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Solving Bernoulli Differential Equations Using the Adomian Laplace Decomposition Method

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A B S T R A CT

Bernoulli differential equation is one form of first order ordinary differential equation. Because Bernoulli differential equation is a non-linear equation with a fairly complex form, this study uses the Adomian Laplace decomposition method to find its solution. This method is a semi-analytical method that combines the Laplace transform and the Adomian decomposition method. The steps for solving it include applying the Laplace transform to the Bernoulli differential equation, defining the solution as an infinite series, using the Adomian polynomial to solve the non-linear part, and applying the inverse Laplace transform. The simulation results and error analysis show that the Adomian Laplace decomposition method can provide an accurate approach to the exact solution for values $0 \leq t \leq 0.2$. Meanwhile, for values $t \geq 0.2$ the resulting solution tends to move away from the exact solution.

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INTRODUCTION

Differential equations are one of the important topics in mathematics and its applications in various fields such as physics, engineering, and economics. One type of differential equation that is often encountered is the Bernoulli differential equation. The Bernoulli differential equation is one form of a first-order ordinary differential equation that has the general form:

$$\frac{dy}{dt} + A(t)y = B(t)y^n \quad (1)$$

Where n is a real number. (Brannan and Boyce, 2015)

Because the Bernoulli differential equation is a non-linear equation that is quite complex in form, a semi-analytical method with an alternative approach can be used to find the solution. The Adomian Laplace Decomposition Method is a semi-analytical method that combines the Laplace transform and the Adomian decomposition method (Abdy et al., 2018). This method has been widely used to solve various linear and non-linear differential equations.

Based on previous research conducted by (Sari, 2017), namely solving the Riccati differential equation using the Adomian Laplace decomposition method, in his research stated that the calculation results of the Adomian Laplace decomposition method are quite effective in approaching exact solutions. This method allows the author to obtain solutions in the form of series that can be calculated numerically, thus providing flexibility in handling complex problems.

Furthermore, research conducted by (Sanusi et al., 2019) namely finding a solution to the Transport equation using the Adomian-Laplace decomposition method which states that the results of the study have the same solution as the analytical method in general, namely a mathematical function in the form of u (x, t) with x and t are the concentration of pollutants in position x and time t. As well as other studies that also use the Adomian Laplace position decom method to find solutions to various equations, namely research by (Abdy et al., 2022) on the Advection-Diffusion equation and (Sari et al., 2023) on the Burgers equation. In addition to finding solutions to an equation, the Adomian Laplace decomposition method is also used in analyzing the fractional differential equation model of the spread of measles and its numerical solution carried out by (Gumelar et al., 2023).

In this study, the author will discuss the application of the Adomian Laplace decomposition method in solving Bernoulli differential equations. This study begins by applying the Laplace transform to the Bernoulli differential equation, defining the solution as an infinite series, stating the nonlinear terms in the adomian polynomial, and applying the inverse Laplace transform to solve it.

LITERATURE REVIEW

Ordinary Differential Equations

An ordinary differential equation is an equation that only involves ordinary derivatives of one or more dependent variables with respect to a single independent variable (Sugiyarto, 2015). Order is the highest derivative in a differential equation. Meanwhile, degree is the power of the highest derivative in a differential equation.

Linear Ordinary Differential Equations

An ordinary differential equation is called linear if the equation is in the form:

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x) \quad (2)$$

Provided that all variables and derivatives of y are first degree and there is only one independent variable, namely x. (Sugiyarto, 2015)

Bernoulli Differential Equation

The Bernoulli Differential Equation is named after Jacob Bernoulli (1654–1705) and was first solved by Leibnitz in 1696. The Bernoulli differential equation is a first-order differential equation that has the following formula:

$$\frac{dy}{dt} + A(t)y = B(t)y^n \quad (3)$$

Where n is a real number. (Brannan and Boyce, 2015)

Laplace Transform

Let F(t) be a function of t which is certain for t > 0. Then the Laplace transform of F(t), which is given by f(s) = L{F(t)} is defined as follows:

$$L\{F(t)\} = \int_0^\infty e^{-st}F(t)\,dt = F(s) \quad (4)$$

with parameter s is a real number. (Sugiyarto, 2015)

Inverse Laplace Transform

If the Laplace transform of a function F(t) is f(s) or can be written as L{F (t)} = f(s), then F(t) is called the inverse Laplace transform of f(s) and can be written as follows:

$$F(t) = L^{-1}\{F(s)\} \quad (5)$$

with L^ (-1) called the inverse Laplace transform operator. (Sugiyarto, 2015)

Adomian Decomposition Method

In the Adomian decomposition method, the equation given in the operator equation is as follows:

$$Ly = g(x) - Ry - Ny \quad (6)$$

The functions y and Ny are the solutions and nonlinear terms solved using An. Thus, the n-term approximation φ_n= ∑_(i=0) ^(n-1) y_i approaches y=∑_(n=0) ^∞ y_n for n→∞. The solution can be written as:

$$y = \sum_{n=0}^{\infty} y_n = y_0 - L^{-1}R\sum_{n=0}^{\infty} y_n + L^{-1}\sum_{n=0}^{\infty} A_n \tag{7}$$

(Astreandini, 2016)

Adomian Laplace Decomposition Method

Revisiting equation (6) and applying the Laplace transform to it, we obtain:

$$L\{Ly\} = L\{g(x)\} - L\{Ry\} - L\{Ny\} \tag{8}$$

Therefore:

$$y = \textstyle\sum_{n=0}^{\infty} y_n \text{ dan } Ny = \sum_{n=0}^{\infty} A_n \tag{9}$$

Substitute equation (9) into equation (8), resulting in:

$$L\{Ly\} = L\{g(x)\} - L\{Ry\} - L\{\sum_{n=0}^{\infty} A_n\} \tag{10}$$

(Wartono dan Muhaijir, 2013)

Error

Error is the difference between the original value and the approximate value. Suppose a ˆ is the approximate value of the true value symbolized by a, then we get:

$$\varepsilon = a - \hat{a} \tag{11}$$

With ε called the error.

If the sign of the error for positive or negative is ignored, then the absolute error can be defined as:

$$|\varepsilon| = |a - \hat{a}| \tag{12}$$

(Munir, 2010)

METHODOLOGY

This study uses a literature study method that focuses on books found in the University of Lampung library, the reading room of the Mathematics Department, Faculty of Mathematics and Natural Sciences, University of Lampung, or public libraries and domestic or foreign journals that support the research being conducted.

RESULT AND DISCUSSION

Laplace Adomian Decomposition Method (LDAM) on Bernoulli Differential Equations

In this section, the Adomian Laplace Decomposition method will be explained in solving Bernoulli differential equations. Bernoulli differential equations in general are as follows:

$$\frac{dy}{dt} + A(t)y = B(t)y^n \tag{13}$$

With initial conditions:

$$y(0) = a \tag{14}$$

The following are the steps for solving the Bernoulli differential equation using the Adomian Laplace decomposition method:

Step 1 Apply the Laplace transform to equation (13)

$$L\{y'\} + L\{A(t)y\} = L\{B(t)y^n\}$$

$$sL\{y\} - y(0) \ = -A(t)L\{y\} + B(t)L\{y\} \tag{15}$$

Step 2 Substitute the given initial conditions

$$sL\{y\} - a \ = -A(t)L\ \{y\} + B(t)L\{y^n\}$$

$$L\{y\} = \frac{a}{s} - \frac{A(t)}{s}L\{y\} + \frac{B(t)}{s}L\{y^n\} \tag{16}$$

Step 3 Express y(t) in the form $\sum_{n=0}^{\infty} y_n(t)$

$$y = \sum_{n=0}^{\infty} y_n(t) \tag{17}$$

Step 4 Express non-linear terms in the form $\sum_{n=0}^{\infty} A_n$

$$y = \sum_{n=0}^{\infty} A_n \tag{18}$$

Substitute equations (17) and (18) into equation (16) so that the equation becomes:

$$L\left\{\sum_{n=0}^{\infty} y_n\right\} = \frac{a}{s} - \frac{A(t)}{s}L\left\{\sum_{n=0}^{\infty} y_n\right\} + \frac{B(t)}{s}L\left\{\sum_{n=0}^{\infty} A_n\right\} \tag{19}$$

For example, $\frac{A(t)}{s}\sum_{n=0}^{\infty} \mathrm{L}\{y_n\}$ and $\frac{B(t)}{s}\sum_{n=0}^{\infty} \mathrm{L}\{A_n\}$ of order λ and for example: $\sum_{n=0}^{\infty} \mathrm{L}\{y_n\}$ dan $\sum_{n=0}^{\infty} \mathrm{L}\{A_n\}$ of order λ^n, obtained:

$$L\{y_0\} + \lambda L\{y_1\} + \lambda^2 L\{y_2\} + \cdots = \frac{a}{s} - \lambda\frac{A(t)}{s}\{L\{y_0\} + \lambda L\{y_1\} + \cdots\} + \lambda\frac{B(t)}{s}\{L\{A_0\} + \lambda L\{A_1\} + \cdots\} \tag{20}$$

Step 5 Determining the Tribe $\mathrm{L}\{y_0\},\ \mathrm{L}\{y_1\},\ \mathrm{L}\{y_2\}, \ldots ,\mathrm{L}\{y_n\}$

$$\mathrm{L}\{y_0\} = \frac{a}{s}$$

$$L\{y_1\} = -\frac{A(t)}{s}L\{y_0\} + \frac{B(t)}{s}L\{A_0\}$$

$$\vdots$$

$$L\{y_n\} = -\frac{A(t)}{s}L\{y_{n-1}\} + \frac{B(t)}{s}L\{A_{n-1}\} \tag{21}$$

Step 6 Applying the Laplace inverse to the term $\mathrm{L}\{y_0\},\ \mathrm{L}\{y_1\},\ \mathrm{L}\{y_2\}, \dots, \mathrm{L}\{y_n\}$

$$y_0 = L^{-1}\{\frac{a}{s}\}$$

$$y_1 = L^{-1}\{-\frac{A(t)}{s}\mathrm{L}\{y_0\} + \frac{B(t)}{s}\mathrm{L}\{A_0\}\}$$

$$\vdots$$

$$y_n = L^{-1}\{-\frac{A(t)}{s}L\{y_{n-1}\} + \frac{B(t)}{s}L\{A_{n-1}\}\} \tag{22}$$

Step 7 Adding up $y_0, y_1, \dots, y_n$ as a solution y

$$y = \sum_{n=0}^{\infty} y_n = y_0 + y_1 + y_2 + \cdots + y_n \tag{23}$$

Example 1 Solving Bernoulli Differential Equations Using the Laplace Adomian Decomposition Method

Determine the solution to the following Bernoulli differential equation.

$$\frac{dy}{dt} + 3y = y^2 \tag{24}$$

$$y(0) = 1$$

After solving equation (24) using the solution steps with the Adomian Laplace decomposition method, the results of the solution y(t) are obtained, namely:

$$y = 1 - 2t + t^2 + t^3 - \frac{5}{4}t^4 - \frac{7}{20}t^5 - \frac{49}{40}t^6 - \frac{49643}{5\times10^5}t^7 - \frac{988839}{10\times10^5}t^8 + \frac{281027}{5\times10^5}t^9 + \frac{633973}{10\times10^5}t^{10} \tag{25}$$

Example 2 Solving Bernoulli Differential Equations Using the Laplace Adomian Decomposition Method

Determine the solution to the following Bernoulli differential equation.

$$\frac{dy}{dt} + 7y = 4y^2 \tag{26}$$

$$y(0) = 1$$

After solving equation (26) using the solution steps with the Adomian Laplace decomposition method, the results of the solution y(t) are obtained, namely:

$$y = 1 - 3t - \frac{3}{2}t^2 + \frac{23}{2}t^3 + \frac{95}{8}t^4 - \frac{2041}{40}t^5 + \frac{395021}{5000}t^6 + \frac{218871}{1000}t^7 + \frac{107562}{125}t^8 + \frac{25821}{25}t^9 + \frac{42459}{20}t^{10} \tag{27}$$

Simulation and Error Analysis

The solution used in this Simulation and Error Analysis is the solution y(t) obtained from the solution using the Adomian Laplace decomposition method in equations (24) and (26) for the order n = 5 and n = 10. This simulation and error analysis is carried out using the MATLAB program at intervals $0 \leq t \leq 1$ and $0 \leq t \leq 10$ to compare the exact solution with the LDAM solution.

The LDAM solution to equation (24) for n = 5 is as follows.

$$y = 1 - 2t + t^2 + t^3 - \frac{5}{4}t^4 - \frac{7}{20}t^5 \tag{28}$$

The LDAM solution to equation (24) for n = 10 is as follows.

$$y = 1 - 2t + t^2 + t^3 - \frac{5}{4}t^4 - \frac{7}{20}t^5 - \frac{49}{40}t^6 - \frac{49643}{5 \times 10^5}t^7 - \frac{988839}{10 \times 10^5}t^8 + \frac{281027}{5 \times 10^5}t^9 + \frac{633973}{10 \times 10^5}t^{10} \tag{29}$$

The LDAM solution to equation (26) for n = 5 is as follows.

$$y = 1 - 3t - \frac{3}{2}t^2 + \frac{23}{2}t^3 + \frac{95}{8}t^4 - \frac{2041}{40}t^5 \tag{30}$$

The LDAM solution to equation (26) for n = 10 is as follows.

$$y = 1 - 3t - \frac{3}{2}t^2 + \frac{23}{2}t^3 + \frac{95}{8}t^4 - \frac{2041}{40}t^5 + \frac{395021}{5000}t^6 + \frac{218871}{1000}t^7 + \frac{107562}{125}t^8 + \frac{25821}{25}t^9 + \frac{42459}{20}t^{10} \tag{31}$$

Simulation and Error Analysis on Interval $0 \leq t \leq 1$

In this first simulation, the equations used are equations (28) and (29) along with their exact solutions in the interval $0 \leq t \leq 1$. The following is a comparison graph of the approximate solution with the exact solution.

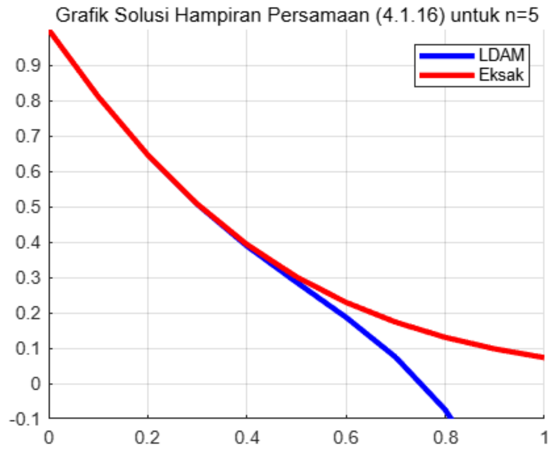

Figure 1. Comparison graph of solution (28) LDAM and its exact solution in the interval $0 \leq t \leq 1$

Figure 2. Comparison graph of solution (29) LDAM and its exact solution in the interval $0 \le t \le 1$

Figures (1) and (2) show that at intervals $t \geq 0.5$, both LDAM solutions move away from the exact solution. The following table is provided to see the error value from the comparison of the LDAM solution and the exact solution.

Table 1. First Simulation Error

t	n=5	n=10
0	0	0
0,1	0	0
0,2	0,00007	0
0,3	0,00080	0,00001
0,4	0,00424	0,00003
0,5	0,01516	0,00035
0,6	0,04224	0,00251
0,7	0,09904	0,01308
0,8	0,20486	0,05418
0,9	0,38533	0,18829
1	0,67287	0,57004

The calculation results in table (1) show the absolute maximum error value at $t = 1$ between the exact solution and the LDAM solution, for $n = 5$ it is 0.67287 and for $n = 10$ it is 0.57004. Thus, it can be said that the error value for order $n = 10$ is smaller than the error for order $n = 5$. In addition, it can also be seen that when the t value approaches zero, the error value becomes smaller or approaches the original value. Furthermore, the second simulation is carried out on equations (30) and (31) along with their exact solutions in the interval $0 \le t \le 1$. The following is a comparison graph of the approximate solution with its exact solution.

Figure 3. Comparison graph of solution (30) LDAM and its exact solution in the interval $0 \leq t \leq 1$

Figure 4. Comparison graph of solution (31) LDAM and its exact solution in the interval $0 \leq t \leq 1$

Figures (3) and (4) show that at intervals t ≥0.2, both LDAM solutions move away from the exact solution. The following table is provided to see the error value from the comparison of the LDAM solution and the exact solution.

Table 2. Second Simulation Error

t	n=5	n=10
0	0	0
0,1	0,00006	0,00001
0,2	0,00165	0,00235
0,3	0,00207	0,08166
0,4	0,05374	1,0385
0,5	0,35758	7,5697
0,6	1, 319	38,82
0,7	3,6323	156,35
0,8	8,3365	527,8
0,9	16,874	1556,6
1	31,152	4125

The calculation results in table (2) show the absolute maximum error value, namely at t = 1 between the exact solution and the LDAM solution, for n = 5 it is 31.152 and for n = 10 it is 4125. Thus, it can be said that the error value for order n = 10 is greater than the error for order n = 5. In addition, it can also be seen that when the t value approaches zero, the error value becomes smaller or approaches the original value.

Simulation and Error Analysis on Interval 0 ≤ t ≤ 10

For this third simulation, the equations used are equations (28) and (29) along with their exact solutions in the interval 0 ≤ t ≤ 10. The following is a comparison graph of the approximate solution with the exact solution.

Figure 5. Comparison Graph of Solution (28) LDAM and its Exact Solution in the Interval 0 ≤ t ≤ 10

Figure 6. Comparison Graph of Solution (29) LDAM and Its Exact Solution in the Interval 0 ≤ t ≤ 10

Figures (5) and (6) show that in the interval t ≥ 1, both LDAM solutions move away from the exact solution. The following table is provided to see the error value from the comparison of the LDAM solution and the exact solution.

Table 3. Third Simulation Error

t	n=5	n=10
0	0	0
1	0,67287	0, 57004

2	22,204	715,78
3	155,3	42334
4	605,4	748.610
5	1734	$69{,}053 \times 10^5$
6	4100,6	$423{,}37 \times 10^5$
7	8504,7	$1960{,}4 \times 10^5$
8	16028	$7394{,}8 \times 10^5$
9	28075	23854×10^5
10	46419	68022×10^5

The calculation results in table (3) show the absolute maximum error value at t = 10 between the exact solution and the LDAM solution, for n = 5 it is 46419 and for n = 10 it is 68022 × 10^5. So, it can be said that the error obtained from both LDAM solutions is very large.

Furthermore, the fourth simulation is carried out on equations (30) and (31) along with their exact solutions at the interval $0 \leq t \leq 10$. The following is a comparison graph of the approximate solution with its exact solution.

Figure 7. Comparison graph of solution (30) LDAM and its exact solution in the interval $0 \leq t \leq 10$

Figure 8. Comparison Graph of Solution (31) LDAM and its Exact Solution in the Interval $0 \leq t \leq 10$

Figures (7) and (8) show that there is no LDAM solution from the interval t ≥ 0 that is close to the exact solution. The following table is provided to see the error value from the comparison of the LDAM solution and the exact solution.

Table 4. Fourth Simulation Error

t	n=5	n=10
0	0	0
1	$0{,}31152 \times 10^2$	$0{,}00004125 \times 10^8$
2	$13{,}618 \times 10^2$	$0, 029446 \times 10^8$
3	$111{,}48 \times 10^2$	$1, 5174 \times 10^8$
4	$485{,}09 \times 10^2$	$25{,}564 \times 10^8$
5	$1506{,}50 \times 10^2$	$231{,}01 \times 10^8$
6	$3789{,}7 \times 10^2$	$1402{,}8 \times 10^8$
7	$8252{,}1 \times 10^2$	$6464{,}9 \times 10^8$
8	16176×10^2	24330×10^8
9	29268×10^2	78405×10^8
10	49724×10^2	223500×10^8

The calculation results in table (4) show the absolute maximum error value at t = 10 between the exact solution and the LDAM solution, for n = 5 it is 49724 × 10^2 and for n = 10 it is 223500 × 10^8. Thus, it can be said that the error values obtained from both LDAM solutions are very large.

Based on the solution above, the following conclusions can be drawn:

1. Results of Completion of $\frac{dy}{dt} + 3y = y^2$ with $y\,(0) = 1$ in series form, namely:

$$y = 1 - 2t + t^2 + t^3 - \frac{5}{4}t^4 - \frac{7}{20}t^5 - \frac{49}{40}t^6 - \frac{49643}{5 \times 10^5}t^7 - \frac{988839}{10 \times 10^5}t^8 + \frac{281027}{5 \times 10^5}t^9 + \frac{633973}{10 \times 10^5}t^{10}$$

2. Results of Completion of $\frac{dy}{dt} + 7y = 4y^2$ with $y(0) = 1$ in series form, namely:

$$y = 1 - 3t - \frac{3}{2}t^2 + \frac{23}{2}t^3 + \frac{95}{8}t^4 - \frac{2041}{40}t^5 + \frac{395021}{5000}t^6 + \frac{218871}{1000}t^7 + \frac{107562}{125}t^8 + \frac{25821}{25}t^9 + \frac{42459}{20}t^{10}$$

3. In the first simulation, figures (1) and (2) show that when the interval t ≥0.5 in both LDAM solutions move away from the exact solution. And the error value at t = 1 is the absolute maximum value for n = 5 of 0.67287 and for n = 10 of 0.57004.
4. In the second simulation, figures (3) and (4) show that when the interval t ≥0.2 in both LDAM solutions move away from the exact solution. And the error value at the time of the absolute maximum value t = 1 for n = 5 is 31.152 and for n = 10 is 4125.
5. In the third simulation, figures (5) and (6) show that when the interval t ≥1 in both LDAM solutions is far from the exact solution. And the error value at the time of the absolute maximum value t = 10 for n = 5 is 46419 and for n = 10 is 68022×10^5.

6. In the third simulation, figures (7) and (8) show that when the interval t ≥0 in both LDAM solutions move away from the exact solution. And the error value at the absolute maximum value of t = 10 for n = 5 is 49724 × 10^2 and for n = 10 is 223500 × $10^{8.}$

CONCLUSION AND RECOMMENDATIONS

Based on the results of the analysis that has been done previously, several conclusions were obtained from this study, including the results of solving the Bernoulli differential equation using the Adomian Laplace decomposition method, namely the solution y(t) in the form of a series. Also, the Bernoulli differential equation can be solved by the Adomian Laplace decomposition method at a value of $0 \leq t \leq 0.2$. However, for a value of $t \geq 0.2$, the LDAM solution moves away from its exact solution and the error value obtained is very large because when the t value moves away from zero, the error value obtained is getting bigger or moving away from its original value.

FURTHER RESEARCH

In this study, the researcher has limitations, the limitations in this study are as follows, the software used only uses the MATLAB program. Suggestions for further research if using programming to solve the Bernoulli differential equation are not just one software but more than one to compare the results obtained such as phyton, mathematical, R, etc. And change the LDMAN method to other methods such as the Homotropy Perturbation (HPM) method, the Iterative Picard method, etc.

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