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Application of Adams-Bashforth-Moulton Method for Predict Population Growth in Banten Province with Logistic Equation

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ABSTRACT

In this study, the Logistic equation is used to solve a non-linear GDP. The Adams Ashforth-Moulton method is a multi-step numerical method consisting of Adams Ashforth as a predictor and Adams-Moulton as a corrector. To obtain four initial solutions, the Runge-Kutta method is used first before proceeding to use the Adams-Bashforth-Moulton method. By using the Adams-Bashforth-Moulton method, it aims to get the prediction results of population growth in Banten Province. Based on the calculation results, a numerical solution of the logistic equation for population growth at the time $t = 2030$, with a step size of $h = 1$, the capacity of Banten Province is $20,000,000$, and the growth rate $is\, m = 0.034$.

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INTRODUCTION

Population growth is an important aspect that affects various sectors of life, including economic, social, and environmental. The fairly high population growth rate in Indonesia and in several other provinces has resulted in increased population density. Especially in Banten Province, reported by the Central Bureau of Statistics (BPS) that the growth rate in Banten Province is 1.16% with a population of 12,431.39 which is ranked as the 5th most populous province in Indonesia. There are many challenges arising from rapid population growth, such as the need for adequate infrastructure, education, and health services. Therefore, accurate population growth prediction is essential for effective planning and decision-making.

One mathematical model that is often used to predict population growth is the logistic equation. The logistic equation is a differential equation model that describes the relationship between population change, current population, growth rate, and carrying capacity (Allen 2008). According to him, this equation helps explain how the population grows exponentially at first, but then slows down and reaches equilibrium when it approaches the carrying capacity (May 1976).

To solve the logistic equation numerically can be with the help of the Adams-Bashforth-Moulton method. Based on previous research conducted by Dewi, Wasono, and Huda (2022), namely applying the Adams-Bashforth-Moulton method with logistic equations to predict population growth in East Kalimantan Province. The results showed that this method is effective in providing accurate numerical solutions by using the initial step of the Runge-Kutta method.

The Adams-Bashforth-Moulton method has proven effective in solving non-linear differential equations, including logistic equations. This method is a multi-step approach that combines AdamsBashforth as a predictor and Adams-Moulton as a corrector. By applying this method, population growth prediction can be done more accurately and efficiently. This study aims to apply the Adams-Bashforth-Moulton method in solving logistic equations to predict population growth in Banten Province.

Banten Province, which is experiencing rapid population growth, needs a model to accurately predict its demographic development. The use of the Adams-Bashforth-Moulton method in projecting population growth in this area is expected to provide a clearer insight into future population trends. The results of this prediction will be very useful for local governments and policy makers in planning programs related to development, health, education, and infrastructure.

Thus, this study aims to apply the Adams-Bashforth-Moulton method in predicting population growth in Banten Province using the logistic equation, and analyse the effectiveness and accuracy of this method compared to other prediction methods.

LITERATURE REVIEW

Ordinary Differential Equation

An ordinary differential equation is an equation that contains one or more derivatives of an unknown function, called $y(x)$ or sometimes $y(t)$ (if the independent variable is in time t) where this equation may also contain y itself, the known function x (or t) and a constant (Kreyszig 2011). The general form of an ordinary differential equation is:

$$\frac{dy}{dx} = f(x, y)$$

Logistic Model of Population Growth

The logistic growth model (Verhulst Model) is a population growth model that reflects the influence of intraspecific competition (Rosyanti et al. 2022). In this method Pierre Verhulst emphasized that in this case it is assumed that the population cannot grow indefinitely where there is a maximum population size, K that can be accommodated (carrying capacity) properly in a sustainable manner by its environment. Here is the logistic model equation:

$$\frac{dP}{dt} = m\left(1 - \frac{P}{K}\right)P$$

m : Population Growth Rate
P : Population
K : Regional Capacity

Numerical Methods

Numerical methods are used to formulate mathematical problems so that they can be solved by ordinary calculation and arithmetic operations, such as add, subtract, multiply, and divide. Literally, “numerical method” means a way of counting using numbers because the word “numerical” refers to numbers. These complex mathematical models sometimes cannot be solved by common analytical methods to obtain their true solutions (Munir 2006).

The Runge-Kutta Method

The Runge-Kutta method is an alternative to the Taylor method which does not require derivative calculations. This method aims to obtain a high degree of accuracy while avoiding the need to find higher derivatives by evaluating the function $f(x, y)$ at a particular point in time by evaluating the function $f(x, y)$ at a certain point on a part interval (Nugroho 2009). The fourth-order Runge-Kutta method is also used as a preliminary to obtain the initial value that will later be needed in the fourth-order Adams-Bashforth-Moulton method. Here is the form of the fourth-order Runge Kutta method (Triatmodjo, 2002).

$$k_1 = hf(x_r, y_r)$$
$$k_2 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_1)$$
$$k_2 = hf\left(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_2\right)$$
$$k_4 = hf(x_r + h, y_r + k_3)$$

$$y_{r+1} = y_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Error

The value that differs between the actual value and that created using numerical methods is known as "error" in numerical methods error. Numerical methods usually do not prioritize achieving the exact answer to the problem. The error shows how close the approximation or solution is to the true solution because the approximation solution usually appears as a result of the solution. Numerical solution is more precise with a smaller error rate.

Adams-Bashforth-Moulton Method

The Adams-Bashforth-moulton method is a method that belongs to the multi-step method. There are two methods for solving ordinary differential equations: one-step method and multi-step method. The first method, which includes Milne's method and Hamming's method, uses the Adams-Bashforth-Moulton method, requiring an initial solution obtained from one-step methods (such as Euler's method, Taylor series method, and Runge-Kutta method). Instead of finding the derivatives of the functions, the Adams-Bashforth-Moulton method can be used by directly using the predictor equation. Because the error cut is smaller than the Adams-Bashforth-Moulton method of order 2 and 3, the fourth-order Adams-Bashforth-Moulton method is accurate enough to solve the initial value problem of non-linear ordinary differential equations (Djojodihardjo 2000).

In this case, the Adams method used is the 4th order AdamsBashfort method as a predictor in the first step and as a corrector in the second step (Kosasih, 2006). Two equations are used in the predictor-corrector method for y_{r+1}. The first equation is known as the predictor to predict (obtain the first approximation) y_{r+1}, and the second equation is known as the corrector to obtain the corrected value (obtain the second approximation) y_{n+1}. he predictor equation of the fourth-order AdamsBashforth-Moulton method is:

$$y_{r+1} = y_r + \frac{h}{24}(55f_r - 59f_{r-1} + 37f_{r-2} - 9f_{r-3})$$

The corrector equation of the fourth-order AdamsBashforth-Moulton method is (Erwin 1999).

$$y_{r+1} = y_r + \frac{h}{24}(f_{r-2} - 5f_{r-1} + 19f_r - 9f_{r+1}^{(0)})$$

Step Size Control (h)

The fourth-order Adams-Bashforth method and the fourth-order Adams-Moulton method can be evaluated using the truncation error to determine or estimate the step size. If h is too small, the rounding error increases and the number of steps increases; if h is too large, the truncation error also increases because the truncation error is proportional to h (Munir 2006). If the step size h is chosen correctly, the numerical solution will be obtained with a small number of iterations.

Population Growth in the Province of Banten

According to Statistics Indonesia (BPS) Banten Province in 2023 the area is 9,352, 77 km2 with a population of 12,307, 73 million people. From 2018 to 2021, the population of Banten Province increased every year. From 2018 to 2019, the population increased by 184,098 people, from 2019 to 2020, the population increased by 136,051 people, and from 2020 to 2021, the population increased by 988,369 people. This shows that in 2018 every 100 (one hundred) people of Banten increased by 1.14 people due to births and migration.

METHODOLOGY

This research uses a literature study method that focuses on books found at the University of Lampung library, the reading room of the mathematics department of the Faculty of Mathematics and Sciences, University of Lampung, or public libraries and domestic or foreign journals that support the research conducted.

RESULT AND DISCUSSION

Data Processing of Total Population in Banten Province

The data used in this study is the population data of Banten Province obtained by downloading from the internet on the official website of the Central Statistics Agency (BPS) of Banten Province in 2010-2019. The data obtained can be seen in table 1.

Tabel 1. Provincial Population Data 2010−2019

Number	Year	Census Result
1	2010	10.632.166
2	2011	11.005.518
3	2012	11.248.947
4	2013	11.452.491
5	2014	11.704.877
6	2015	11.955.243
7	2016	12.203.148
8	2017	12.448.160
9	2018	12.689.736
10	2019	12.927.316

Based from Table 1 data on the population of Banten Province reported by the Banten Provincial Statistics Agency (BPS) has increased every year with a variety of people, then obtained population growth data in 2014-2019 as follows:

Tabel 2. Population Growth Data from 2010—2019

Number	Year	Census Result
1	2010—2011	373.352
2	2011—2012	243.429
3	2012—2013	203.544
4	2013—2014	252.386
5	2014—2015	250.366
6	2015—2016	247.905
7	2016—2017	245.012
8	2017—2018	241.576
9	2018—2019	237.580

Judging from Table 2, it can be seen that the population of Banten Province increases on average every year by 255,017 people. Researchers want to use the Adams-Bashforth-Moulton logistic equation to predict population growth in the following year.

Logistic Equation

The logistic equation that will be used in predicting population growth in Banten Province in the coming year with the Adams-Bashforth-Moulton method is as follows:

$$\frac{dP}{dt} = m\left(1 - \frac{P}{K}\right)P$$

m : Population Growth Rate
P : Population
K : Regional Capacity

Furthermore, to determine the population growth rate in Banten Province, the following formula was used:

$$m = \frac{1}{t} ln\left(\frac{P(t)}{P_0}\right)$$

$$m = \frac{1}{1} ln\left(\frac{11.005.518}{10.632.166}\right) = 0{,}034$$

Based on the value of m = 0.034, the population growth rate is 3,4%. The next step is to determine the capacity of the region, assuming $K = 20{,}000{,}000\ km^2$ /soul.

After obtaining the growth rate from equation which is 3,4% and the capacity of Banten Province $K = 20{,}000{,}000\ km^2$ /soul with $P_0 = 10.632.166$, population growth for the next 20 years will be predicted, so the interval $[0, 20]$ is used. Next, determine the step size with many iterations $n = 20$ with the following formula:

$$h = \frac{b - a}{n}$$

$$h = \frac{20 - 0}{20} = 1$$

The step size h = 1 is obtained from the calculation of the formula above. Next, substitute the values that have been obtained into the initial logistic equation above to get:

$$\frac{dP}{dt} = m\left(1 - \frac{P}{K}\right)P$$

$$\frac{dP}{dt} = 0{,}034\left(1 - \frac{P}{20.000.000}\right)P$$

A logistic equation of population growth in Banten Province was produced.

Determination of Four Initial Value Solutions with 4th Order Runge-Kutta

Calculating four initial value solutions P_0, P_1, P_2, P_3 by the fourth-order RungeKutta method on the interval [0,20], initial value $P_0 = 10.632.166$ and with step size $h = 1$.

For $r = 0$, $P_0 = 10.632.166$ the values of k_1, k_2, k_3, and k_4 are calculated to get the initial solution value of P_1:

$$k_1 = h.f(t_r, P_r)$$
$$k_1 = h.f(t_0, P_0) = h.f(0; 10.632.166)$$
$$k_1 = 1\left[0{,}034\left(1 - \frac{10.632.166}{20.000.000}\right)10.632.166\right] = 169.320{,}6225$$
$$k_2 = h.f\left(t_r + \frac{1}{2}h; P_r + \frac{1}{2}k_1\right)$$
$$k_2 = h.f\left(t_0 + \frac{1}{2}h; P_0 + \frac{1}{2}k_1\right)$$
$$k_2 = h.f\left(0 + \frac{1}{2}; 10.632.166 + \frac{1}{2}169.320{,}6225\right)$$
$$k_2 = h.f(0{,}5; 10.716.826{,}31125)$$
$$k_2 = 1\left[0{,}034\left(1 - \frac{10.716.826{,}31125}{20.000.000}\right)10.716.826{,}31125\right]$$
$$k_2 = 169.126,4721$$
$$k_3 = h.f\left(t_r + \frac{1}{2}h; P_r + \frac{1}{2}k_2\right)$$
$$k_3 = h.f\left(t_0 + \frac{1}{2}h; P_0 + \frac{1}{2}k_2\right)$$
$$k_3 = h.f\left(0 + \frac{1}{2}; 10.632.166 + \frac{1}{2}169.126,4721\right)$$
$$k_3 = h.f(0{,}5; 10.716.729,23605)$$
$$k_3 = 1\left[0{,}034\left(1 - \frac{10.716.729{,}23605}{20.000.000}\right)10.716.729,23605\right]$$
$$k_3 = 169.126,7086$$
$$k_4 = h.f(t_r + h, P_r + k_3)$$

$$k_4 = h.f(t_0 + h, P_0 + k_3) = 1(0 + 1;\ 10.632.166 + 169.126,7086)$$
$$k_4 = 1(0,5;\ 10.801.292,7086) = 168.908,4810$$

Next, substitute it into the fourth-order Runge-Kutta equation

$$P_{r+1} = P_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$P_1 = P_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$P_1 = 10.632.166 + \frac{1}{6}(169.320,6225 + 2(169.126,4721)$$
$$+2(169.126,7086) + 168.908,4810)$$
$$P_1 = 10.801.288{,}58$$

The value of $P_1 = 10.801.288{,}58$ is obtained.

For $r = 1$, $P_1 = 10{,}801{,}288.58$ the values of k_1, k_2, k_3, and k_4 are calculated to get the initial solution value of P_2:

$$t_{r+1} = t_r + h$$
$$t_1 = t_0 + h$$
$$t_1 = 0 + 1 = 1$$

$$k_1 = h.f(t_r, P_r)$$
$$k_1 = h.f(t_1, P_1) = h.f(0; 10.801.288,58)$$
$$k_1 = 1\left[0{,}034\left(1 - \frac{10.801.288{,}58}{20.000.000}\right)10.801.288,58\right] = 168.908,4922$$
$$k_2 = h.f\left(t_r + \frac{1}{2}h; P_r + \frac{1}{2}k_1\right)$$
$$k_2 = h.f\left(t_1 + \frac{1}{2}h; P_1 + \frac{1}{2}k_1\right)$$
$$k_2 = h.f\left(1 + \frac{1}{2}; 10.801.288,58 + \frac{1}{2}168.908,4922\right)$$
$$k_2 = h.f(1{,}5; 10.885.742,8261)$$
$$k_2 = 1\left[0{,}034\left(1 - \frac{10.885.742{,}8261}{20.000.000}\right)10.885.742,8261\right]$$
$$k_2 = 168.666,2814$$
$$k_3 = h.f\left(t_r + \frac{1}{2}h; P_r + \frac{1}{2}k_2\right)$$
$$k_3 = h.f\left(t_1 + \frac{1}{2}h; P_1 + \frac{1}{2}k_2\right)$$
$$k_3 = h.f\left(1 + \frac{1}{2}; 10.801.288,58 + \frac{1}{2}168.666,2814\right)$$
$$k_3 = h.f(1{,}5;\ 10.885.621,7205)$$
$$k_3 = 1\left[0{,}034\left(1 - \frac{10.885.621{,}7205}{20.000.000}\right)10.885.621,7205\right]$$
$$k_3 = 168.666,6461$$
$$k_4 = h.f(t_r + h, P_r + k_3)$$
$$k_4 = h.f(t_1 + h, P_1 + k_3) = 1(1 + 1;\ 10.801.288,58 + 168.666,6461)$$
$$k_4 = 1(2;\ 10.969.955,2261) = 168.400,6177$$

Next, substitute it into the fourth-order Runge-Kutta equation

$$P_{r+1} = P_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$P_2 = P_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$P_2 = 10.801.288,58 + \frac{1}{6}(168.908,4922 + 2(168.666,2814) \\ +2(168.666,6461) + 168.400,6177)$$

$$P_2 = 10.969.951,07$$

The value of $P_2 = 10.969.951,07$ is obtained.

For $r = 2$, $P_2 = 10.969.951,07$ the values of k_1, k_2, k_3, and k_4 are calculated to get the initial solution value of P_3:

$$t_{r+1} = t_r + h$$
$$t_2 = t_1 + h$$
$$t_1 = 1 + 1 = 2$$

$$k_1 = h.f(t_r, P_r)$$
$$k_1 = h.f(t_2, P_2) = h.f(0; 10.969.951,07)$$
$$k_1 = 1\left[0,034\left(1 - \frac{10.969.951,07}{20.000.000}\right)10.969.951,07\right] = 168.400,6314$$
$$k_2 = h.f\left(t_r + \frac{1}{2}h; P_r + \frac{1}{2}k_1\right)$$
$$k_2 = h.f\left(t_2 + \frac{1}{2}h; P_2 + \frac{1}{2}k_1\right)$$
$$k_2 = h.f\left(2 + \frac{1}{2}; 10.969.951,07 + \frac{1}{2}168.400,6314\right)$$
$$k_2 = h.f(2,5; 10.885.742,8261)$$
$$k_2 = 1\left[0,034\left(1 - \frac{11.054.151,3857}{20.000.000}\right)11.054.151,3857\right]$$
$$k_2 = 168.110,9002$$
$$k_3 = h.f\left(t_r + \frac{1}{2}h; P_r + \frac{1}{2}k_2\right)$$
$$k_3 = h.f\left(t_2 + \frac{1}{2}h; P_2 + \frac{1}{2}k_2\right)$$
$$k_3 = h.f\left(2 + \frac{1}{2}; 10.969.951,07 + \frac{1}{2}168.110,9002\right)$$
$$k_3 = h.f(2,5; 11.054.006,5201)$$
$$k_3 = 1\left[0,034\left(1 - \frac{11.054.006,5201}{20.000.000}\right)11.054.006,5201\right]$$
$$k_3 = 168.111,4194$$
$$k_4 = h.f(t_r + h, P_r + k_3)$$
$$k_4 = h.f(t_2 + h, P_2 + k_3) = 1(2 + 1; 10.969.951,07 + 168.111,4194)$$
$$k_4 = 1(3; 11.138.062,4894) = 167.798,1834$$

Next, substitute it into the fourth-order Runge-Kutta equation

$$P_{r+1} = P_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$P_3 = P_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$P_3 = 10.969.951,07 + \frac{1}{6}(168.400,6314 + 2(168.110,9002) \\ +2(168.111,4194) + 167.798,1834)$$

$$P_3 = 11.138.058,31$$

The value of $P_3 = 11.138.058,31$ is obtained.

Determination of Four Initial Value Solutions with 4th Order Runge-Kutta

To find the necessary values, the Adams-BashforthMoulton method requires the Runge-Kutta method as a preliminary method. The Adams-Moulton corrector equation is used to change the value of the Adams-Bashforth predictor equation. The values obtained from the 4th order Runge-Kutta Method are $P_0 = 10.632.166$, $P_1 = 10.801.288{,}58$, $P_2 = 10.969.951{,}07$ dan $P_3 = 11.138.058, 31$, in the interval $[0, 20]$.

Numerical Solution with Adams-Bashforth Method

After obtaining four initial value solutions then calculate the values of $f_r, f_{r-1}, f_{r-2}, f_{r-3}$ with $r = 3,4,\ldots,n$ by substituting in the equation:

$$\frac{dP}{dt} = 0{,}034\left(1 - \frac{P}{20.000.000}\right)P$$

Next, the values obtained are substituted into the Adams-Bashforth equation with a step size of $h = 1$

$$\begin{aligned} f_r = f_3(t_3, P_3) &= f_3(3; 11.138.058{,}31) \\ &= 0{,}034\left(1 - \frac{11.138.058{,}31}{20.000.000}\right) 11.138.058{,}31 \\ &= 167.798{,}1996 \end{aligned}$$

$$\begin{aligned} f_{r-1} = f_2(t_2, P_2) &= f_2(2; 11.138.058{,}31) \\ &= 0{,}034\left(1 - \frac{10.969.951{,}07}{20.000.000}\right) 10.969.951, 07 \\ &= 168.400, 6314 \end{aligned}$$

$$\begin{aligned} f_{r-2} = f_1(t_1, P_1) &= f_1(1; 10.801.288, 58) \\ &= 0{,}034\left(1 - \frac{10.801.288{,}58}{20.000.000}\right) 10.801.288, 58 \\ &= 168.908, 492 \end{aligned}$$

$$\begin{aligned} f_{r-3} = f_0(t_0, P_0) &= f_0(0; 10.632.166) \\ &= 0{,}034\left(1 - \frac{10.632.166}{20.000.000}\right) 10.632.166 \\ &= 169.320, 6224 \end{aligned}$$

Tabel 3. Four Initial Value Solutions by Runge Kutta Method in Logistic Equation

r	t_r	$h = 1$	
		P_r	$P' = f(t, P) = 0{,}034\left(1 - \frac{P}{20.000.000}\right)P$
0	0	10.632.166	$169.320, 6224$
1	1	10.801.288,58	$168.908, 4922$
2	2	10.969.951,07	$168.400, 6314$
3	3	11.138.058,31	$167.798{,}1996$

After the value of $f_r\ (x_r, y_r)$ has been obtained, it is then substituted into the Adams-Bashforth equation.

Untuk $r = 3, P_3 = 11.138.058,31$

$$t_{r+1} = t_r + h$$
$$t_4 = t_3 + h$$
$$t_4 = 3 + 1 = 4$$

$P_{r+1}^{(0)} = P_r + \frac{h}{24}(55f_r - 59f_{r-1} + 37f_{r-2} - 9f_{r-3})$

$P_{3+1}^{(0)} = P_3 + \frac{h}{24}(55f_3 - 59f_2 + 37f_1 - 9f_0)$

$P_4^{(0)} = 11.138.058,31 + \frac{1}{24}(55(167.798,1996) - 59(168.400,6314) + 37(168.908,4922) - 9(169.320,6224))$

$P_4^{(0)} = 11.305.516,33$

Solution of Adams-Moulton Corrector

With the Adams-Moulton corrector, after the value of f_{r+1} is added to the equation, its relative error is calculated and compared with the stopping criterion.

$$f_4^{(0)}\left(t_4, P_4^{(0)}\right) = f_4(4; 11.305.516,32)$$
$$= 0,034\left(1 - \frac{11.305.516,32}{20.000.000}\right) 11.305.516,32$$
$$= 167.102,5661$$

Untuk $r = 3, t_4 = 4,\ P_3 = 11.138.058,31$

$P_{r+1}^{(1)} = P_r + \frac{h}{24}\left(f_{r-2} - 5f_{r-1} - 19f_r + 9f_{f_{r+1}}^{(0)}\right)$

$P_{3+1}^{(1)} = P_3 + \frac{h}{24}\left(f_1 - 5f_2 + 19f_3 + 9f_4^{(0)}\right)$

$P_4^{(1)} = 11.138.058,31 + \frac{1}{24}(168.908,4922 - 5(168.400,6314) + 19(167.798,1996) + 9(167.102,5661))$

$P_4^{(1)} = 11.305.516,41$

The relative error is calculated and then compared with the stopping criterion, $\varepsilon = 9 \times 10^{-9}$

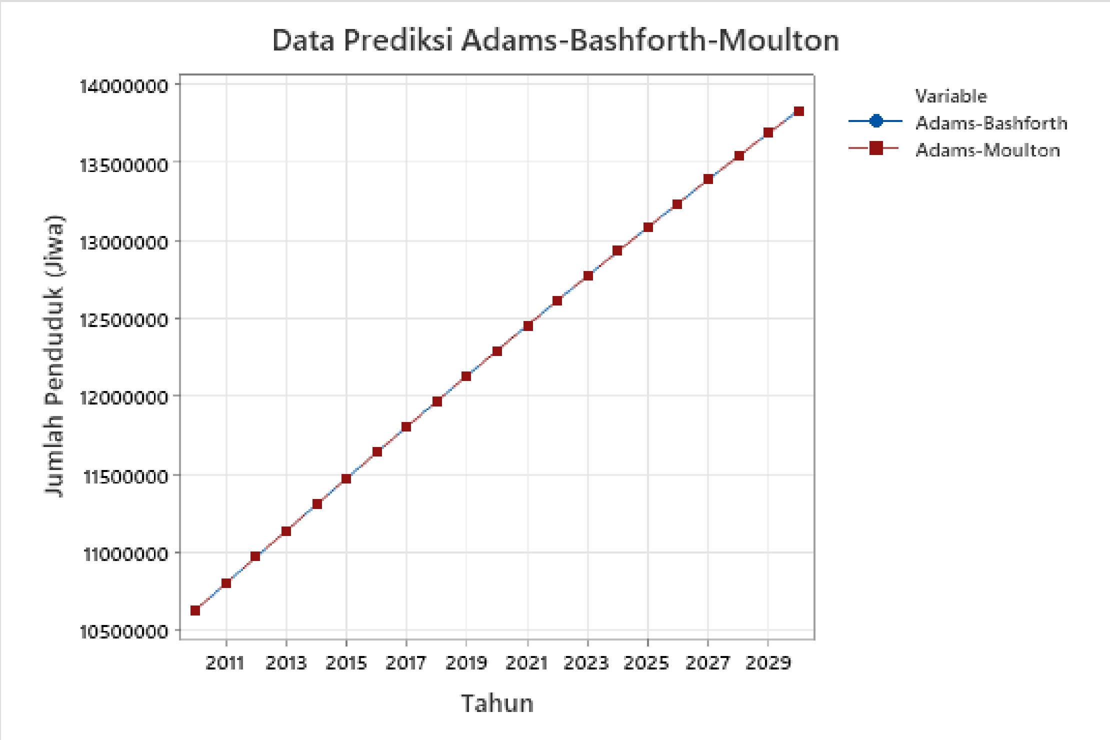

Figure 1. Numerical Solution with Adams-Bashforth-Moulton Method

$$\frac{\left|P_{r+1}^{(1)} - P_{r+1}^{(0)}\right|}{\left|P_{r+1}^{(1)}\right|} = \frac{\left|P_{4}^{(1)} - P_{4}^{(0)}\right|}{\left|P_{4}^{(1)}\right|}$$
$$= \frac{|11.305.516{,}41 - 11.305.516{,}33|}{|11.305.516{,}41|}$$
$$= 7{,}07619 \times 10^{-9}$$

It is then analyzed with the selection criteria of the h-step size:

$$\frac{19}{270} \times \frac{\left|P_{r+1}^{(1)} - P_{r+1}^{(0)}\right|}{\left|P_{r+1}^{(1)}\right|} = \frac{19}{270} \times \frac{|11.305.516{,}41\ -\ 11.305.516{,}33|}{|11.305.516{,}41|}$$
$$= 4{,}97954 \times 10^{-10}$$

Obtained a value of $4{,}97954 \times 10^{-10} < 9 \times 10^{-9}$, then continue the calculation with the steps as before for $r = 4, 5, , \ldots, 20$or until the 20th iteration.

Furthermore, it is displayed in the form of a graph of the Adams-Bashforth-Moulton prediction results.

It can be seen that as the years increase, the population in Banten Province is increasing every year. The following table shows the results of the prediction of the population of Banten Province in 2010—2025:

Table 4. Population prediction for 2010−2025

Number	Year	Total Population
1	2010	10.632.166
2	2011	10.801.289
3	2012	10.969.951
4	2013	11.138.058
5	2014	11.305.516
6	2015	11.472.233
7	2016	11.638.117
8	2017	11.803.080
9	2018	11.967.035
10	2019	12.129.897
11	2020	12.291.584
12	2021	12.452.016
13	2022	12.611.116
14	2023	12.768.811
15	2024	12.925.028
16	2025	13.079.699
17	2026	13.232.759
18	2027	13.384.146
19	2028	13.533.802
20	2029	13.681.670
21	2030	13.827.699

Based on the table obtained, it can be concluded:

1. To predict the population in Banten Province, the AdamsBashforth-Moulton method can be used.
2. The results showed that the population of Banten Province increased every year. In 2025, the population was 13,079,699 people with a population increase percentage of $1,2\%$, in 2026 it was 13,232,759 people with a population increase percentage of $1,17\%$, in 2027 it was 13,384,146 people with a population increase percentage of $1,14\%$, in 2028 it was 13,533,802 people with a population increase percentage of $1,12\%$, in 2029 it was 13,681,670 people with a percentage increase in population of $1,09\%$, and in

2030 it was 13,827,699 people with a percentage increase in population of $1,07\%$.

CONCLUSIONS AND RECOMMENDATIONS

Based on the results of the existing discussion and the results obtained in the study, it can be concluded that to predict the population in Banten Province, the Adams-Bashforth-Moulton method can be used. Then the results showed that the population of Banten Province increased every year with a stable population increase.

FURTHER RESEARCH

For further research, it is suggested to conduct additional research on nth-order non-linear differential equations in everyday life using the many-step method or the AdamsBashforth-Moulton method. In addition, it is suggested to use various computer programs such as R Studio, and Matematica, to help solve the problem.

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