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Bandarlampung, 22 September 2008

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halaman

THE TREE $T_{n,k}$ CONFIGURATIONS USING POLYA'S POLYNOMIAL OF ORDER 2 AND 3

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Abstract

In graph theory, tree plays an important role because it used to represent many problems such as in desain the telecommunication networks, distribution networks and so on. The $T_{n,k}$, is the tree with n vertex and k branches and Polya's polynomial is a polynomial in the form of cycle index. In this paper we will discuss the configuration of $T_{n,k}$ using Polya's polynomial for order 2 and 3.

Keywords: *Tree $T_{n,k}$, cycle index, Polya's polynomial*

Introduction

Graph theoretic concepts have proven to be useful in studying problems arising in network design and analysis. The graph structure is used to model or design complex network systems. Usually the nodes or vertices represent the components (stations, cities, computers, depots, etc) and the edges represent the relationship or interconnection between the components (railways, roads, cables, etc). On designing the network, there is some nonstructural information in the network that must be considered such as: cost, capacity, distance, reliability, time delay, equipment capacity, traffic density, etc. By assigning weights to the vertices and edges of the graph representing the network, these factors can be incorporated into the graph theoretical models (Wamiliana, 2002).

A graph G is defined as G =(V,E), where V = {1,2,...,n} is the set of vertices , $V \neq \phi$, and $E = \{e_{ij} \mid i,j \in V\}$. Two graphs *G* and *H* are called adalah isomorphics if there exists a one-one correspondence between the edges and vertices of G and H that preserve incidency (Deo, 1989).

Pőlya's theorem uses some concepts in combinatorics such as permutation group and cycle index. Pőlya's theorem is one formulae that can be used to count the orbital of the chemical compounds based on the cycle structure of the permutation group. (Harary, 1994).

This paper is organized as follow: in Section 2, literature review, we will give some terms, definitions and theorems used throughout the paper, in Section 3 we will give some observations concerning tree $T_{n,k}$ and it configurations using Pőlya's polynomial, and in Section 4 we give conclusion.

1. Literature review

Tree $T_{n,k}$

Tree is a connected graph which not containing cycle and tree with *n* vertex and *k* branches is denoted with $T_{n,k}$ (Deo, 1989).

Automorphism

Isomorphism from graph *G* to itself is called authomorphism (Gross and Yellen,1999).

Below is six automorphisms by rotating the vertex clockwise direction and then reflexing it to the edge given.

Table 1. Permutation on $K_{1,3}$ graph

Symmetry	Vertex permutation	Edge permutation
Identity	(*u*) (*v*) (*w*) (*x*)	(*a*) (*b*) (*c*)
120º rotation	(*x*) (*uvw*)	(*abc*)
240º rotation	(*x*) (*uwv*)	(*acb*)
Reflected to *a*	(*x*) (*u*) (*vw*)	(*a*) (*bc*)
Reflected to *b*	(*x*) (*v*) (*uw*)	(*b*) (*ac*)
Reflected to c	(*x*) (*w*) (*uv*)	(*c*) (*ab*)

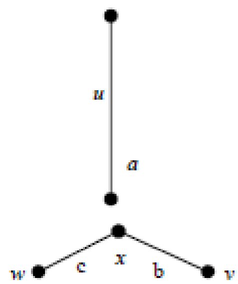

Figure 1. $K_{1,3}$ graph with 6 automorphisms

Automorphism group

Automorphism group from graph *G* is a group permutation of all automorphisms on graph *G* denoted as *Aut*(*G*). Automorphism of V_G denoted as $Aut_V(G)$ and for E_G denoted as $Aut_E(G)$ (Gross and Yellen,1999).

Grup Automorphism trivial and nontrivial

Automorphism that only consists of identity permutation is called as trivial automorphism. If there is exist otther permutation except identity permutation then that permuation is called as nontrivial group automorphism (Biggs,1993)

Cycle Structure

Suppose that $\mathcal{P} = [P : Y]$ is a permutation group on set *Y* consisting of *n* objects, and let $\pi \in P$. *Cycle structure monomial* of π is *monomial n*-variabel

$$\zeta(\pi) = \prod_{k=1}^{n} z_k^{r_k} = z_1^{r_1} z_2^{r_2} \dots z_n^{r_n}$$

Where z_k is a formal variabel and r_k is the number of *k*-*cycle* on the *disjoint cycle form* π (Gross and Yellen,1999).

The figure below give the cycle structure for all vertices permutation and edge permutation on Aut_V (K_4-K_2) , and the table provides the results.

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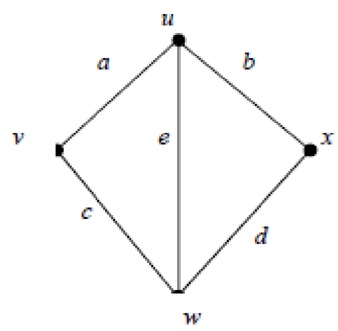

Figure 2. K_4-K_2 graph

Table 1. Cycle structure on Aut_V (K_4-K_2)

Symmetry	π ∈ $Aut_V(g)$	Cycle structure	π ∈ $Aut_E(g)$	Cycle structure
Identity	*(u)(v)(w)(x)*	z_1^4	*(a)(b)(c)(d)(e)*	z_1^5
Reflected on the ordinat	*(u)(w)(v x)*	$z_1^2 z_2$	*(e)(a b)(c d)*	$z_1 z_2^2$
Reflected on the absis	*(v)(x)(u w)*	$z_1^2 z_2$	*(e)(ac)(bd)*	$z_1 z_2^2$
180º rotation	*(u w)(v x)*	z_2^2	*(e)(a d)(b c)*	$z_1 z_2^2$

Cycle Index

Suppose that $\mathcal{P}$ = [*P* : *Y*] is a permutation group on a set with *n* objects then cycle index $\mathcal{P}$ is a polynomial $Z_{\mathcal{P}}(z_1,..., z_n)= \frac{1}{|P|}\sum_{\pi \in P} \zeta(\pi)$

(Gross and Yellen,1999).

Pőlya theorem

The sequence of configuration is resulted by substituting the sequence of numbers into the *cycle index* permutation group *P*,

$$Z_{\mathcal{P}}(x_1 + ... + x_k) = Z(P, \sum_{i=1}^{k} x_i) \text{ (Harary, 1994).}$$

Proof :

Suppose that $\mathcal{P}$ = [*P* : *Y*] is a group permutation on set *Y* that consists of *n* object, and π ∈ *P*. Also assume that z_k is a variabel and r_k is a number of *k-cycle* on *disjoint cycle form* π so that *cycle structure monomial* of π is *monomial n*-variabel

$$\zeta(\pi) = \prod_{k=1}^{n} z_k^{r_k} = z_1^{r_1} z_2^{r_2} \ldots z_n^{r_n}$$

Add the two sides of this equation with all permutations permutasi $\pi \in P$, and then divide it by $|P|$, we get

$$\frac{1}{|P|}\sum_{\pi \in P} \zeta(\pi) = \frac{1}{|P|}\sum_{\pi \in P} \prod_{k=1}^{n} z_k^{r_k}$$

This result is *cycle index* $Z_P(z_1, \ldots, z_n)$. By changing every variableof *cycle index* z_j with *polynomial k-variate* $x_1^j + x_2^j + \ldots + x_k^j$ then we get:

$$Z_P(x_1 + \ldots + x_k) = Z(P, \sum_{i=1}^{k} x_i)$$

Pőlya's substitution and Pőlya counting polynomial of order k

Suppose that $Z_P(z_1, \ldots, z_n)$ is the cycle index polynomial of the permutation group $\mathcal{P}$ = [P : Y] on set Y with n objects. Pőlya's substitution on order k of cycle index variable z_j is a k-variate polynomial $x_1^j + x_2^j + \ldots + x_k^j$ and Pőlya counting polynomial of order k is a polynomial found by changing each cycle index variable z_j with Pőlya's substitution of order k, and it is $Z_P(x_1 + \ldots + x_k)$

(Gross and Yellen,1999).

2. The Tree $T_{n,k}$ Configuration

As we know that tree $T_{n,k}$ which we discussed in this paper is a tree with n vertices and k branches and also has property that it only can have trivial automorphism group. The number of minimal vertices of the tree with k branches that can form trivial automorphism group is :

$$n = \frac{k(k+1)}{2} + 1, \quad 3 \le k < \infty \quad \text{(Nisa', 2005).}$$

The configuration that found from $T_{n,k}$ which only formed trivial automorphism group by using Pőlya's theorms for order 2 and 3 can be seen on Tabel 2.

Tabel 2. Cycle structure and Cycle Index on Tree $T_{n,k}$

Number of branches	Number of vertices	Cycle structure	Cycle Index	2 order configuration	3 order configuration
3	7	z_1^7	z_1^7	$(x_1+x_2)^7$	$(x_1+x_2+x_3)^7$
4	11	z_1^{11}	z_1^{11}	$(x_1+x_2)^{11}$	$(x_1+x_2+x_3)^{11}$
5	16	z_1^{16}	z_1^{16}	$(x_1+x_2)^{16}$	$(x_1+x_2+x_3)^{16}$
6	22	z_1^{22}	z_1^{22}	$(x_1+x_2)^{22}$	$(x_1+x_2+x_3)^{22}$
...	...	...	...	...	...

n	$\frac{k(k+1)}{2}+1$	$z_1^{\frac{k(k+1)}{2}+1}$	$z_1^{\frac{k(k+1)}{2}+1}$	$(x_1+x_2)^{\frac{k(k+1)}{2}+1}$	$(x_1+x_2+x_3)^{\frac{k(k+1)}{2}}$

Conclusion

Tree $T_{n,k}$ which only formed trivial automorphism group has 1 cycle structure because it only consists of identity permutation so that the second order configuration of tree $T_{n,k}$ is

$$(x_1+x_2)^{\frac{k(k+1)}{2}+1},\ 3 \le k < \infty$$

and third order configuration of tree $T_{n,k}$ is

$$(x_1+x_2+x_3)^{\frac{k(k+1)}{2}+1},\ 3 \le k < \infty$$

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