REPRESENTING DEGREE RESTRICTED TREE USING AUGMENTED ADJACENCY MATRIX

Wamiliana, Admi Syarif, and Didik Kurniawan

Department of Mathematics, Faculty of Mathematics and Natural Sciences, Lampung University

ABSTRACT

In graph theory, tree plays an important role due to its structure that can be used to represent many problems such as in desain the telecommunication networks, distribution networks and so on. There are some type of tree representations such as vector representation, adjacency list, permutation, and Prufer’s numbers. For problems that use tree as the key structure, like the DCMST problem, the tree representation by Prufer number is an advantage because in the Prufer number representation we can get information about the degree of the vertex in the tree. The vertex in the tree that has degree *r*, in the Prufer number representation will appear *r*-1 times. In this paper we will propose an alternative method for representing degree restricted tree, that is the augmented adjacency matrix.

Keywords : *degree restricted tree, adjacency matrix, augmented*

1. INTRODUCTION.

Graph theoretic concepts have proven to be useful in studying problems arising in network design and analysis. The graph structure is used to model or design complex network systems. Usually the nodes or vertices represent the components (stations, cities, computers, depots, etc), and the edges represent the relationship or interconnection between the components (railways, roads, cables, etc). On designing the network, there is some nonstructural information in the network that must be considered such as: cost, capacity, distance, reliability, time delay, equipment capacity, traffic density, etc. By assigning weights to the vertices and edges of the graph representing the network, these factors can be incorporated into the graph theoretical models. (Caccetta, 1989).

Tree, as one concept in graph, plays an important role due to its specific properties. Thus, some techniques to represent tree had been proposed such as Prufer’s number, arc vector, adjacency list, and others. According to Palmer and Kershenbaum (1994) there is only a small probability of $2^{[n(\frac{n}{2} \ \log 2(n))]}$ that a random presentation of arc vector type will be a tree. For arc list representation, the weakness lies on the use of the arc list. After a swap based neighborhood search, the procedure cannot guarantee that the resulting set will continue to remain a tree. For predecessor and oriented tree representations, the tree encoding is very computationally expensive (Wamiliana, 2002).

One Efficient method for representing, encoding and decoding trees that lends itself well to a neighborhood search type algorithm is based on Cayley' Theorem. The resulting encoding is called Prufer number. Moon (1967) gave a full description of the properties and Cayley's method of representing a tree. In this paper we propose an alternative way to represent degree restricted tree, which is the augmented adjacency matrix.

Next, in Section 2 we will discuss about some tree representations that already available in literatures.

2. Some tree representations proposed in literature

The Simulated Annealing method has been applied to the DCMST problem by Krishnamoorthy et al. (2001). In their Simulated Annealing procedure, they also used the Prufer (1918) number and a permutation of the nodes in the representation of solutions. The search strategy used is a *cut and paste* neighborhood transformation.

This process starts by choosing randomly a vertex *i* from the spanning tree and then deleting the edge from *i* to *j*. This deletion of edge (*i*,*j*) creates two disconnected components. In their paper they did not mention which edge should be chosen if there are some edges incidence to vertex *i*. The two disconnected components are then re-connected by randomly choosing another node to connect *i*. To do this cut and paste method, they employed the permutation of the nodes, besides the representation of the tree by the Prufer number. They did this in order to avoid having to decode a number in each iteration.

For problems that use tree as the key structure, like the Degree Constrained Minimum Spanning Tree Problem (DCMST), the tree representation by Prufer number seems as an advantage because in the Prufer number representation we can get information about the degree of the vertex in the tree. The vertex in the tree that has degree *r*, in the Prufer number representation will appear *r*-1 times. However, on this specific problem, Caccetta and Wamiliana (2001) concluded that this representation not really attractive when the problem size grows. Some researchers that already used this representation in their research such as: Zhou and Gen (1997), Krishnamoorthy et al (1998, 2001),etc. Below we give the process how to encode and decode a tree using Prufer number as in Wamiliana (2002).

Encoding tree

1. Let *i* be the lowest numbered leaf in*T*. Let *j* be the predecessor vertex of *i*. Then *j* is the rightmost in the Prufer number representation *P(T)*.

2. Remove *i* and (*i*,*j*) so that thus *i* is no longer be considered. If *i* is the only successor of *j*, *j* become a leaf.
3. If only two vertices remain to be considered, stop. *P(T)* is found. If not, go back to Step 1.

Decoding tree

1. Let *P(T)* be the Prufer number and let all vertices not part in *P(T)* be designated as eligible for consideration.
2. If no digits remains in *P(T)*, there are exactly two vertices , *i* and *j* still eligible for consideration. Add edge (*i*,*j*) to *T* and stop.
3. Let *i* be the lowest numbered eligible vertex. Let *j* be the leftmost digit of *P(T)*. Add the edge (*i*,*j*) to T. Remove the leftmost digit from *P(T)*. Designate *i* as no longer eligible. If *j* does not occur anywhere in what remains of *P(T)*, designate *j* as eligible.
4. Return to Step 2

Example:

Suppose we have a tree as follow:

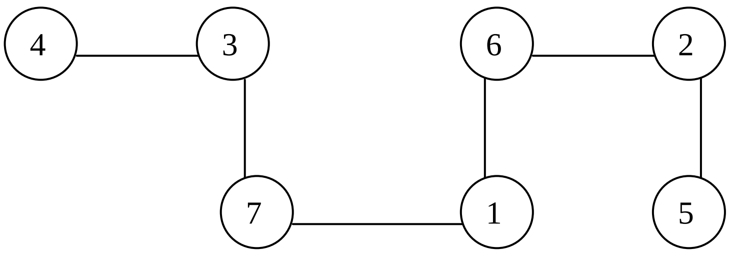

Figure. 1 The tree for example of encoding Prufer number

Using the encoding step, the Prufer number for that tree is P(T) = 37261. To decode the tree with the similar example, we give details in the Table 2.1 Applying the steps in encoding tree we have P(T)= 37261.

	Vertices to be considered	Added edge	The tree construction
37261	4, 5	(3,4)	T = {(3,4)}
7261	3,5	(3,7)	T={(3,4),(3,7))
261	5,7	(2,5)	T={(3,4),(3,7),(2,5)}
61	2,7	(2,6)	T= {(3,4),(3,7),(2,5),(2,6)}
1	6,7	(1,6)	T={(3,4),(3,7),(2,5),(2,6),(1,6)}
∅	1,7	(1,7)	T={(3,4),(3,7),(2,5),(2,6),(1,6),(1,7)}

Table 1. Table of the Prufer number construction

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Next, in Section 3 we discuss the our algorithm for representing tree.

3. The Augmented Adjacency Matrix

Encoding tree

Initialization:: Input : Tree with n vertices

R= ∅ (R is the set of the index of the row)

C= ∅ (C is the set of the index of the column)

Step 1. Make the adjacency n x n matrix.

Step 2. Augment one column to the right and one row below. Set the value of cell of this row as the sum of the value for every entry in that row, and the value of the cell of the column as the sum of the value for every entry in that column (as counter).

Step 3. Let D as the bottom rightmost entry, SLR as the sum of the element in the last row and CLC as the sum of the element in the last column.

Step 4. Choose the smallest index of vertices in tree T (suppose that it *i*). Put *i* in R and C.

Step 5. Find all vertices adjacent to *i* (vertices whose common edges with *i*).

Step 6. Assign value 1 to entry a_{ij} and a_{ji}, where j is the adjacent vertex to i, put j as an element in R and C.

Repeat it to all vertices that adjacent to *i*.

Step 7. Resetting counter to augmented row and column.

Step 8. Repeat

Select the next edge which is not in R or C, and suppose that edge is e_{ik}

Assign value 1 to entry a_{ik} and a_{ki} .Put i and k as an element in R and C.

Assign value 0 (and set it from 'temp' to' fix') to entry a_{jk} and a_{kj}

Resetting counter to augmented row and column.

Until

The value in the counter every row and column is at most *b* (*b* is the degree bound), all entries already set as 'fix', and value of the bottom rightmost entry is 2n-2.

Decoding Tree.

To do the decoding tree from augmented adjacency matrix to the common tree representation using vertices and edges, see the upper triangular matrix or lower triangular of the augmented adjacency matrix (due to its form as symetry matrix).

Step 1. Represent every row or column (just row or column, but not both simultaneously) as vertices, except the last row/column.

Step 2. Draw a line that connect vertices whose entry in the matrix is 1.

To give the illustration of the algorithm, look at the example below

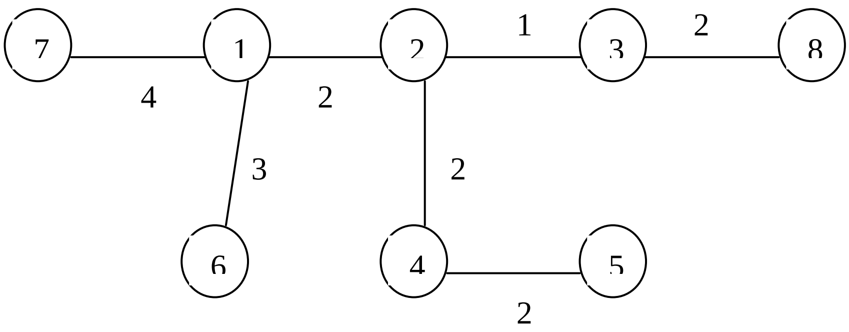

Figure 2. Example for tree representation using Prufer number

After encoding that tree we get the following matrix:

	1	2	3	4	5	6	7	8	LC
1	0	1	0	0	0	1	1	0	3
2	1	0	1	1	0	0	0	0	3
3	0	1	0	0	0	0	0	1	2
4	0	1	0	0	1	0	0	0	2
5	0	0	0	1	0	0	0	0	1
6	1	0	0	0	0	0	0	0	1
7	1	0	0	0	0	0	0	0	1
8	0	0	1	0	0	0	0	0	1
LR	3	3	2	2	1	1	1	1	14

Notice that the matrix above gives the degree information of every vertex in the last column or row, and the value of the bottom rightmost entry indicated the sum of the degree in that tree.

This algorithm can be enhanced to represent the Degree Constrained Minimum Spanning Tree (DCMST), and in our research we found that this representation is quite efficient.

4. CONCLUSION

In this paper we propose another alternative for representing tree that uses adjacency matrix and augment one row and one column in the last. For the degree restricted tree, this representation can be used because it also provides the degree information on the last column or row.

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