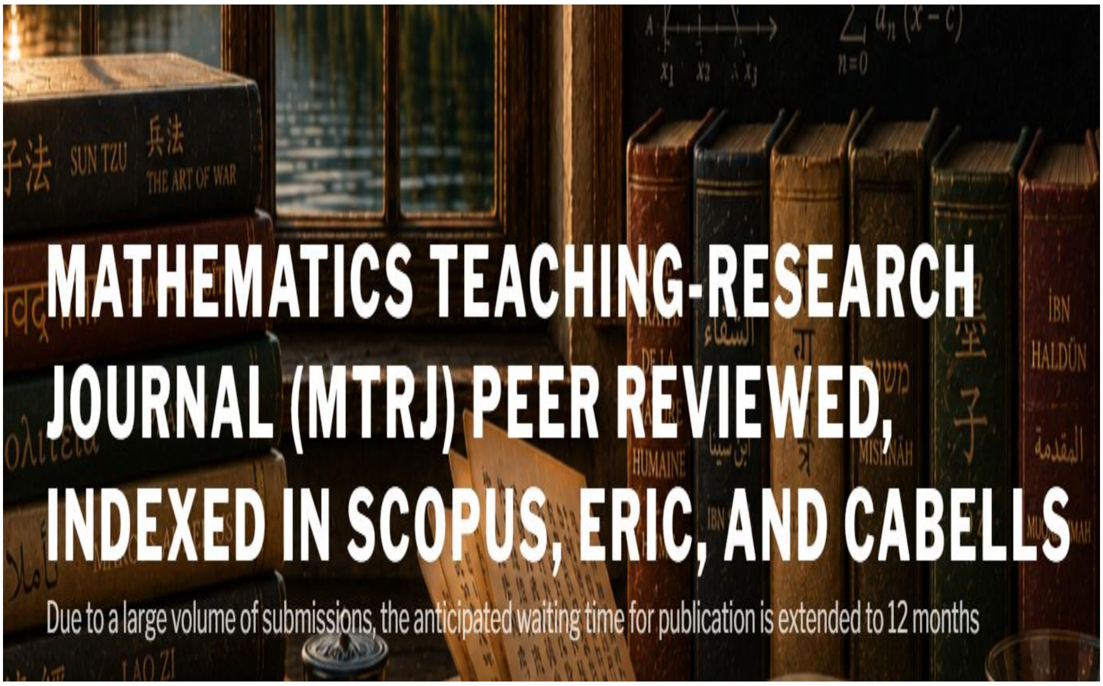

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The Role of Realistic Mathematics Education with Scaffolding Teaching Model in Enhancing Mathematical Representation Ability

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Abstract: Mathematics education in higher learning often faces difficulties in developing students' mathematical representation skills, with many struggling in verbal, visual, and symbolic expressions. This study explored the impact of the Realistic Mathematics Education (RME) approach combined with scaffolding on improving students' ability to represent mathematical concepts. A quasi-experimental design was employed, involving 50 students across three experimental groups and 16 students in a control group, all selected at random. The data were analyzed using both inferential and descriptive statistics, with mathematical representation assessed through a specific test. Hypothesis testing was conducted using one-way ANOVA and post hoc comparisons. The results revealed significant differences between the experimental and control groups, demonstrating that the RME model with scaffolding effectively enhanced students' mathematical representation skills. These findings highlight that integrating RME can lead to substantial improvements in mathematical understanding within higher education.

Keywords: Realistic Mathematics Education, Scaffolding, Mathematical Representation Ability

INTRODUCTION

Mathematics is a core subject taught from elementary to higher education levels (Langoban & Langoban, 2020). Its primary purpose is to develop essential mathematical competencies. According to Ferrini-Mundy (2001) these include: (1) problem-solving, (2) reasoning and proof, (3) connections, (4) communication, and (5) representation. Among these, mathematical representation is crucial for helping students understand, communicate, and apply mathematical ideas effectively.

Mathematical representation refers to the ability to express mathematical concepts using various forms such as graphs, symbols, and written language (Mainali, 2021). Tang (2023) classifies representation into three types: visual (e.g., diagrams, graphs), symbolic (e.g., formulas, notations), and verbal (e.g., written explanations). Vincent-Lancrin (2023) highlight that this ability allows students to translate concepts across different forms, enabling deeper understanding. McCoy et al. (1996) emphasize the importance of encouraging students to construct their own representations as tools for thinking and communication. This skill helps students concretize abstract concepts, making mathematics more accessible and meaningful (Santiago & Alves, 2022). However, many university students struggle with developing mathematical representations, particularly in translating between verbal, visual, and symbolic forms. Studies by Hwang & Ham (2021), students tend to demonstrate lower levels of mathematical literacy when they are more frequently exposed to applied mathematics reasoning tasks. Krawec (2014) suggests that this approach might have unintentionally hindered some students. For instance, certain students may have been capable of solving several problems without relying on visual representations, leading to incomplete visual outputs and, in turn, weakening the correlation between visual representation and problem-solving accuracy.

To address this, various instructional models have been explored. One promising approach is Realistic Mathematics Education (RME), which emphasizes contextual learning through real-life situations (Fredriksen, 2021). RME is a learning approach that people naturally apply in everyday life, often unconsciously (Fredriksen, 2021). It situates mathematical concepts within contexts that are closely connected to students' real-world experiences and environments. According to Van Den Heuvel-Panhuizen & Drijvers (2020), one of the seven key principles that guide RME design is the interactivity principle. This principle emphasizes that mathematics learning should be a collaborative process, involving group work and class discussions, rather than being solely an individual activity. A study by Kaba & Şengül (2017) demonstrated that using RME-based instruction significantly outperformed traditional methods in improving students' measurement estimation skills, underscoring the practical benefits of applying RME principles in mathematics education.

Despite its benefits, RME in higher education still faces challenges, particularly in supporting students with diverse learning needs. To address this gap, scaffolding can be introduced. Scaffolding, introduced by Wood et al. (1976), is a method of providing tailored, temporary support to help students master new concepts. As a result, the scaffolding construct is now being broadly used, becoming "a proxy for any cultural practices associated with performance, knowledge and skills" (Pea, 2018).

While both RME and scaffolding have independently shown effectiveness in improving learning outcomes, limited research has explored their integration in the context of higher education, especially regarding mathematical representation skills. Previous studies (Fredriksen, 2021; Laurens et al., 2017; Yuanita et al., 2018) have validated RME's effectiveness in enhancing conceptual understanding, while scaffolding has supported structured learning (Brush & Saye, 2002; Kim & Lim, 2019). However, a combined model remains underexplored.

Therefore, integrating RME with scaffolding presents a novel approach to address the limitations of each model. This study investigates the effectiveness of combining RME and scaffolding to improve students' mathematical representation skills in Mathematics Education at Universitas Islam Negeri Raden Intan Lampung, Indonesia. It is expected to contribute to more effective instructional practices and serve as a reference for educators and curriculum developers aiming to enhance the quality of mathematics education.

We formulate the research questions: Tow hat extent does the integration of RME with scaffolding improve undergraduate students' mathematical representation abilities—verbal, visual, and symbolic—compared to conventional teaching methods in higher education?

LITERATURE REVIEW

Realistic Mathematics Education

RME is an instructional approach that focuses on connecting mathematical concepts with real-life contexts to foster a deeper understanding of mathematics among students. RME, originally developed in the Netherlands by Hans Freudenthal in the 1970s, is based on the idea that mathematics is an activity that humans engage in to understand and solve problems related to their environment (Kösece & Doğanay, 2023). RME views mathematics as a human activity that should be rooted in problem-solving related to students' everyday experiences, making abstract concepts more accessible and meaningful (Fitria et al., 2022; Susanto et al., 2024). A key feature of RME is the distinction between horizontal and vertical mathematization.

Horizontal mathematization involves translating real-world problems into mathematical models, while vertical mathematization refines these models into formal mathematical reasoning (Hermandra et al., 2022; Izzah & Ekawati, 2023). This progression helps students develop deeper problem-solving skills by moving from concrete to abstract understanding. Collaboration and interaction are essential in RME, fostering a learning environment where students discuss, share strategies, and build upon each other's ideas. This social interaction deepens cognitive engagement and enhances understanding (Nermin & Kapucu, 2022; Umar & Zakaria, 2022). Additionally, RME encourages students to develop their own models, serving as bridges between informal reasoning and formal mathematics, enabling a smoother transition to abstract thought (Nurfadilah et al., 2021). RME aligns well with 21st-century educational goals, promoting critical thinking, creativity, and collaboration. Research shows that students taught through RME demonstrate greater proficiency in mathematical representation and practical problem-solving (Ningsi et al., 2024). By anchoring learning in real-world contexts, RME prepares students to apply mathematics both academically and effectively in everyday life (Altiner et al., 2023).

Previous studies have shown that RME can enhance students' mathematical understanding and problem-solving skills, though challenges remain in aligning real-world contexts with formal models (Laurentiz, 2021; Payadnya et al., 2023; Sembiring et al., 2008). These mixed outcomes indicate that the success of RME depends on factors such as instructional design, scaffolding, and learner context. Therefore, while RME holds strong potential for higher education, its effective implementation requires well-prepared teachers, equitable learning opportunities, and contextually adapted assessment strategies.

Scaffolding

Scaffolding was introduced by educational psychologist Jerome Bruner in the 1970s (Bruner, 1978). This concept is closely related to constructivist learning theory and refers to the way educators provide support to students during the learning process to help them develop new understanding and skills (Wood et al., 1976). In an educational context, scaffolding is a strategy where teachers offer assistance tailored to students' needs as they engage with new or challenging material (Hogan & Pressley, 1997). This support may include hints, encouragement, guiding questions, or additional explanations (Van de Pol et al., 2010). As students begin to grasp the content or skills being taught, the level of assistance is gradually reduced until they are able to complete tasks independently (Pea, 2004).

Scaffolding is an instructional strategy that offers temporary, structured support to learners as they acquire new skills or understanding. Originally introduced by Jerome Bruner and later expanded upon by Lev Vygotsky, the concept is closely tied to Vygotsky's theory of the Zone of Proximal Development (ZPD) (see Figure 1)—the range of tasks a learner can perform with guidance but not yet independently (Fokong, 2023; Rokhmat et al., 2019). This assistance may take the form of breaking down complex tasks into manageable steps, posing guiding questions, offering strategic hints, or giving clarifying explanations (Richardson et al., 2021). The ultimate goal is to bridge the gap between what students currently know and what they can achieve with temporary support, fostering independence over time (Safa & Motaghi, 2021).

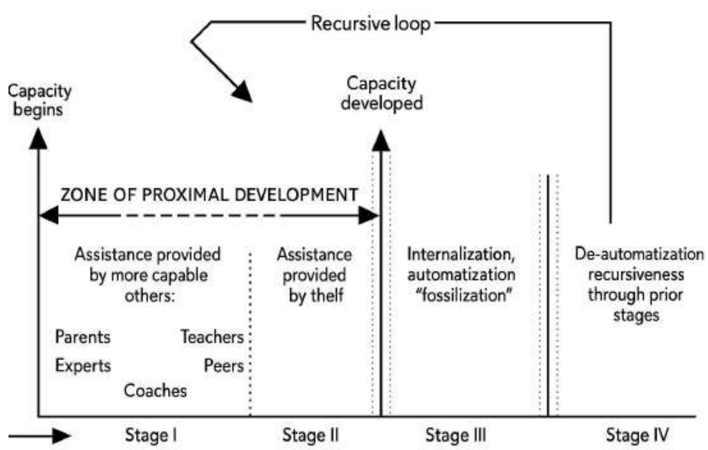


Figure 1: Model of four stages in the ZPD (Gallimore, 1990)

The process begins with Stage I, where learners rely on guidance from more capable individuals. In Stage II, they begin to support themselves after internalizing the help received. By Stage III, knowledge becomes automatic—often called "fossilization." In Stage IV, learners may revisit earlier stages to adjust their understanding when facing new challenges.

In classroom settings, scaffolding is especially valuable in helping students master difficult concepts. In mathematics education, for example, scaffolding supports learners in linking abstract ideas to familiar, real-world contexts, thereby deepening their conceptual understanding (Johnson et al., 2022).

The gradual release of responsibility from teacher to student ensures that learners gain confidence and internalize essential skills, eventually becoming capable of completing tasks independently. Scaffolding can be implemented through various forms, including modeling (demonstrating how to perform a task), questioning (encouraging critical thinking), and feedback (offering corrections and suggestions). Each type of support plays a crucial role in advancing cognitive development and promoting self-regulated learning. In essence, scaffolding is a powerful educational tool that supports learners as they transition from dependence to independence. By providing appropriate, responsive assistance and gradually removing it, educators can help students achieve higher levels of understanding and academic success across disciplines.

Realistic Mathematics Education Supported Scaffolding

The integration of RME and scaffolding provides a powerful instructional approach for strengthening students' mathematical understanding and representation skills. RME, pioneered by Freudenthal (2002), emphasizes connecting mathematical concepts to real-life contexts, making learning meaningful and relevant to students' experiences. However, engaging with such authentic problems often requires support, which aligns with Vygotsky (1978) Sociocultural Theory of Cognitive Development. Central to this theory is the Zone of Proximal Development (ZPD), defined as the gap between what a learner can accomplish independently and what they can achieve with guidance from adults or collaboration with more capable peers. The ZPD highlights not only completed abilities but also emerging skills that are "in the process of maturing," emphasizing learning as a dynamic, forward-looking process.

Within the Zone of Proximal Development (ZPD), scaffolding serves as a vital instructional strategy that provides temporary, structured support to help learners tackle tasks they cannot yet complete independently (Wood et al., 1976). While distinct from the ZPD itself, scaffolding operates within it by enabling gradual engagement with complex problems and reducing support as learners gain autonomy (Smagorinsky, 2018). In the context of Realistic Mathematics Education (RME), scaffolding facilitates both horizontal mathematization—translating real-world problems into mathematical models—and vertical mathematization—refining these into formal reasoning. Strategies such as modeling, prompting, and feedback foster confidence, active engagement, and deeper understanding, ultimately enhancing students' mathematical representation skills (Ariska et al., 2021; Bostic et al., 2020). By integrating RME with scaffolding, educators can provide a coherent framework that nurtures independence, problem-solving ability, and meaningful application of mathematics in real contexts.

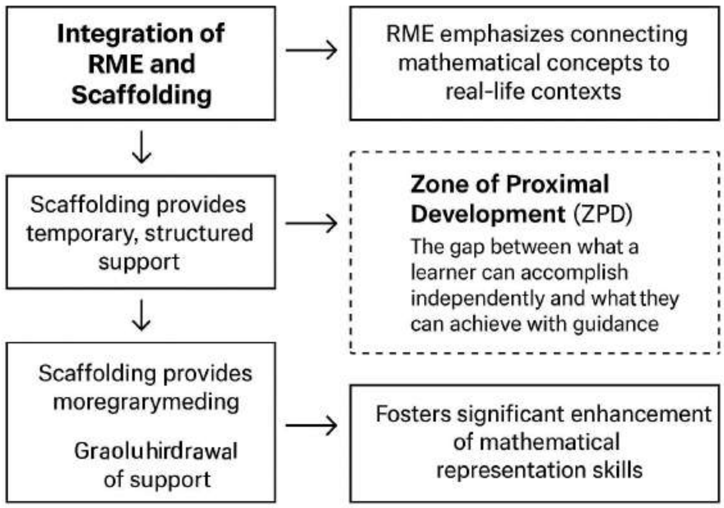


Figure 2: RME with Scaffolding Theory of Cognitive Development for ZPD (create by authors)

The RME with Scaffolding approach differs from traditional instruction by emphasizing real-world problem contexts, where students engage in horizontal mathematization (modeling everyday situations) and vertical mathematization (developing formal reasoning). Scaffolding strategies—such as guided questioning, modeling, collaboration, and gradual release of responsibility—support learners in moving from informal to formal understanding. Unlike conventional teacher-centered methods that stress procedures, rote practice, and abstract problems with little connection to students' experiences, RME with Scaffolding fosters conceptual understanding, multiple representations, and stronger problem-solving skills.

METHODS

Participants

The participants in this study were undergraduate students from the Mathematics Education Study Program at Universitas Islam Negeri Raden Intan Lampung, enrolled in courses covering secondary school mathematics topics such as probability, arithmetic sequences and series, and three-dimensional geometry. This study was carried out in collaboration with several mathematics education lecturers from Universitas Islam Negeri Raden Intan Lampung, Indonesia, who teach high school mathematics courses. Each lecturer was responsible for supervising a different class, which was assigned to either the experimental group or the control group. The experimental groups included Group A with 16 students (2 male, 14 female), Group B with 15 students (all female), and Group C with 19 students (2 male, 17 female). These groups received instruction using the Realistic Mathematics Education (RME) model integrated with scaffolding. The control group consisted of Group

D, which included 16 students (6 male, 10 female) and followed conventional teaching methods without the RME-scaffolding integration.

Research Model

This study employed a quasi-experimental design within the Mathematics Education Study Program at Universitas Islam Negeri Raden Intan Lampung, involving undergraduate students enrolled in secondary mathematics courses. The intervention covered probability, arithmetic sequences and series, three-dimensional geometry, and combinatorics, all contextualized through real-life problems such as drawing marbles, savings patterns, calculating water tank volumes, and arranging books or forming committees. These tasks were designed to link abstract concepts with practical applications and strengthen students' mathematical reasoning. The research model is presented in Table 1.

Class	*Pretest*	Treatment	*Posttest*
Group A	O_1	XR_1	O_2
Group C	O_3	XR_2	O_4
Group D	O_5	XR_3	O_6
Group B	O_7	C	O_8

Table 1: Research Design. *Note:* XR = RME model with scaffolding; C = Control; O = test

This study employs a five-step instructional model integrating RME with scaffolding—contextual problem presentation, guided questioning, collaborative discussion, multi-representation exploration, and gradual release of support—to enhance students' conceptual understanding and representation skills. In contrast, the traditional model follows a teacher-centered sequence of concept explanation, formula memorization, worked examples, routine practice, and answer correction, emphasizing procedural fluency over meaningful engagement. The instructional syntax is summarized in Appendix A (modified from).

Research Procedures

The research was conducted in three experimental classes and one control class. Each experimental class followed the steps of the Realistic Mathematics Education (RME) model integrated with scaffolding, while the control class used traditional teaching. The learning process began with a preparation phase, in which students in the experimental groups took a pretest to assess their initial mathematical representation skills in verbal, visual, and symbolic forms. Lesson plans were carefully designed to include real-life problems relevant to the students' local context, with a focus on developing mathematical representation abilities.

The intervention was carried out over 12 class meetings from July to September 2024. During this implementation phase, students participated in weekly 90-minute sessions centered on problem-solving activities aligned with RME and scaffolding principles. In the first two weeks, students were introduced to the learning context and objectives, with teachers presenting problems rooted in local culture, such as traditional craft patterns. From weeks three to nine, students engaged in guided explorations of these problems, working collaboratively in groups with ongoing teacher support, gradually progressing toward formal mathematical understanding. Weeks 10 and 11 were dedicated to reflection and presentations, during which students shared their work and discussed their insights. In the final week, a posttest was administered to measure improvements in students' mathematical representation skills.

Instrument

The study employed an essay-based test to evaluate students' mathematical representation skills across three forms: verbal, visual, and symbolic. The test comprised five items in total both pre-post test (see Appendix B and C). For verbal representation, students were asked to articulate their reasoning in written form. Visual representation required them to interpret problems through graphs or diagrams, while symbolic representation involved translating real-life situations into mathematical expressions. The following is the scoring rubric for mathematical representation skills.

Score	Verbal	Visual	Expression
0	No answer, or if there is, it only shows misunderstanding of the concept so the information provided is meaningless		
1	Only a small part of the explanation is correct	Only a small part of the diagram or drawing is correct	Only a small part of the mathematical model is correct
2	The mathematical explanation makes sense but is only partially complete and correct	Draws a diagram or picture, but it is incomplete and correct	Finds the mathematical model correctly, but makes mistakes in obtaining the solution
3	The mathematical explanation makes sense, although not logically structured or contains minor language errors	Draws a diagram or picture completely and correctly	Finds the mathematical model correctly, then performs calculations or obtains the solution correctly and completely
4	The mathematical explanation makes sense, is clear, and logically and systematically structured	Draws a diagram or picture completely, correctly, and systematically	Finds the mathematical model correctly, then performs calculations or obtains the solution correctly, completely, and systematically

Table 2: The scoring rubric for mathematical representation ability

An example of the test item is illustrated.

Given a rectangular prism ABCD.EFGH as shown in the figure, the dimensions are as follows: AB = 10 cm, BC = 4 cm, CG = 8 cm, AS = 2 cm, and GM = 3 cm. A small ant starts at point S and moves along the surface of the prism toward a piece of food located at point M. What is the shortest distance the ant must travel from S to M? Draw the route the ant would take to reach the food.

The validity and reliability of the test instrument were evaluated to ensure it met standard criteria. The construct validity of the five test items ranged from 0.49 to 0.86, indicating moderate to high validity. Reliability analysis using Cronbach's alpha yielded a coefficient of 0.75, which suggests that the instrument had acceptable internal consistency. The difficulty level of the items varied from difficult to easy, with values ranging between 0.30 and 0.81. Meanwhile, the item discrimination indices ranged from 0.34 to 0.50, indicating a fair to good ability to distinguish between high- and low-performing students. Overall, these results confirm that the test instrument is both valid and reliable for assessing students' mathematical representation skills.

Data Analysis

This study utilized SPSS 25 to conduct descriptive statistical analyses and examine the correlations among variables. To evaluate the impact of the RME learning model integrated with a scaffolding approach, a one-way ANOVA was employed, along with the N-Gain Multiple Comparison Test to analyze the results in greater detail. To better understand the practical significance of the findings, Cohen’s d was calculated to determine the effect size, indicating the strength of the RME model with scaffolding on students' mathematical representation abilities. Additionally, R software was used to generate visualizations of the relationships between scale variables, offering deeper insight into students’ performance.

The normalized gain (N-Gain): $N\text{-}Gain = \frac{Posttest\ score\ -\ Pretest\ score}{Maximum\ score\ -\ Pretest\ score}$

Based on the resulting scores, N-Gain was categorized as low (N-Gain < 0.3), medium (0.3 ≤ N-Gain < 0.7), or high (N-Gain ≥ 0.7).

RESULTS

Statistical Description

An initial pretest was administered before the intervention to gather baseline data from both the experimental and control groups. Following the implementation of the treatment, a posttest was conducted to assess students' progress. A detailed summary of the test scores is presented in Table 3 below.

Class	Pretest					Posttest				
	n	*M*	SD	Min	Max	*n*	*M*	SD	Min	Max
Experiment 1	16	38.75	17.81	5	75	16	76.40	13.03	55	97.5
Experiment 2	15	38	15.06	15	62.5	15	75	12.46	55	97.5
Experiment 3	19	38.55	11.55	7.5	55	19	75.26	11.42	55	97.5
Control	16	38.59	10.56	12.5	57.5	16	51.40	12.64	32.5	75
Total	66	38.48	13.60	5	75	66	69.69	15.95	32.5	97.5

Table 3: Description of observed Pretest and posttest scores

Table 3 presents the descriptive statistics of pretest and posttest scores for three experimental classes and one control class. In the pretest, all groups had similar average scores, with means around 38, indicating a comparable baseline in mathematical representation skills. However, notable differences emerged in the posttest results. The experimental groups, which received the RME learning model with scaffolding, showed substantial improvements. Experiment 1 increased from a mean of 38.75 to 76.40, Experiment 2 from 38 to 75, and Experiment 3 from 38.55 to 75.26, with corresponding increases in minimum and maximum scores across the board. By contrast, the control group taught through conventional instruction exhibited only a slight improvement, with the mean rising from 38.59 to 51.40 and a narrower distribution of scores. The results show a noticeable difference between the experimental and control groups.

Normality and Homogeneity Data

A normality test was conducted to assess whether the data from the four classes were normally distributed. This analysis is essential to determine the appropriate statistical tests for further data interpretation. The Lilliefors test was employed for this purpose, using a significance level of 5%. The results of the normality test for each class are presented in Table 4 below.

Class	Kolmogorov-smirnov		
	Statistic	*df*	Sig.
Experiment 1	0.154	16	0.200*
Experiment 2	0.116	15	0.200*
Experiment 3	0.176	19	0.125
Control	0.199	16	0.089

Table 4: The normality results data.

Table 4 displays the results of the normality test using the Kolmogorov-Smirnov method for each class. The significance values (Sig.) for all groups are greater than the 0.05 threshold, indicating that the data are normally distributed. Specifically, Experiment 1 and Experiment 2 both yielded a significance value of 0.200, which is the maximum value reported by SPSS, suggesting strong evidence of normality. Experiment 3 had a slightly lower significance value of 0.125, and the control group showed a value of 0.089—both still above the 0.05 level. Overall, these results indicate that the assumption of normality was met for all four groups, supporting the use of parametric statistical tests in subsequent analyses.

The homogeneity test was conducted to determine whether the four classes had similar or different variances. The test was performed using Levene's test for homogeneity of variances, and the results are presented in Table 5. For all conditions—based on the mean, median, adjusted degrees of freedom, and trimmed mean—the significance values (sig.) were all well above the 0.05 threshold. Specifically, the significance values were 0.937, 0.930, 0.930, and 0.928, respectively. These results indicate that there is no significant difference in the variances between the groups, suggesting that the assumption of homogeneity of variances was met for the data.

Data	Levene Statistic	*df1*	*df2*	sig.
Based on Mean	0.138	3	62	0.937
Based on Median	0.148	3	62	0.930
Based on Median and with adjusted *df*	0.148	3	55	0.930
Based on Trimmed mean	0.152	3	62	0.928

Table 5: Homogeneity test results

Hypothesis Analysis

This analysis was conducted following the normality and homogeneity tests, which confirmed that the N-gain data from the four classes were normally distributed and homogeneous. The test was carried out using a one-way ANOVA, and the results are presented in Table 6. The ANOVA test showed a significant difference between the groups, with an F-value of 17.442 and a significance value (Sig.) of 0.000, which is below the 0.05 threshold. This indicates that there is a statistically significant difference in the N-gain scores between the four groups. The sum of squares between groups was 1.901, and within groups, it was 2.252. The results suggest that the treatment applied in the experimental groups had a meaningful effect on the students' performance compared to the control group.

Data	Sum of Squares	*df*	Mean Square	F	Sig.
Between Groups	1.901	3	0.634	17.442	0.000
Within Groups	2.252	62	0.036		
Total	4.153	65			

Table 6: One-way Anova test result data

Multiple Comparison Data

The analysis was conducted following the results of the one-way ANOVA, which indicated a significant difference between the groups. To further explore these differences, a multiple comparison test was performed using the Bonferroni method in SPSS. The results of the multiple comparison test are presented in Table 7.

Class (I)	Group (J)	Mean Difference (I-J)	S.E.	Sig.	96% CI	
					Lower Bound	Upper Bound
Experiment 1	Experiment 2	0.043	0.068	< 0.001	-0.143	0.229
	Experiment 3	0.044	0.065	1	-0.131	0.221
	Control	0.424*	0.067	< 0.05	0.240	0.607
Experiment 2	Experiment 1	-0.043	0.068	1	-0.229	0.143
	Experiment 3	0.002	0.065	1	-0.177	0.181
	Control	0.381*	0.068	< 0.05	0.194	0.567
Experiment 3	Experiment 1	-0.045	0.064	1	-0.221	0.131
	Experiment 2	-0.002	0.065	1	-0.181	0.177
	Control	0.378*	0.064	< 0.05	0.202	0.555
Control	Experiment 1	-0.424	0.067	< 0.001	-0.607	-0.240
	Experiment 2	-0.381	0.068	< 0.001	-0.567	-0.194
	Experiment 3	-0.379*	0.064	< 0.05	-0.555	-0.202

Table 7: Multiple comparison test result data. *Note:* * = significance level at $p < 0.05$

The table shows the mean differences between the groups. For example, in the comparison between Experiment 1 and the Control group, the mean difference was 0.424, with a significance value of $p < 0.05$, suggesting a significant difference between these two groups. Similar significant differences were found between the Control group and each of the experimental groups (Experiment 1, Experiment 2, and Experiment 3), with all comparisons showing $p < 0.05$. However, no significant differences were found within the experimental groups themselves, as all comparisons within the experimental groups (Experiment 1 vs. Experiment 2, Experiment 1 vs. Experiment 3, etc.) had $p > 0.05$. These results highlight that the RME model with scaffolding had a significant positive effect compared to the control group, but the differences between the experimental groups were not statistically significant.

Students Performance in Mathematical Representation Indicators

Figure 3 presents the final results of students' mathematical representation abilities, measured through three key indicators: verbal, visual, and symbolic expression. A clear upward trend from pretest to posttest is evident across all experimental groups.

In verbal representation, the most substantial gains were recorded, with Experiment 1 showing the highest increase from 38.54 to 93.23 (N-Gain = 0.890), followed closely by Experiment 3 (39.91 to 91.23; N-Gain = 0.854) and Experiment 2 (42.22 to 91.11; N-Gain = 0.846). This suggests a strong impact of the RME-scaffolding approach in developing students' ability to articulate mathematical reasoning verbally.

The visual representation indicator also exhibited a positive trend, though to a lesser degree. Experiment 1 improved from 62.5 to 87.5 (N-Gain = 0.667), Experiment 2 from 70 to 86.67 (N-Gain = 0.556), and Experiment 3 from 84.21 to 89.47 (N-Gain = 0.333), indicating that students were better able to translate problems into graphical or diagrammatic forms after the intervention.

In symbolic expression, while the pretest scores were initially lower across all groups, noticeable improvements were achieved. Experiment 1 increased from 29.69 to 61.88 (N-Gain = 0.458), Experiment 2 from 22.67 to 60.67 (N-Gain = 0.491), and Experiment 3 from 19.47 to 60.00 (N-Gain = 0.503), suggesting moderate gains in students' ability to convert real-world problems into mathematical symbols.

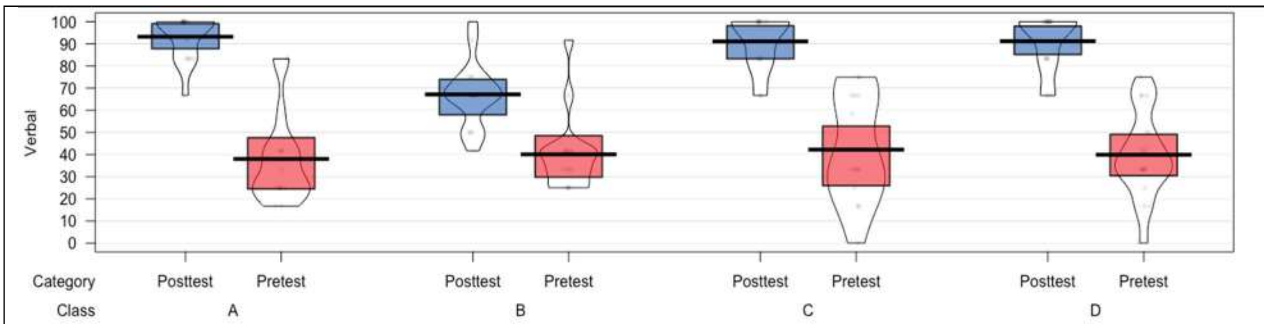


Figure 3a: Verbal

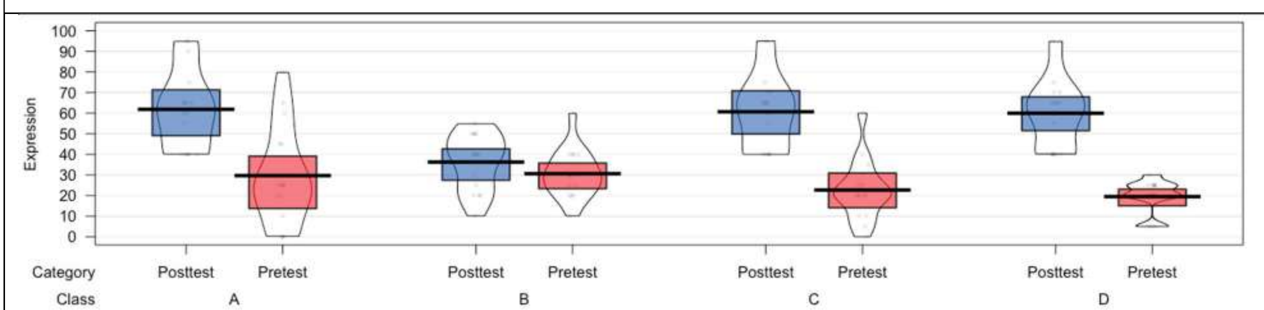


Figure 3b: Expression

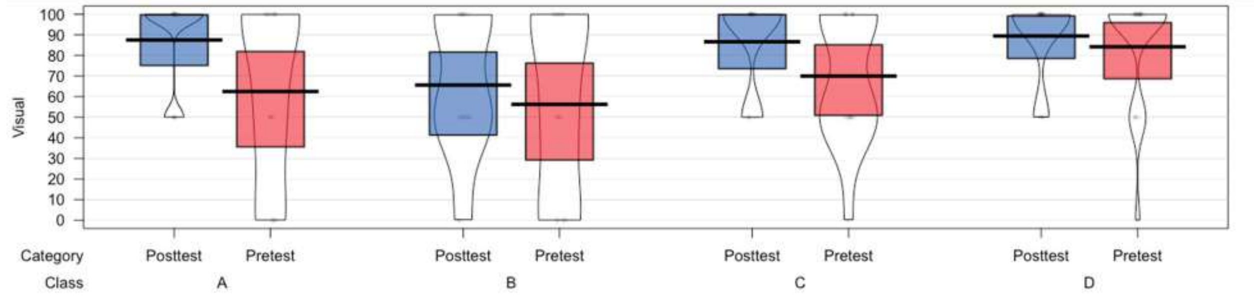


Figure 3c: Visual

Figure 3: Visualization of the students' performance (R software output)

In contrast, the control group showed limited progress. Verbal representation increased modestly from 40.10 to 67.19 (N-Gain = 0.452), visual representation saw a small rise from 56.25 to 65.63 (N-Gain = 0.214), and symbolic expression remained unchanged at 36.25 (N-Gain = 0.081). Overall, the data show a strong positive trend in posttest performance for the experimental groups, highlighting the effectiveness of the RME-scaffolding model in enhancing students' mathematical representation skills across all indicators. The students' answer is presented in Figure 4 and Figure 5.

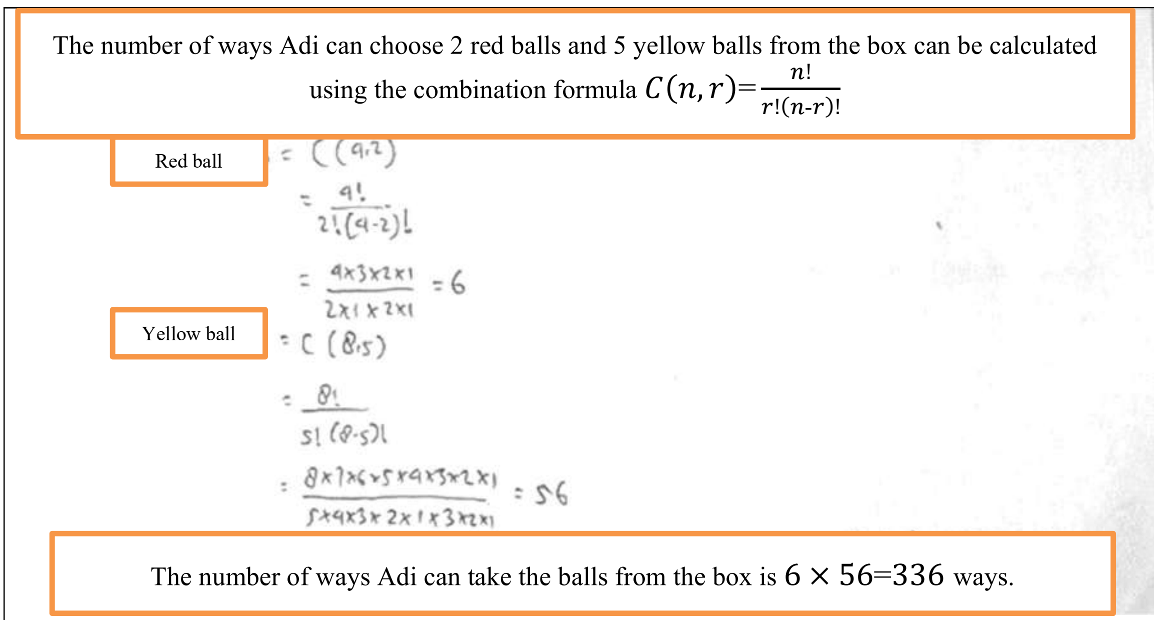


Figure 4a: Verbal Representation–Experimental Class

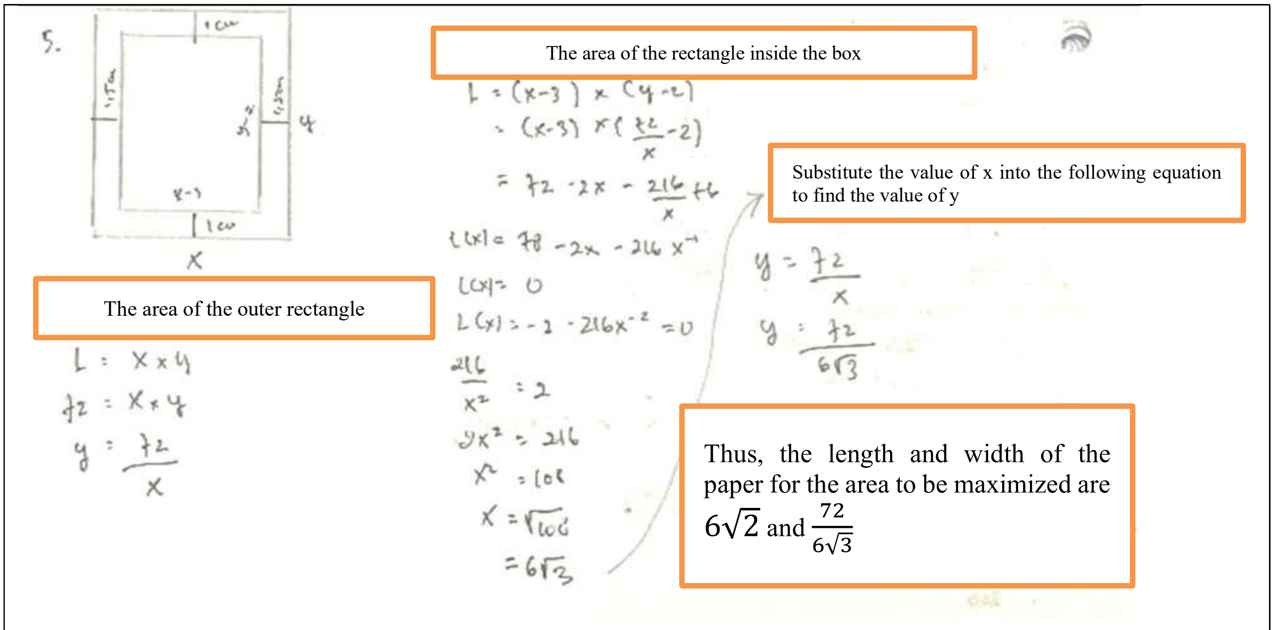


Figure 4b: Visual Representation–Experimental Class

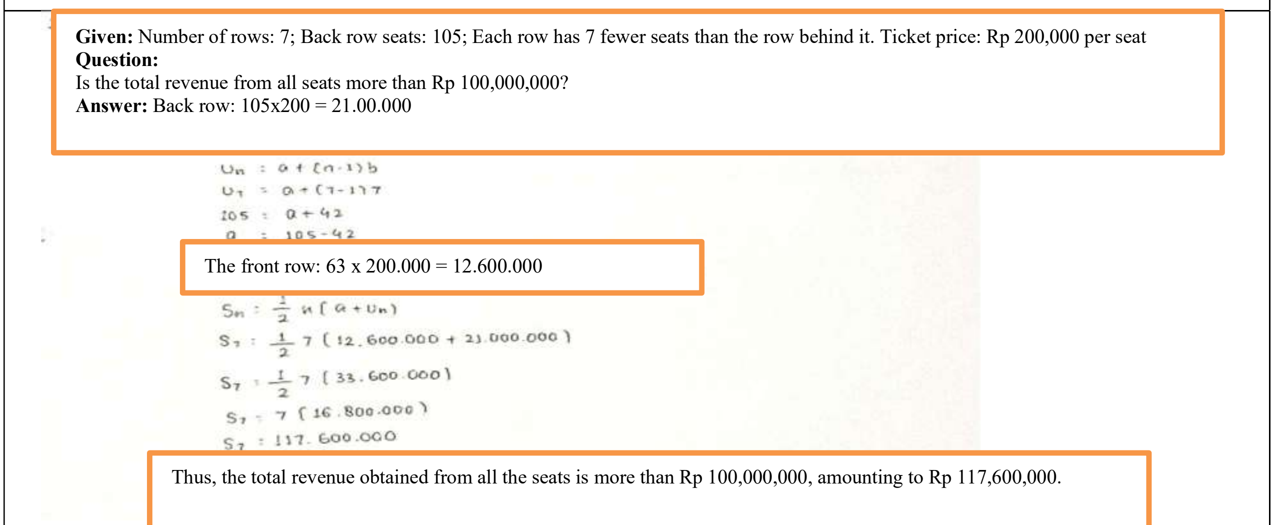


Figure 4c: Expressive Representation–Experimental Class

Figure 4: Students' Responses in Mathematical Representations Categories in experiment class (students answer)

Figure 4a, 4b, and 4c illustrate students in the experimental class engaging with verbal, visual, and expressive representation tasks, respectively. In Figure 4a, students articulate mathematical ideas through discussion, demonstrating their ability to translate abstract concepts into verbal explanations. Figure 4b shows students using diagrams and models to solve problems, reflecting the use of visual tools to enhance comprehension. Figure 4c highlights students creating expressive representations, such as written solutions, sketches, or symbolic formulations, to communicate their understanding.

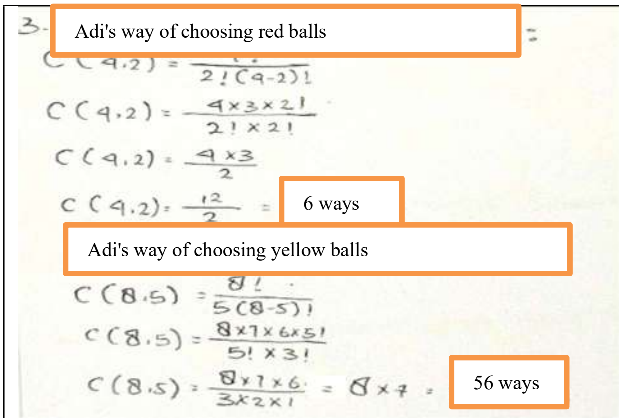


Figure 5a: Visual Representation–Control Class

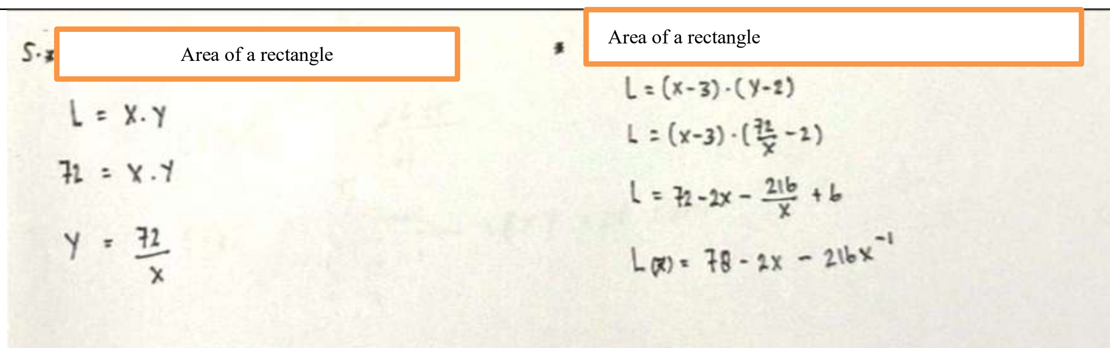
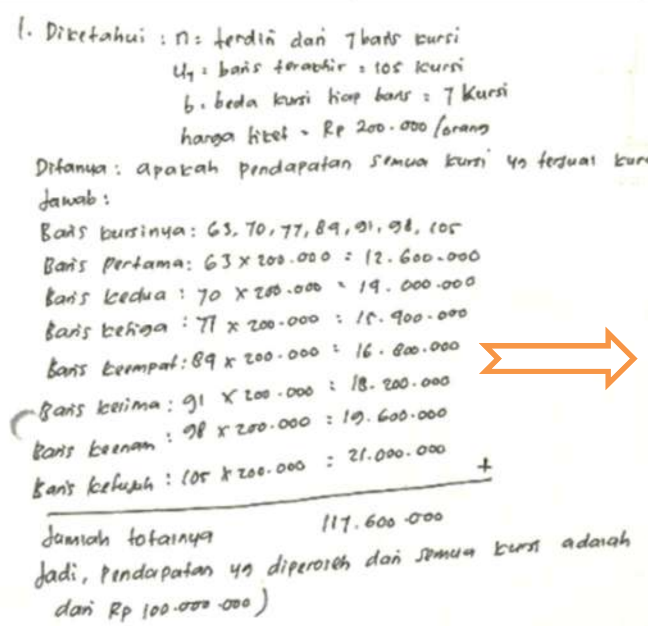


Figure 5b: Visual Representation–Control Class



1. Diketahui : n: terdiri dari 7 baris kursi
U1 : baris terakhir : 105 kursi
b. beda kursi tiap baris : 7 kursi
harga tiket - Rp 200.000 /orang
Ditanya : apakah pendapatan semua kursi yg terjual kur
jawab :
Baris kursinya: 63, 70, 77, 84, 91, 98, 105
Baris pertama: 63 x 200.000 : 12.600.000
Baris kedua : 70 x 200.000 : 14.000.000
Baris ketiga : 77 x 200.000 : 15.400.000
Baris keempat: 84 x 200.000 : 16.800.000
Baris kelima : 91 x 200.000 : 18.200.000
Baris keenam : 98 x 200.000 : 19.600.000
Baris ketujuh : 105 x 200.000 : 21.000.000 +
Jumlah totalnya 117.600.000
Jadi, pendapatan yg diperoleh dari semua kursi adalah
dari Rp 100.000.000)

Given:
- n consists of 7 rows of seats
- U_1 = number of seats in the first row = 105 seats
- The difference in the number of seats between each row = 7 seats
- Ticket price = Rp 200,000 (per seat)

Question:
Is the revenue from all seats less than Rp 100,000,000?

Answer:
Number of seats per rows: 63, 70, 77, 84, 91, 98, 105
- First row: 63 × 200,000 = 12,600,000
- Second row: 70 × 200,000 = 14,000,000
- Third row: 77 × 200,000 = 15,400,000
- Fourth row: 84 × 200,000 = 16,800,000
- Fifth row: 91 × 200,000 = 18,200,000
- Sixth row: 98 × 200,000 = 19,600,000
- Seventh row: 105 × 200,000 = 21,000,000

Total revenue = 117,600,000
Thus, the total revenue from all the seats is Rp 117,600,000; which is more than Rp 100,000,000

Figure 5c: Expressive Representation–Control Class

Figure 5: Students' Responses in Mathematical Representations Categories in control class (students answer)

Figure 5 presents the representation performance of the control class across different tasks. In the control class, students' answers reflected variations in how they represented their understanding. For the visual representation–control class, learners tended to use the mathematical symbol as a representation of visual. In the second visual representation–control class, responses were also delivered through mathematical symbol, but the ideas appeared to be more structured and clearer and sometimes incomplete. On the other hand, in the expressive representation–control class, students relied more on mathematical symbol or verbal explanations rather than visual forms, which highlighted their ability to describe concepts but often without strong visual support.

DISCUSSIONS

The integration of RME with scaffolding has been shown to be an effective instructional approach for enhancing students' mathematical representation skills. This approach situates mathematical learning in real-world contexts (Vos, 2018), helping students relate abstract concepts to everyday experiences (Farida et al., 2023). Scaffolding, in this context, provides graduated support that enables students to gradually develop their reasoning and mathematical argumentation skills (Belland et al., 2008; Khairunnisak et al., 2021). Initially guided by lecturers, students learn to recognize patterns, formulate problem-solving strategies, and draw logical conclusions independently over time. The instructional design includes structured scaffolding through contextual problems, reflective questioning, collaborative learning, and exploration of multiple representations (Belland et al., 2011). This structure facilitates deeper conceptual understanding and encourages the application of knowledge across diverse contexts. Students in the experimental groups exhibited superior abilities in articulating mathematical ideas, converting visual data into mathematical expressions, and generalizing patterns when compared to their peers in the control group.

In the experimental class, students engaged directly with real-world problems using RME combined with scaffolding. Lecturers provided step-by-step guidance, posed reflective questions, and facilitated group discussions, allowing students to progressively take ownership of their learning and represent mathematical concepts through visual, symbolic, and written forms. In contrast, the control class followed a conventional teaching approach, relying mainly on lectures and routine exercises. Students in the control group tended to focus on memorizing procedures and solving problems mechanically, with limited opportunities to explore the relationships between concepts or use multiple forms of representation. As a result, students in the experimental class demonstrated greater flexibility in thinking, problem-solving, and expressing mathematical ideas compared to their peers in the control group.

The effectiveness of this approach is statistically supported by the results of a one-way ANOVA conducted on the N-Gain scores of the posttest data, which revealed a significant difference between the experimental and control groups. The Bonferroni multiple comparison test further confirmed that students in all three experimental classes outperformed those in the control group. Notably, more than 80% of students in the experimental groups achieved scores above the minimum threshold of 56, a clear indication of learning gains not observed in the control group. These findings affirm the advantage of using an RME-based scaffolding model over conventional teaching strategies.

Visual representation, a critical component of mathematical understanding, also showed notable differences. Experimental group students consistently created accurate and contextually relevant diagrams, graphs, or schematics that complemented their problem-solving processes. The control group, however, displayed limited use of visuals, and when attempted, the representations were often disconnected from the problem context, reducing their effectiveness in communicating understanding.

Similarly, in terms of symbolic or mathematical expression, students exposed to the RME-scaffolding model demonstrated a higher degree of precision and fluency in using mathematical symbols and equations. Their expressions were clearly linked to conceptual understanding and problem context. In contrast, control group students used symbols inconsistently and often failed to connect them meaningfully to underlying concepts, reflecting a more superficial understanding.

The RME approach with scaffolding fosters the gradual development of mathematical reasoning, moving students from basic comprehension to autonomous application of concepts. As teacher support is reduced, students begin to demonstrate more complex and independent uses of mathematical representation, particularly in visual forms (Gravemeijer, 1994). This transition highlights the effectiveness of the approach in promoting self-regulated learning and higher-order thinking.

The contribution of this study extends beyond empirical results, offering theoretical insights into mathematics education. It validates the integration of RME and scaffolding as a powerful model for teaching representation skills and supports the constructivist paradigm, which holds that learners construct knowledge through meaningful experiences (De Lange, 1995). By embedding mathematical concepts within real-life contexts, students not only build deeper theoretical understanding but also develop practical competencies for real-world problem-solving. Moreover, the study aligns with Vygotsky's theory of the ZPD, illustrating the role of scaffolding in advancing students' learning beyond their current level of competence. As students receive tailored support and feedback, they are enabled to perform tasks that would otherwise be beyond their reach (Suherman & Vidakovich, 2022). The gradual removal of support leads to the internalization of concepts, resulting in more autonomous and sophisticated mathematical thinking (Anggoro et al., 2024; Suherman & Vidákovich, 2024; Supriadi et al., 2024; Zakaria et al., 2024).

The integration of RME and scaffolding presents a robust pedagogical model for enhancing mathematical representation in higher education. It effectively supports students' development across verbal, visual, and symbolic domains, enabling them to construct, apply, and generalize mathematical ideas with confidence and depth. This approach contributes meaningfully to the refinement of mathematics education theory and practice, advocating for instructional designs that are both contextually rich and cognitively supportive.

CONCLUSIONS

The findings of this study indicate that integrating Realistic Mathematics Education (RME) with scaffolding effectively enhances students' mathematical representation skills, with consistent outcomes across classes underscoring its reliability. This approach supports students in connecting abstract concepts to real-world contexts, fostering deeper understanding compared to conventional methods. However, the study's limited sample, single-institution scope, and short intervention duration restrict the generalizability of the results. Future research should extend this work by exploring long-term effects, application across diverse contexts, and its impact on broader mathematical competencies such as problem-solving and reasoning.

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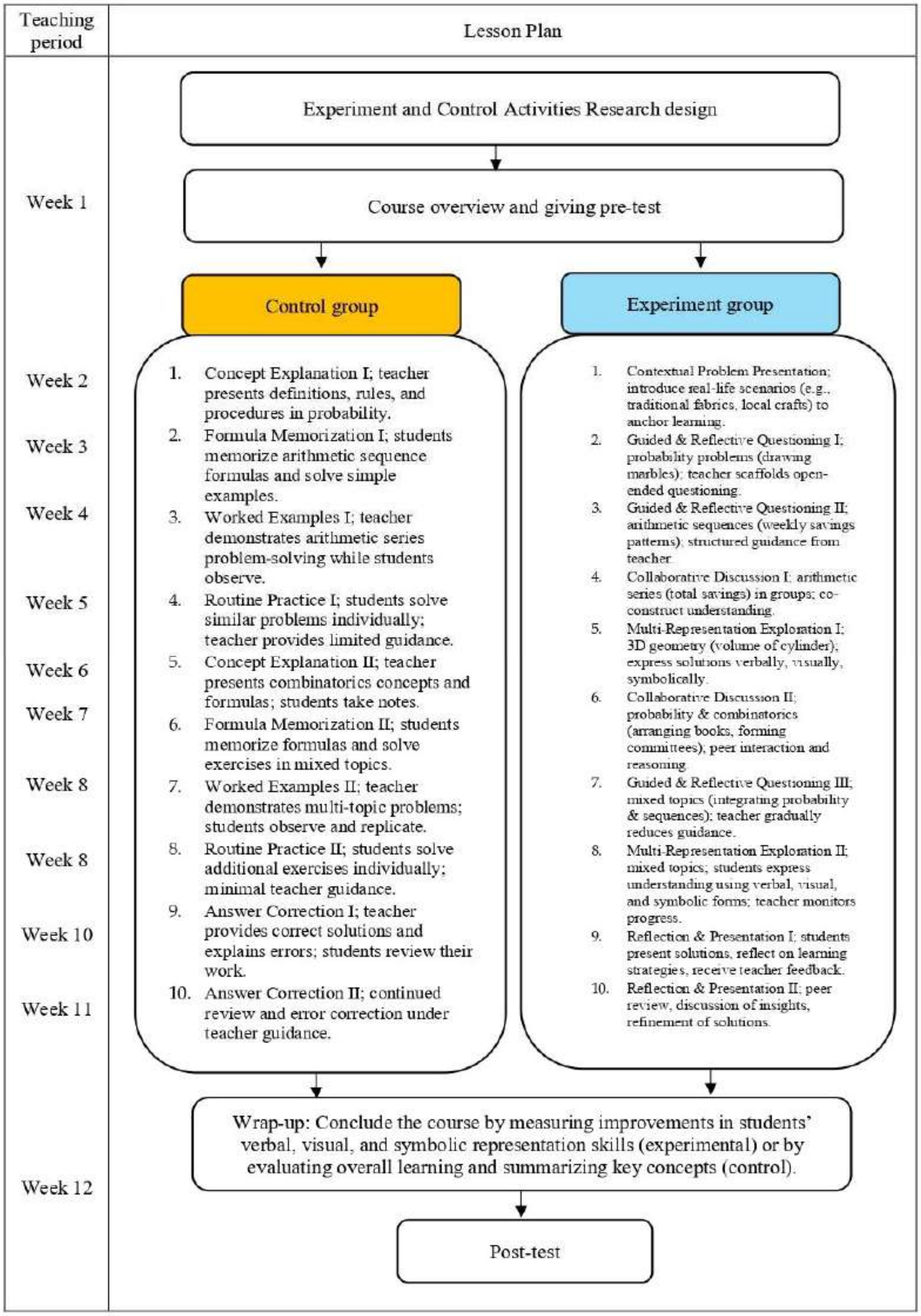
APPENDIX A

Syntax teaching



APPENDIX B

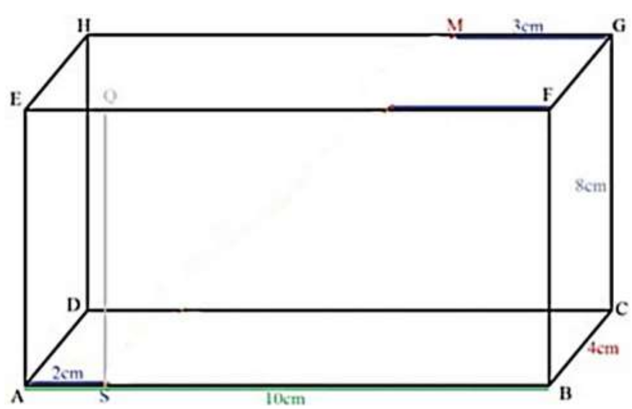
Mathematical Representation Skills Pretest

1. At the premiere screening of a film, the cinema provides a room for the audience that is divided into 6 rows of seats. The back row consists of 90 seats, and each row toward the front has 6 fewer seats than the row behind it. The ticket price for each seat is Rp100.000,00 and all tickets are sold out. If the total revenue from the front row is Rp6.000.000,00 determine whether the total revenue obtained from all the rows in the cinema is less than Rp50.000.000,00. Provide reasons and calculations!
2. In a survey, 200 adults were asked about the number of books they read in a year. The results of the survey are as follows:

Number	Time Spent Reading Books	Number of People
1	0–1	30
2	2–3	50
3	4–5	60
4	6–7	40
5	8–9	20

Determine the average number of books read per person!

3. In a soccer team, there are 11 players consisting of 3 goalkeepers and 8 field players. How many ways can the coach select 2 goalkeepers and 4 field players for the next match?
4. A cuboid ABCD.EFGH is shown in the figure below.



The cuboid has dimensions AB = 10 cm, BC = 4 cm, CG = 8 cm, AS = 2 cm, GM = 3 cm. If an ant walks on the surface of the cuboid from point S to food located at point M, what is the shortest distance from the ant (S) to the food (M)? Draw the route from the ant to the food!

5. A sheet of HVS paper has an area of 54 cm². Sukardi will use the paper to type an invitation letter. If the typing margins are 1 cm for the top and bottom, and 1.5 cm for the left and right sides, what is the maximum length and width of the typing area? Illustrate your answer!

APPENDIX C

Mathematical Representation Skills Posttest

1. At a music concert, the committee provided a room for the audience which was divided into 7 rows of seats. The back row consisted of 105 seats and each row decreased by 7 seats until the front row. The ticket price for each seat was Rp200,000.00 and all seats were sold out. If the total money obtained from the front row was Rp12,600,000.00, was the total revenue from all seats in the concert hall less than Rp100,000,000.00? Provide reasons along with the calculation!

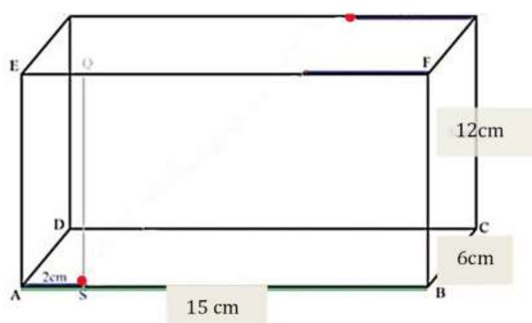
2. In a survey, 200 adults were asked about how many hours per day they use their mobile phones. The results are as follows:

No.	Time (hours)	Number of People
1	0–1	15
2	2–3	20
3	4–5	35
4	6–7	60
5	8–9	70

   Determine the average daily duration of mobile phone usage!

3. In a box, there are 12 balls consisting of 4 red balls and 8 yellow balls. How many ways can Adi choose 2 red balls and 5 yellow balls?

4. Given a cuboid ABCD.EFGH as shown in the figure:



   The cuboid has dimensions AB = 15 cm, BC = 6 cm, CG = 12 cm, AS = 2 cm, GM = 3 cm. If an ant walks on the surface of the cuboid from point S to the food located at point M, what is the shortest distance from the ant's starting point S to the food at M? Illustrate the ant's path to the food!

5. A sheet of HVS paper has an area of 72 cm². Doni will use this paper to type an invitation letter. If the typing margin is 1 cm for the top and bottom, and 1.5 cm for the left and right sides, what are the maximum length and width of the typing area? Illustrate your answer!