The solving of Calculus problem based on Polya's steps: An investigation on pre-service teachers with low self-efficacy

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Abstract. In some researches, self-efficacy was found to relate to students' mathematics achievement. This present research was aimed to describe the ability of pre-service teachers with low self-efficacy in solving the mathematics problems based on Polya's steps, that are understanding the problem, devising a plan, carrying out the plan, and looking back. This descriptive qualitative research was done on first-year of pre-service teacher in Mathematics Education, Faculty of Teacher Training and Education, University of Lampung. The subjects were 26 students with low self-efficacy. The problem-solving ability was measured by Calculus Problem Solving Test and interview guidelines as the triangulation. This research results showed that pre-service teachers with low self-efficacy were not able to understand the problem. It implied students did mistakes in devising and carrying out the plan, and were not able to look back. The conclusion of this research was that pre-service teachers with low self-efficacy were not able to solve the mathematics problems based on Polya's steps. The recommendation of this research was how to understand the problem should get full attention in the learning process to increase the ability of pre-service teacher in solving mathematics problems.

1. Introduction

Calculus is one of the compulsory course for all students in department of mathematics education. This course is one of the basic course for the first-year students. In this course, all students learn about real functions, the derivative of certain function, and the using of derivative to solve the problem, including make a sketch of the graph of function. To reach the good achievement in this course, all students should have good ability in solving the problems. Problem solving ability will be very useful to solve the non-routine problems. Problem solving is an approach to find the best solution of a certain problems [1].

Problem solving is a mental process that requires to think critically and creatively in finding alternative ideas and specific steps to overcome any obstacles [2]. It is a hallmark of mathematics and media for developing mathematical understanding [3]. Related to mathematics, it is the basic of various activities [4], as well as cognitive activities involving the processes and strategies [5]. The implementation of problem solving is a systematic process and requires critical and creative thinking [6]. Without the strategy to implement the plan of problem solving, the problems would not be solved. Thus, the implementation of problem solving is a very important activities in solving the problem.

Many experts have explained the process in solving the problems, including mathematics. One of them is Polya. Polya suggests that there are four steps in solving the problem [7]. These steps are (1) understanding the problem. The problem solver should understand about all information or conditions that are known in the problems. Is it possible to satisfy the condition, whether the condition is sufficient to show any other information that is unknown, or may be insufficient, or may be excessive or even contradictory; (2) devising a plan of the problem solving. In this step, the problem solver should find the relation between the known information and the question. Sometimes, the problem solvers need additional information that must be determined if the known information is not directly linked to the question. The end of this step is the problem solvers should find a plan to get the best solution of the problem; (3) carrying out the plan. The problem solver implement a plan based on what has been planned in the second stage. The problem solver should carry out the plan of the problem solving and check each steps in solving the problem. In this step, the problem solver should able to see clearly that the step is correct or not. The problem solver also should able to prove that all steps is correct or not; (4) looking back. The final step in solving the problems is re-examining the solutions that have been obtained by all steps that was done in solving the problem. The problem solver should check the result or the argument obtained. The problem solver should able to determine that the result or the method can be used for some other problem or find out other method that possible to use in solving the problems.

The Polya's steps guide the problem solver to find the best solution of the problems. To solve the non-routine problem in Calculus, students also can use the Polya's steps. By using these steps, students are expected get the maximum achievement. Not only these steps, many factors that can be influenced student's mathematical achievement. One of them is student's self-efficacy.

Many studies have concluded that self-efficacy affects mathematical achievement. Self-efficacy is a belief in the ability possessed. Student's judgments about their capabilities to organize and execute courses of action required to attain designated types of performance [8]. This belief will be affects the process in solving the problems. In the real life, every students has a different level of self-efficacy. This difference is influenced by four factors, that are enactive attainment and performance accomplishment, vicarious experience, verbal persuasion, and physiological state and emotional arousal [8]. Because of this factors, there are student who has high level of self-efficacy, otherwise there are also student who has low level of their belief to the capabilities possessed. The difference in self-efficacy lies in three aspects, namely magnitude, strength, and generality [8]. By having the higher belief of the abilities, it is possible for to be more creative in finding the solutions of the problem. This study aims to describe the ability of students with low self-efficacy in solving the Calculus problem based on Polya's steps.

2. Research Method

This is qualitative descriptive research that describes the problem solving ability based on Polya's steps of pre-service teacher with low self-efficacy. This study was conducted at the first-year of pre-service teacher in Department of Mathematics Education, Faculty of Teacher Training and Education, University of Lampung. The subjects were 26 students (13 male and 13 female) with low self-efficacy. The selecting subjects was done by setting criteria of subject that is, they have a strong tendency in the low self-efficacy and able to express opinions, both written and oral clearly viewed by gender. One of the characteristics of qualitative research is the nature of the natural research setting, which is the source of data sought and collected directly by the researcher, not through the questionnaire [9]. In this study, the data are collected directly by the researcher, so the main instrument of this study is the researcher himself. The researcher was assisted by the Calculus problem solving test and interview guidelines.

The subjects are given the opportunity to work on a written test of Calculus problem-solving ability. The test that are given as follows:

Please, sketch the graph of $f(x) = \frac{1+x^2}{1-x^2}$.

The testing of students was carried out separately. Each student was given a maximum of 60 minutes. Separate work was intended for students to be more concentrated in working on the problem and if there are questions about the given problem, researchers can explain it directly. Next, the researchers analyzed the results of the written tests in the answer sheet. Based on the results of this analysis, we obtain the writing data.

After a few days, the researchers gave the same Calculus problem solving test to students by using the interview. The list questions of interview guidelines as follows:

Question 1: In your opinion, what is known in the matter? (*understanding the problem*)
Question 2: What does the question ask? (*understanding the problem*)
Question 3: In your opinion, is the information sufficient to solve the problem? (*understanding the problem)*
Question 4: How can you solve the problem? (*devising the plan)*
Question 5: In your opinion, is there any other way that can be used for solve the problem? (*looking back)*

The interviews are made to the students separately. Problem given is a matter that previously been done by students. Based on the results of this analysis, it obtains the orally data. Furthermore, triangulation method is done that is comparing the data in writing and orally. The valid data is then used to determine the Calculus problem-solving ability based on Polya's steps. The process of analysis through the data reduction, presentation of data, conclusions and verification. To obtain valid data, in qualitative research can be done through various ways. In this research, the way is done by triangulation method [10], that is comparing result of data obtained through test and interview. The same triangulation data is valid data. While the different data will be reduced or made other findings in the study.

3. Result and Discussion

3.1. Result

Based on the results of analysis, it obtained data as follows.

Tabel 1. Student's Problem Solving Ability

Polya's Steps	**Subjects**	
	Male	**Female**
Understanding the problem	Not able	Not able
Devising a plan	Did mistake	Did mistake
Carrying out the plan	Did mistake	Did mistake
Looking back	Not able	Not able

3.1.1. Male

Understanding The Problems

The student has not been able to understand the information that is known and asked in the matter. Based on the results of the interview, he has not been able to determine sufficient and conditions terms that need to be contained in the problem and have not been able to determine whether the sufficient requirements are able to answer the questions asked. Thus, he has not been able to understand the problem. This is supported by interviews between researcher (R) and student (S) as follows.

R : In your opinion, what is known in the matter?
S : A function.
R : What function?
S : Quadratic function, Sir.
R : Which is known only quadratic function?

S : Yes, Sir.
R : What does the question ask?
S : The graph sketch of function.
R : In your opinion, is the information sufficient to solve the problem?
S : Yes, Sir.

Devising The Plan
The student failed to develop a settlement plan in accordance with the known and asked beforehand. He has not been able to determine the relationship between sufficient terms and conditions necessary. In addition, he has not been able to use the information available to plan a solution. He has not been able to plan the problem solving properly and correctly. This is supported by interviews between researcher (R) and student (S) as follows.
R : How can you solve the problem?
S : I try to determine the domain of function, symmetry, intersection points with axis coordinate, critical points, increase or decrease, concave up, and asymptotes.
R : Is it enough to solve the problem?
S : Yes, I think.

The student did mistakes in devising the plan to solve the problem. He should also determine the extreme point and down concave of the function but he did not it.

Carrying out The Plan
He did mistakes in carrying out the plan. Because problem-solving planning is not yet mature, the student has also been unable to use the correct troubleshooting steps and solve the problem appropriately. This is supported by students worksheet as follows.

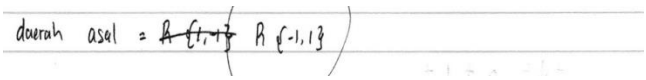

Figure 1. Male student in finding the domain of function

The translate:
The domain of fuction = $R\ \{-1, 1\}$ (*unmeaning answer*)

Figure 2. Male student in finding the symmetry of function

The translate:
Because $f(x) = f(-x)$, $f(x)$ is even function
(*he has not able to determine the symmetrical of function*).

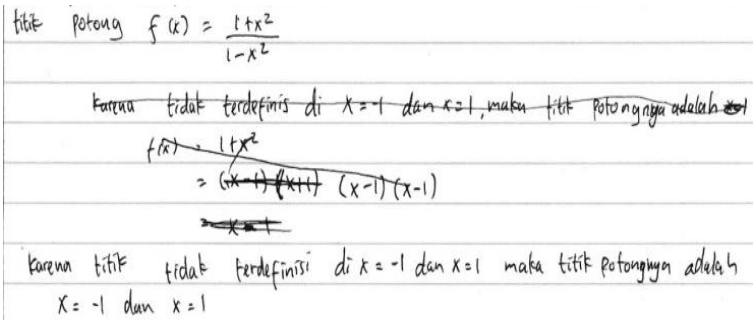

Figure 3. Male student in finding the intersection points of axis coordinate

The translate:
Because the point is undefined at $x = -1$ and $x = 1$, the intersection point are $x = -1$ and $x = 1$ (*unmeaning answer*)

Figure 4. Male student in finding the critical points

The translate:
The critical points of f are $x = -1, x = 1, x = 0$ (*unmeaning answer*).

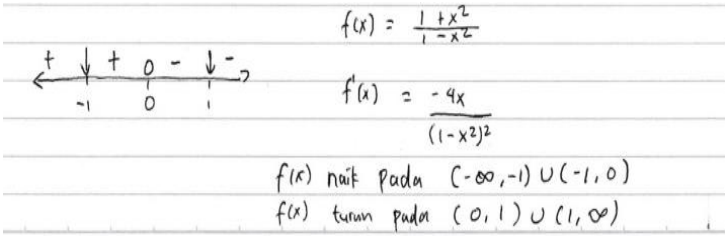

Figure 5. Male student in determining the increase or decrease interval

The translate:
The $f(x)$ increases on $(-\infty,-1) \cup (-1,0)$.
The $f(x)$ decreases on $(0,1) \cup (1,\infty)$.

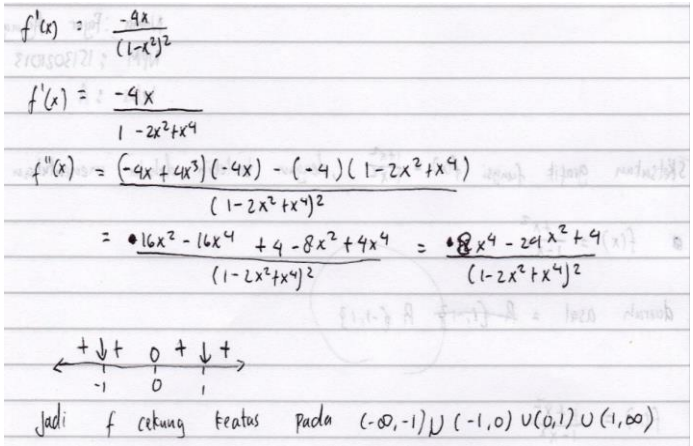

Figure 6. Male student in determining the concave up interval

The translate:
Thus, f concave up on $(-\infty,-1) \cup (-1,0) \cup (0,1) \cup (1,\infty)$

Figure 7. Male student in determining the asymptote

The translate:
The vertical asymptote of f are $x = -1$ and $x = 1$

Finally, the subject has not able to sketch the graph of function. It because he did mistakes in devising the plan.

Looking Back
The student has not been able to re-examine the answer. He has not been able to determine the linkage between the methods or problem solving used to apply to other problems. The following is presented on the interview of the researcher (R) and student (S).
R : In your opinion, is there any other way that can be used for solve problem?
S : I do not have any idea.

3.1.2. Female
Understanding The Problems
The student has not been able to understand the information that is known and asked in the matter. She has not been able to determine sufficient and conditions terms that need to be contained in the problem and has not been able to determine whether the sufficient requirements are able to answer the questions asked. Thus, the student has not been able to understand the problem. This is supported from the results of interviews between researchers (R) with student (S) as follows.
R : In your opinion, what is known in the matter?
S : Function.
R : That's it? Any other?
S : That's it, Sir.
R : What does the question ask?
S : The sketch of graph.
R : Do you think that information is enough to solve the problem?
S : Enough, Sir.

Devising The Plan
The student has not been able to determine the linkage between sufficient terms and necessary conditions. In addition, she has not been able to use the information available to plan a solution. Thus, she has not been able to plan the problem solving well and correctly. This is supported by interviews between researcher (R) and student (S) as follows.
R : How can you solve the problem?
S : I need to find the domain of function, symmetry, intersection points with axis coordinate, and critical points.
R : Is it enough to solve the problem?
S : Yes, Sir.

The student did mistakes in devising the plan to solve the problem. He should devise to determine the increase or decrease, concave up or down, extreme points, asymptotes of the function. But, she did not it.

Carrying Out the Plan
The student failed in carrying out the plan. Because problem-solving planning is immature, she has not been able to use the correct troubleshooting steps and solve the problem correctly. This is supported by students worksheet as follows.

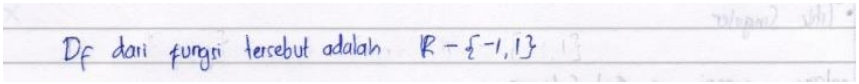

Figure 1. Female student in finding the domain of function

Translate:
The D_f of the function is $R - \{-1, 1\}$

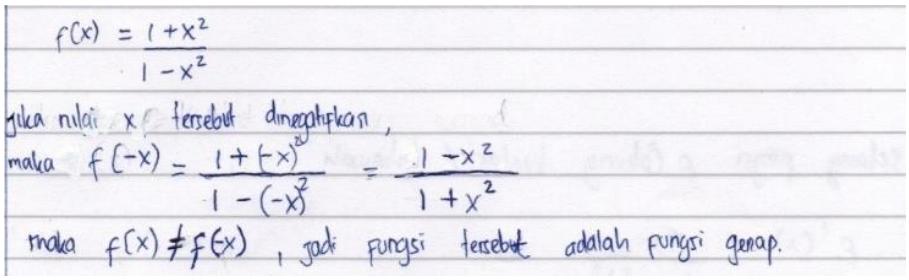

Figure 2. Female student in finding the symmetry of function

Translate:
Because $f(x) \neq f(-x)$, it is even function
(*she has not able to determine the symmetrical of function*).

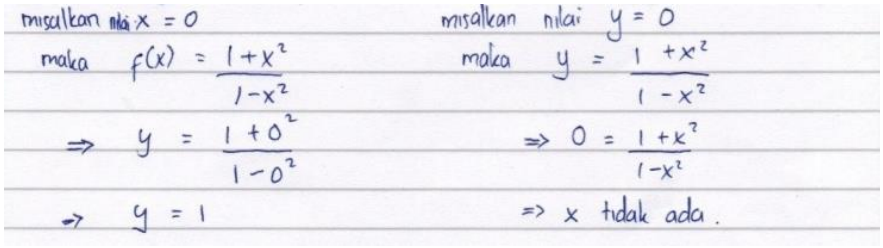

Figure 3. Female student in finding the intersection points of axis coordinate

Translate:
Suppose the value of $= 0$, then $y = 1$.
Suppose the value of $= 0$, then there is no x.

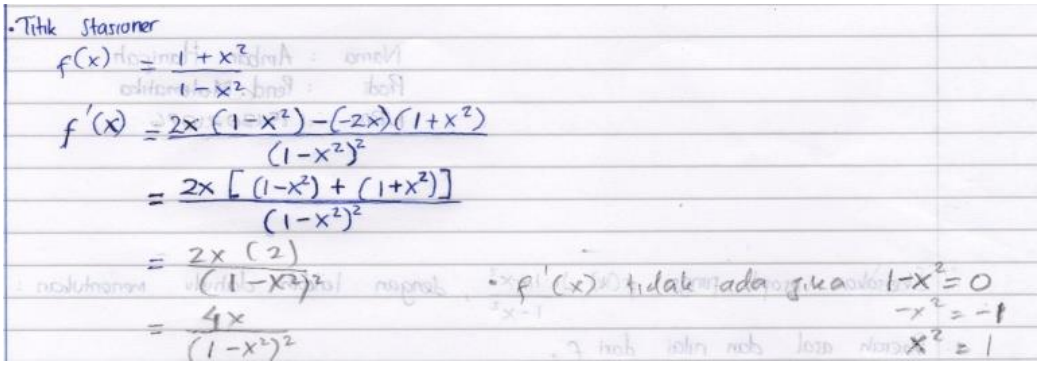

Figure 4. Female student in finding the critical points

Translate:
The stationary point
There is no $f'(x)$ if $x^2 = 1$.

Finally, the subject has not able to sketch the graph. It because he did mistakes in devising the plan.

Looking Back
The student has not been able to re-examine the answer. She has not been able to determine the linkage between the methods or problem solving used to apply to other problems. The following is presented on the interview of the researcher (R) and student (S).
R : In your opinion, is there any other way that can be used for solve the problem?
S : I do not know, Sir.

3.2 Discussion
An important first step in solving mathematics problems is understanding the problem [11]. Students must clearly know what the problems means, what are students looking for the answer. Students should get the key point and the context of problem. Based on the result of this study, in understanding the problem, the students have not been able to determine sufficient terms and conditions necessary. Both students have not been able to determine the adequacy of requirements to be able to solve a problem. In this step, both students have difficulty in understanding the problem. Low self-efficacy affects students to organize a problem. The students will have low ability to organize the problem. It begins with the ability to understand the problem.

In devising problem-solving plans, students should clearly know the relationship between the point of problem, select the suitable approach and devise the plan for solving the problems, which is the most major task in solving the problem [11]. The result of this study showed that students have not been able to undertake a careful, orderly, and detailed planning. Students have not been able to associate the known information with the question. Students are unable to use the information obtained to plan a problem-solving. The students with low self-efficacy also claimed that have low ability to remember the concepts. It is also can be happened because the students don't have any good understanding of the concepts.

In carrying out the plan, students should able to follow step 1 and 2, and practically calculate by themselves, and also find the solution of the problems. This study found that the students have not shown that the problem solved well. Failure to plan results in the inability of students to solve problem. The inability of students to understand the information obtained in the matter makes students unable to use the information to solve the problem. Both students have difficulty in solving the problem. This difficulty starts from the previous step. Student failure in planning a solution results in his inability to resolve the problem.

Looking back is step necessarily emphasized in solving the problems and also the important step compared with the obtained results [12]. These results clarified that the majority students did not make some review of their solutions they had made [13], while for non-routine problems, students had some weaknesses in solving them [14]. While in general, problem solving using the Polya's steps could be made well [15]. In the last step of re-examining the answers that have been obtained, the students have not been able to work in different ways. In addition, the students are also unable to associate between the way of solution used to apply to other solving problems of the same model of completion. Both students showed that students have not been able to re-examine the answers that have been obtained. Errors in calculations and algorithms are not re-examined by students. The tendency of students only to accept the organization that has been given but unable to reorganize the information that has been given.

Based on these results, students with low efficacy are not able to solve Calculus problems based on Polya's steps. Students with low efficacy may even fail to understand the problem, devise a plan,

carry out the plan, dan look back. The solving of mathematics problems would nevertheless inform social cognitive theory and its claims about self-efficacy in general. Student's judgement about their capability to solve mathematics problems were more predictive of their ability to solve those problem. Self-efficacy mediated the effect of mathematics problem solving performance[16].

4. Conclusion

Based on the analysis and discussion, the ability of pre-service teacher with low self-efficacy to solve mathematical problems based on Polya's steps describes as follows:

a. In understanding the problem, students has not been able to determine the sufficient and necessary condition and can not yet be able to determine the adequacy of the requirement to solve the problem.
b. In devising a plan, students did mistake to determine the linkage between sufficient terms and necessary requirements, and have not been able to determine other unknown information on the matter to plan the settlement.
c. In carrying out the plan, students did mistakes to use the steps correctly and skillfully in the algorithm and accuracy in answering questions.
d. In looking back, students have not been able to reuse the information obtained to develop a new plan that is different from the previous one.

To improve the ability of mathematical problem solving based on Polya's steps is suggested that in the less stressful learning of students to understand the problem so as to be able to use the information contained in the problem to plan a problem solving.

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