sign bias	1.208 0.227892
Joint effect	7.563 0.055973*

Table 9: Adjusted pearson goodness-of-fit test

Groups	Statistic	p-values	(g-1)
1	20	45.77	0.000535
2	30	55.06	0.002435
3	40	64.04	0.006961
4	50	72.80	0.015272

Table 10: The results of mean model and variance model

Parameter	Coefficient	SE	p-values
δ	-11.979360	11.377870	0.292403
ϕ_1	-0.116093	0.052343	0.026559
ω	2.521682	1.192470	0.034458
λ_1	0.007322	0.050178	0.883990
β_1	0.809723	0.090358	0.000000
ς_1	0.237119	0.076937	0.002056

Table 11: Forecasting data weekly share price of PTBA

					95% confidence Interval of forecasting	
Period	Date	Real Data	Forecast	Variance	Lower bound	Upper bound
374	07-03-2016	6275	5705.336	659.3053	4413.1	6997.574
375	15-03-2016	6575	5702.946	676.4968	3084.77	8321.118
376	21-03-2016	6500	5689.841	690.7427	1717.81	9661.869
377	28-03-2016	6200	5677.981	702.4955	329.062	11026.9

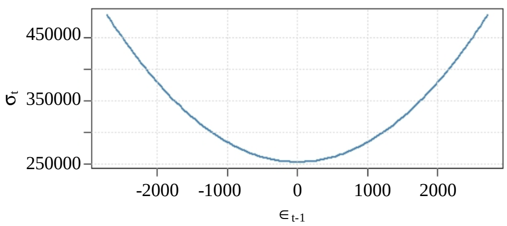

Fig. 5: News impact curve data PTBA

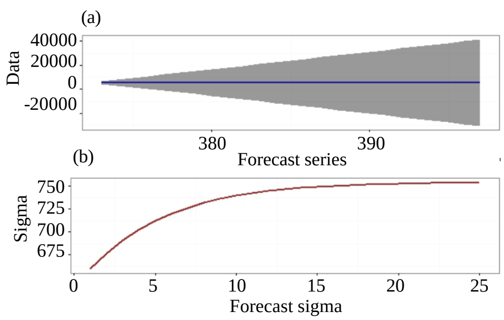

Fig. 6: a, b) The graph of forecasting data weekly share price of PTBA

$$\ln(\sigma_t^2) = 2.521682+0.809723\ \ln(\sigma_{t-1}^2)+$$

$$0.237119\ \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}+0.007322\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}}-\sqrt{\frac{2}{\pi}}\right]$$

Fore casting: Below are the results of forecasting data share price of PTBA for 4 periods. The graph of the forecasting is given Fig. 5.

Table 11 and Fig. 6 show the results of forecasting data weekly share price of PTBA for 4 periods. From Table 10 the values of forecasting are very close to the real values. All the values of forecasting are within the 95% confidence interval. Thus, the EGARCH (1,1) Model is a good model for forecasting the weekly data share price of PTBA for some periods in the future.

CONCLUSION

Based on the analysis data, the best model to analyze data weekly share price of PTBA is Exponential GARCH (EGARCH (1,1)). The application of EGARCH Model for forecasting data weekly share price of PTBA, the best model conditional mean is ARIMA (1,1,0) and the best model conditional variance is EGARCH (1,1). Model conditional mean, ARIMA (1,1,0)

$$x_t = -11.97936-0.116093\ x_{t-1}+\epsilon_t$$

Model conditional variance: EGARCH (1,1)

$$\ln(\sigma_t^2) = 2.521682+0.809723\ \ln(\sigma_{t-1}^2)+$$

$$0.237119\ \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}+0.007322\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}}-\sqrt{\frac{2}{\pi}}\right]$$

The forecasting values for four periods are found very close to the real data and all the forecasting values are within the 95% confidence interval. Thus, this model is very appropriate for the data weekly share price of PTBA.

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