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untuk Mewujudkan Kemakmuran Bangsa

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KATA SAMBUTAN
KETUA PELAKSANA
SEMINAR NASIONAL METODE KUANTITATIF 2017

Seminar Nasional Metode Kuantitatif 2017 diselenggarakan oleh Jurusan Matematika Fakultas Matematika dan Ilmu Pengetahuan Alam (FMIPA) Universitas Lampung yang dilaksanakan pada tanggal 24 – 25 November 2017. Seminar terselenggara atas kerja sama Jurusan Matematika FMIPA, Lembaga Penelitian dan Pengabdian Masyarakat (LPPM) Unila, dan Badan Pusat Statistik (BPS).

Peserta dari Seminar dihadiri lebih dari 160 peserta dari 11 institusi di Indonesia, diantaranya : Kementrian Pendidikan dan Kebudayaan, Badan Pusat Statistik, Universitas Indonesia, Institut Teknologi Bandung, Universitas Sriwijaya, Universitas Jember, Universitas Islam Negeri Sunan Gunung Djati, Universitas Cendrawasih, Universitas Teknokrat Indonesia, Universitas Malahayati, dan Universitas Lampung. Dengan jumlah artikel yang disajikan ada sebanyak 48 artikel hal ini merefleksikan pentingnya seminar nasional metode kuantitatif dengan tema "pengunaan matematika, statistika dan computer dalam berbagai disiplin ilmu untuk mewujudkan kemakmuran bangsa".

Kami berharap seminar ini menjadi tempat untuk para dosen dan mahasiswa untuk berbagi pengalaman dan membangun kerjasama antar ilmuan. Seminar semacam ini tentu mempunyai pengaruh yang positif pada iklim akademik khususnya di Unila.

Atas nama panitia, kami mengucapkan banyak terima kasih kepada Rektor, ketua LPPM Unila, dan Dekan FMIPA Unila serta ketua jurusan matematika FMIPA Unila dan semua panitia yang telah bekerja keras untuk suksesnya penyelenggaraan seminar ini.

Dan semoga seminar ini dapat menjadi agenda tahunan bagi jurusan matematika FMIPA Unila`

Bandar Lampung, Desember 2017

Prof. Mustofa Usman,Ph.D

Ketua Pelaksana

DAFTAR ISI

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Bayesian Inference of Poisson Distribution using Conjugate and Non-Informative Priors

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ABSTRACT

Poisson distribution is one of the most important and widely used statistical distributions. It is commonly used to describe the frequency probability of specific events when the average probability of a single occurrence within a given time interval is known. In this paper, Bayesian inference of Poisson distribution parameter (μ) is presented. Two Bayesian estimators of μ using two different priors are derived, one by using conjugate prior by applying gamma distribution, and the other using non-informative prior by applying Jeffery prior. The two priors yield the same posterior distributions namely gamma distribution. Comparison of the two Bayesian estimators is conducted through their bias and mean square error evaluation.

***Keywords**: Bayesian, conjugate prior, Jeffrey prior, marginal distribution, asymptotic variance.*

1.INTRODUCTION

Parameter estimation is a method to estimate parameter value of population using the statistical values of sample [1]. To estimate the population parameter, a representative sample should be taken. Before making any predictions, the population of the random variable must be known such as the distribution form and its parameters [2]. In the theory of statistical inference, the estimation can be done by the classical method or Bayesian method. The classical method relies entirely on inference process on the sample data, while the Bayesian method not only relying on the sample data obtained from the population but also taking into account an initial distribution [3].

The Bayesian method assumes the parameter(s) as a variable that describes the initial knowledge of the parameter before the observation is performed and expressed in a distribution called the prior distribution. Prior selection is generally made on the basis of whether or not the information about parameter is available. If information about parameter is known then it is called as informative prior, meaning that the prior affects the posterior distribution and is subjective. Whereas if the information about the parameter is unknown then the non-informative prior is used and does not give significant effect to the posterior distribution so that the information obtained is more objective [4]. If the posterior distribution is in the same family as the prior probability distribution, the prior and posterior are then called conjugate distributions**,** and the prior is called a conjugate prior for the likelihood function.

Studies on Bayesian inference for distribution parameter estimation have been done by many authors. For example Fikhri *et al.* discussed the Bayesian estimator for Poisson distribution and compared it to the maximum likelihood estimator [7]. Noor and Soehardjoepri investigated the Bayesian method approach for the assessment of log-normal distribution parameter estimates using non-informative prior [8]. Hazhiah *et al*. discussed the estimation of two parameters Weibull distribution using the Bayesian method [9]. Prahutama *et al.* discussed the statistical inference from normal distribution using Bayesian method with non-informative prior [10].

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In this paper, we study the estimation of the Poisson distribution parameters using Bayesian method with conjugate and non-informative prior. The conjugate prior is a special case of informative prior that leads to the posterior having the same form of the prior distribution. The conjugate prior of Poisson distribution is gamma distribution. For the non-informative prior, the well-known Jeffrey's prior is applied.

2. BAYESIAN INFERENCE FOR POISSON DISTRIBUTIONS

Poisson distribution was introduced by Siméon Denis Poisson in his book which explains the applications of probability theory. The book was published in 1837 with the title *Recherchessur la probabilité des jugements en matiérecriminelleet en matiérecivile* (Investigations into the Probability of Verdicts in Criminal and Civil Matters) [5].

Let $X_1, X_2, ..., X_n$, be random sample from Poisson distributed population with parameter μ. The probability density function for Poisson distribution with parameter μ is given by:

$$f(x;\mu) = \begin{cases} \frac{e^{-\mu}\mu^{x}}{x!} & ; x = 0,1,2,\dots \\ 0 & ; x \text{ lainnya} \end{cases}$$

Now consider the general problem of inferring a Poisson distribution with parameter μ given some data x. From Bayes' theorem, the posterior distribution is equal to the product of the likelihood function $L(\mu) = f(x|\mu)$ and prior $f(\mu)$, divided by the probability of the data $f(x)$:

$$f(\mu|x) = \frac{f(\mu)f(x|\mu)}{\int_0^\infty f(\mu)f(x|\mu)} \tag{1}$$

Let the likelihood function be considered fixed; the likelihood function is usually well-determined from a statement of the data-generating process. It is clear that different choices of the prior distribution $f(\mu)$ may make the integral more or less difficult to calculate, and the product $f(\mu)\times f(x|\mu)$ may take one algebraic form or another. For certain choices of the prior, the posterior has the same algebraic form as the prior (generally with different parameter values). Such a choice is a *conjugate prior*.

The form of the conjugate prior can generally be determined by inspection of the probability density or probability mass function of a distribution. The usual conjugate prior of Poisson distribution is the gamma distribution with parameters (α, β), namely:

$$f(\mu;\alpha,\beta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}\mu^{\alpha-1}e^{\frac{-\mu}{\beta}} \tag{2}$$

where $\Gamma(.)$ is the gamma function defines as: $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$.

There has been a desire for a prior distribution that plays a minimal in the posterior distribution. These are sometime referred to a non-informative prior. While it may seem that picking a non-informative prior distribution might be easy, (e.g. just use a uniform distribution), it is not quite that straight forward. One approach of non-informative prior is by using Jeffrey's method. Jeffrey's principle states that any rule for determining the prior density $f(\mu)$ should yield an equivalent result if applied to the transformed parameter (posterior). Applying this principle, Jeffrey's prior can be expressed as the square root of Fisher information as follow:

$$f(\mu) = \sqrt{I(\mu)}$$

where $I(\mu)$ is the Fisher information for μ, i.e.

$$I(\mu) = -E\,(\frac{\partial^2 \log L(\mu)}{\partial \mu^2}).$$

Then for the Poisson parameter μ we calculate the Jeffery's prior as the following. First we calculate the likelihood function

$$L(\mu) = \prod_{i=1}^{n} f(x_i)$$

$$= \prod_{i=1}^{n} \frac{e^{-\mu} \mu^{Xi}}{x!}$$

$$= \frac{e^{-n\mu} \mu^{\sum_{i=1}^{n} xi}}{\prod_{i=1}^{n} x!}$$

The log of the likelihood function is given by

$$Log\ L(\mu) = log\ \frac{e^{-n\mu} \mu^{\sum_{i=1}^{n} xi}}{\prod_{i=1}^{n} x!}$$

$$= log\ e^{-n\mu} + log\ \mu^{\sum_{i=1}^{n} xi} - log \prod_{i=1}^{n} x!$$

$$= -n\mu + \sum_{i=1}^{n} xi\ log\ \mu - log \prod_{i=1}^{n} x!$$

The first and second derivatives of *log L(μ)* with respect to *x* are

$$\frac{\partial\ log\ L(\mu)}{\partial x} = \frac{\partial}{\partial \mu}(-n\mu) + \frac{\partial}{\partial \mu}\left(\sum_{i=1}^{n} xi\ log\ \mu\right) - \frac{\partial}{\partial \mu}\left(log \prod_{i=1}^{n} x!\right)$$

$$= -n + \frac{1}{\mu}\sum_{i=1}^{n} xi - 0$$

$$= -n + \frac{1}{\mu}\sum_{i=1}^{n} xi,$$

and

$$\frac{\partial^2\ log\ L(\mu)}{\partial \mu^2} = \frac{\partial}{\partial \mu}\left(-n + \frac{1}{\mu}\sum_{i=1}^{n} xi\right)$$

$$= \sum_{i=1}^{n} xi \frac{\partial}{\partial \mu}(\mu^{-1})$$

$$= \frac{-\sum_{i=1}^{n} xi}{\mu^2}.$$

Fisher information for μ is given by:

$$I(\mu) = -E\left(\frac{\partial^2\ log\ L(\mu)}{\partial \mu^2}\right)$$

$$= -E\left(\frac{-\sum_{i=1}^{n} xi}{\mu^2}\right)$$

$$= \frac{1}{\mu^2}\ E\left(\left(\sum_{i=1}^{n} xi\right)\right)$$

$$= \frac{1}{\mu^2}(n\mu)$$

$$= \frac{n}{\mu}$$

The Jeffrey's prior then is obtained as

$$f(\mu) = \sqrt{I(\mu)} = \sqrt{\frac{n}{\mu}}, n \text{ constant}$$

$$= \frac{1}{\sqrt{\mu}} \tag{3}$$

In the next section we use the gamma and Jeffrey's priors in Equation (2) and (3) respectively for obtaining the Bayesian estimator of Poisson parameter μ.

3. RESULT AND DISCUSSION

A. *Bayesian Inference of Poisson using Conjugate prior*

Applying the Bayesian law of probability as expressed in Equation (1), we can calculate the posterior of μ as follow. The joint distribution and the marginal distribution are given by

$$f(\mu)f(x|\mu) = \frac{\mu^{\alpha-1} e^{\frac{-\mu}{\beta}} e^{-n\mu} \mu^{\sum_{i=1}^{n} xi}}{\beta^{\alpha}\Gamma(\alpha)\prod_{i=1}^{n} x!}$$

$$= \frac{\mu^{\alpha-1+\sum_{i=1}^{n} xi} e^{-\mu(n+\frac{1}{\beta})}}{\beta^{\alpha}\Gamma(\alpha)\prod_{i=1}^{n} x!},$$

and

$$\int_0^{\infty} f(\mu)f(x|\mu) = \int_0^{\infty} \frac{\mu^{\alpha-1+\sum_{i=1}^{n} xi} e^{-\mu(n+\frac{1}{\beta})}}{\beta^{\alpha}\Gamma(\alpha)\prod_{i=1}^{n} x!}$$

$$= \frac{1}{\beta^{\alpha}\Gamma(\alpha)\prod_{i=1}^{n} x!} \int_0^{\infty} \mu^{\alpha-1+\sum_{i=1}^{n} xi} e^{-\mu(n+\frac{1}{\beta})} d\mu$$

$$= \frac{1}{\beta^{\alpha}\Gamma(\alpha)\prod_{i=1}^{n} x!} \Gamma\,(\alpha+\sum xi)((n+\frac{1}{\beta})^{-1})^{\alpha+\sum xi}$$

$$= \frac{\Gamma\,(\alpha+\sum xi)((n+\frac{1}{\beta})-1)\alpha+\sum xi}{\beta^{\alpha}\Gamma(\alpha)\prod_{i=1}^{n} x!}.$$

Then the posterior distribution can be written as

$$f(\mu|x) = \frac{f(\mu)f(x|\mu)}{\int_0^{\infty} f(\mu)f(x|\mu)}$$

$$= \frac{\mu^{\alpha-1+\sum_{i=1}^{n} xi} e^{-\mu(n+\frac{1}{\beta})}}{\beta^{\alpha}\Gamma(\alpha)\prod_{i=1}^{n} x!} \cdot \frac{\beta^{\alpha}\Gamma(\alpha)\prod_{i=1}^{n} x!}{\Gamma\,(\alpha+\sum xi)((n+\frac{1}{\beta})-1)\alpha+\sum xi}$$

$$= \frac{\mu^{\alpha-1+\sum_{i=1}^{n} xi} e^{-\mu(n+\frac{1}{\beta})}}{\Gamma\,(\alpha+\sum xi)((n+\frac{1}{\beta})-1)\alpha+\sum xi} \tag{4}$$

The posterior distribution expressed in Equation (4) is gamma distribution with parameter $(\alpha + \sum x_i)$ and $\left(n + \frac{1}{\beta}\right)^{-1}$, or μ ~ gamma $\left(\alpha + \sum x_i, \left(n + \frac{1}{\beta}\right)^{-1}\right)$. The Bayesian estimate is given by the mean of the posterior. For the final step of Bayesian Estimation we need to recall the following property of gamma distribution.

Proposition 1

If X is a gamma distributed random variable with parameters (α, β), the expected value and variance of X are given by E(X) = αβ and Var (X) =$\alpha\beta^2$ respectively. (The proof of this property can be seen in e.g. [6]).

Applying Proposition 1 we obtain the Bayesian estimate of μusing gamma prior as follow

$$\widehat{\mu}_g = \frac{\alpha+\sum xi}{n+\frac{1}{\beta}} .$$

B. Bayesian Inference using non-Informative prior

To obtain Jeffrey's posterior we apply the prior in Equation (3) to Equation (1). First we calculate the joint and the marginal distributions as follow

$$f(x;\mu) = f(x|\mu)f(\mu)$$

$$= \frac{e^{-n\mu}\mu^{\sum_{i=1}^{n} xi}}{\prod_{i=1}^{n} x!}\frac{1}{\sqrt{\mu}}$$

$$= \frac{e^{-n\mu}\mu^{\sum_{i=1}^{n} xi}\,\mu^{-1/2}}{\prod_{i=1}^{n} x!}$$

$$=\frac{e^{-n\mu}\mu^{-\frac{1}{2}+\sum_{i=1}^{n} xi}}{\prod_{i=1}^{n} x!}$$

and

$$\int f\,(x;\mu)\,d\mu = \int_0^\infty \frac{\mu^{-1/2+\sum_{i=1}^{n} xi}e^{-n\mu}}{\prod_{i=1}^{n} x!}d\mu$$

$$= \frac{1}{\prod_{i=1}^{n} x!}\int_0^\infty \mu^{-1/2+\sum_{i=1}^{n} xi}e^{-n\mu}\,d\mu$$

$$=\frac{1}{\prod_{i=1}^{n} x!}\Gamma(\frac{1}{2}+\ \sum_{i=1}^{n} xi)\ (n^{-1})^{\sum_{i=1}^{n} xi+1/2}$$

The Jeffrey's posterior is given by

$$f(\mu|x) = \frac{f\,(x;\mu)}{\int f\,(x;\mu)}$$

$$=\frac{e^{-n\mu}\mu^{-\frac{1}{2}+\sum_{i=1}^{n} xi}}{\prod_{i=1}^{n} x!}\frac{\prod_{i=1}^{n} x!}{\Gamma(\frac{1}{2}+\sum_{i=1}^{n} xi)\ (n^{-1})^{\sum_{i=1}^{n} xi+1/2}}$$

$$=\frac{e^{-n\mu}\mu^{-\frac{1}{2}+\sum_{i=1}^{n} xi}}{\Gamma(\frac{1}{2}+\sum_{i=1}^{n} xi)\ (n^{-1})^{\sum_{i=1}^{n} xi+1/2}}$$

$$= \frac{e^{-n\mu}\mu^{\sum_{i=1}^{n} xi+1-1/2}}{\Gamma(\frac{1}{2}+\sum_{i=1}^{n} xi)\ (n^{-1})^{\sum_{i=1}^{n} xi+1/2}} \tag{5}$$

The posterior distribution expressed in Equation (4) is gamma distribution with parameter $\left(\frac{1}{2}+\sum x_i\right)$ and $\frac{1}{n}$, or μ ~ gamma $\left(\frac{1}{2}+\sum x_i, \frac{1}{n}\right)$. By Proposition 1 we obtain the Bayesian estimate of μ using Jeffrey's prior as follow

$$\hat{\mu}_J = \frac{\frac{1}{2}+\sum xi}{n}$$

C. Evaluation of the estimators

The resulted Bayesian estimates using gamma and Jeffrey's priors are compared by evaluating their bias and variance. The bias of an estimator is the difference between this expected value of the estimator and the true value of the parameter being estimated. Below we show that the two estimates are bias but asymptotically unbiased.

The expected value of Bayesian estimate using gamma prior is

$$E\,(\hat{\mu}_g) = E\left(\frac{\alpha+\sum xi}{n+\frac{1}{\beta}}\right) = \frac{1}{n+\frac{1}{\beta}} E\,(\alpha + \sum xi\,)$$

$$= \frac{1}{n+\frac{1}{\beta}} E\,(\alpha\,) + E\,(\,\sum xi\,)$$

$$= \frac{1}{n+\frac{1}{\beta}} (\alpha + \sum E\,(\,xi)\,)$$

$$= \frac{1}{n+\frac{1}{\beta}}\ (\alpha + n\mu) \tag{6}$$

Since $E(\hat{\mu}_g) \neq \mu$ then $\hat{\mu}_g$ is a biased estimator of μ, now we calculate the bias for $n \to \infty$ as follow

$$lim_{n\to\infty} \frac{1}{n+\frac{1}{\beta}}\ (\alpha + n\mu) = lim_{n\to\infty} \frac{(\alpha+n\mu)}{n+\frac{1}{\beta}}$$

$$= lim_{n\to\infty} \frac{(\alpha/n+n\mu/n)}{n/n+\frac{1}{n\beta}}$$

$$= lim_{n\to\infty} \frac{(\alpha/n+\mu)}{1+\frac{1}{n\beta}}$$

$$= \mu$$

Since $lim_{n\to\infty}\, E\,(\hat{\mu}_g) = \mu$ then $\hat{\mu}_g$ an asymptotically unbiased estimator of μ.

Now consider the Bayesian estimate of μ using Jeffrey's prior $\hat{\mu}_J = \frac{\frac{1}{2}+\sum xi}{n}$, the expected value is given by

$$E\,(\hat{\mu}_J) = E\left(\frac{\frac{1}{2}+\sum xi}{n}\right) = \frac{1}{n} E\,(½ + \sum xi)$$

$$= \frac{1}{n} E\,(½\,) + E\,(\,\sum xi\,)$$

$$= \frac{1}{n}\,(½ + \sum E\,(\,xi)\,)$$

$$= \frac{1}{n}\ (½ + n\mu)$$

$$= \frac{1}{2n} + \mu \tag{7}$$

Since $E\ (\hat{\mu}_J) \neq \mu$ then $\hat{\mu}_g$ is an unbiased estimator of μ. For $n \to \infty$ as we obtain

$$lim_{n\to\infty} \frac{1}{n} + \mu = lim_{n\to\infty} \frac{(1+2n\mu)}{2n}$$

$$= lim_{n\to\infty} \frac{(1/n+2\mu)}{2}$$

$$= \mu,$$

then $\hat{\mu}_J$ is an asymptotically unbiased estimator of μ.

The consistency analysis of the two estimators is described through their means square errors (MSEs) as follow

$$MSE = Var\ (\hat{\mu}) + (Bias\ (\hat{\mu}\ ))^2$$

Where $Bias\ (\hat{\mu}) = E\ (\hat{\mu}) - \mu$ and $Var(\hat{\mu})$ is the variance of $\hat{\mu}$.

For $\hat{\mu}_g$,

$$Bias\ (\hat{\mu}_g) = \frac{\alpha+n\mu}{n+\frac{1}{\beta}} - \mu = \frac{\alpha+n\mu-\left(n+\frac{1}{\beta}\right)\mu}{n+\frac{1}{\beta}} = \frac{\alpha - \frac{\mu}{\beta}}{n+\frac{1}{\beta}},$$

and

$$Var\ (\hat{\mu}_g) = Var\left(\frac{\alpha+\sum xi}{n+\frac{1}{\beta}}\right) = \frac{1}{(n+\frac{1}{\beta})^2}\ Var\ (\ \alpha + \sum x_i\ )$$

$$= \frac{1}{(n+\frac{1}{\beta})^2}[Var\ (\alpha) + Var\ (\sum x_i)]$$

$$= \frac{1}{(n+\frac{1}{\beta})^2}[\ 0 + Var\ (\sum x_i)]$$

$$= \frac{1}{(n+\frac{1}{\beta})^2}[\sum Var\ (x_i\ )]$$

$$= \frac{1}{(n+\frac{1}{\beta})^2}\ n\alpha\beta^2$$

$$= \frac{n\alpha\beta^2}{(n+\frac{1}{\beta})^2}.$$

Then

$$MSE(\hat{\mu}_g) = \frac{n\alpha\beta^2}{(n+\frac{1}{\beta})^2} + \left[\frac{\alpha - \frac{\mu}{\beta}}{n+\frac{1}{\beta}}\right]^2 = \frac{n\alpha\beta^2 + (\alpha - \frac{\mu}{\beta})^2}{(n+\frac{1}{\beta})^2}$$

For $\hat{\mu}_J$,

$$Bias\ (\ \widehat{\mu_J}\ ) = E\ (\ \widehat{\mu_J}\ ) - \mu = \frac{1}{2n} + \mu - \mu = \frac{1}{2n},$$

and

$$Var\ (\ \widehat{\mu_J}\ ) = Var\left(\frac{\frac{1}{2}+\sum xi}{n}\right)$$

$$= \frac{1}{n^2}\ Var\ (\ 1/2 + \sum x_i\ )$$

$$= \frac{1}{n^2}[Var\ (1/2\ ) + \ Var\ (\sum x_i)]$$

$$= \frac{1}{n^2}[\ 0 + \ \sum Var\ (x_i)]$$

$$= \frac{1}{n^2}[\sum Var\ (x_i\ )]$$

$$= \frac{1}{n^2}\ n\alpha\beta^2$$

$$= \frac{n\alpha\beta^2}{n^2}.$$

Then,

$$MSE(\widehat{\mu_J}) \ = \ Var\ (\ \widehat{\mu_J}\ ) + \left(Bias\ \ (\ \widehat{\mu_J}\ )\right)^2$$

$$= \frac{n\alpha\beta^2}{n^2} \ + \ \left[\frac{1}{2n}\right]^2$$

$$= \frac{4n\alpha\beta^2+1}{4n^2}$$

For comparing the MSE of $\hat{\mu}_g$ and $\widehat{\mu}_J$, we conducted a simulation study and we present the result in the following section.

4. SIMULATION STUDY

The objective of the data simulation is to compare empirically the MSE of Bayesian estimates using gamma and Jeffrey's priors. The simulation was done using R software. We generated data from Poisson distributions with parameter μ = 3, 5, 10 and sample sizes n=20, 50, 100, 300, 500, 1000, 5000 dan 10000. For the Bayesian estimate using gamma prior we fix gamma parameters: α =1 and β = 5. The MSE of the estimates of the Bayesian estimators is provided in the following tables.

Table 1. *MSE of $\hat{\mu}$ obtained using the conjugate (gamma) and non-informative (Jeffrey's) priors forμ= 3*

n	MSE	
	Conjugate Prior (gamma)	Non-informative Prior (Jeffrey's)
20	7.839709	8.235576
50	1.250809	1.276125
100	0.270299	0.272759
300	0.032169	0.032266
500	0.011520	0.011539
1000	0.002854	0.002861
5000	0.000128	0.000127
10000	0.000033	0.000033

Tabel 2. *MSE of $\hat{\mu}$ obtained using the conjugate (gamma) and non-informative (Jeffrey's) priorsμ= 5*

n	MSE	
	Conjugate Prior (gamma)	Non-informative Prior (Jeffrey's)

20	12.469016	13.091211
50	2.135610	2.177006
100	0.468095	0.472870
300	0.056265	0.056477
500	0.019575	0.019611
1000	0.005426	0.005412
5000	0.000197	0.000197
10000	0.000050	0.000050

Tabel 3. *MSE of $\hat{\mu}$ obtained using the conjugate (gamma) and non-informative (Jeffrey's) priorsμ= 10*

n	MSE	
	Conjugate Prior (gamma)	Non-informative Prior (Jeffrey's)
20	26.030662	27.275180
50	3.902299	3.970509
100	1.009183	1.017402
300	0.116299	0.116489
500	0.037545	0.037528
1000	0.010257	0.010206
5000	0.000384	0.000383
10000	0.000098	0.000097

The MSEs presented in Table 1-3 of the two Bayesian estimators are very similar. When the sample size (*n)* increases the MSEs become smaller and closer to zero.We notice that for small sample size (*n*=20) the gamma prior produces much better estimates than the Jeffrey's prior (the differences of the MSEs are around one unit), but they become more similar for larger *n* (>20). From the simulation result we indicate that the two estimators are consistent estimators for Poisson parameter μ.

5. CONCLUSIONS

From the analitical and empirical results presented in this paper we obtain the following conclusions :

1. The Bayesian estimate of Poisson parameter μusing gammaprior (notated by $\hat{\mu}_g$) and Jeffrey's prior (notated by $\hat{\mu}_J$) are :

$$\hat{\mu}_g = \frac{\alpha+\sum xi}{n+\frac{1}{\beta}} \quad \text{and} \quad \hat{\mu}_J = \frac{\frac{1}{2}+\sum xi}{n}$$

2. Theoretically, $\hat{\mu}_g$and $\hat{\mu}_J$ are biased estimators of μ, but we proved that they are asymptotically unbiased.
3. Based on the simulation result, the larger the sample size the smaller the MSE value of the two estimators (and closer to zero) then they both are consistent.

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