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Comparison of Gauss-Seidel Method, Newton-Raphson Method, and Broyden Method in Solving Nonlinear Equation Systems

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ABSTRACT

A nonlinear equation system is a set of nonlinear equations that tend to be difficult to solve analytically. One common approach used to solve a nonlinear equation system is numerically in the form of an iteration method, which produces solutions in the form of approximate values or approximations. There are many numerical methods that can be applied to solve a nonlinear equation system, such as the Gauss-Seidel Method, the Newton-Raphson Method, and the Broyden Method. To obtain an effective and efficient solution, the selection of the right method is required. Therefore, this study will compare the performance of the Gauss-Seidel Method, the Newton-Raphson Method, and the Broyden Method in solving a nonlinear equation system. This study uses MATLAB software to assist in the process of solving a nonlinear equation system. The results of the study show that the Newton-Raphson Method is more effective in solving a nonlinear equation system compared to the Gauss-Seidel Method and the Broyden Method.

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INTRODUCTION

Mathematics is an exact science related to formulas and calculations. Using mathematics can help present, understand, analyze, and solve problems more easily. An approach in mathematics used to solve a problem using algebraic formulas is called the analytical method.

The analytical method is a method that focuses on solving problems using mathematical rules and principles systematically. However, not all calculation problems can be solved using the analytical method. There is a numerical method which is an alternative calculation in solving a mathematical problem that cannot be solved analytically. The numerical method is a technique in solving problems that are formulated mathematically. This technique uses arithmetic operations in the form of addition, subtraction, division, and multiplication. Some numerical methods used in solving equation problems include the Gauss-Seidel method, the Newton-Raphson method, the Broyden method, and other methods.

Several previous studies that examined the comparison of numerical methods include the study "Application of Jacobi and Gauss-Seidel Iteration Methods in Solving Complex Linear Equation Systems" (Ihsan et al., 2024), stating that the Gauss-Seidel method is better used to solve complex linear equation systems because it has fewer iterations compared to the Jacobi method. The study "Comparison of the Speed of Convergence of Nonlinear Equation Roots Using the Fixed-Point Method with the Newton Raphson Method Using Matlab" (Ritonga & Suryana, 2019) states that the Newton-Raphson method converges faster to the roots than the Fixed-Point method. In addition, in the study "Application of Numerical Solutions for Finding Roots of Nonlinear Equations and Their Application in Solving Break Even Point Analysis" (Sutrinso, 2023). The results of this study show that the Newton-Raphson method is the most efficient as indicated by the smallest error value compared to other methods. In this study, the author will focus on comparing the Gauss-Seidel method, the Newton-Raphson method, and the Broyden method, for solving nonlinear equation systems using 3 case examples.

LITERATURE REVIEW

- **Nonlinear System of Equations**

A system of nonlinear equations can be understood as a collection of two or more equations, where one of the equations cannot represent a straight line on the resulting graph. The following is the form of a nonlinear equation:

$$f_1(x_1, x_2, \cdots, x_n) = 0$$

$$f_2(x_1, x_2, \cdots, x_n) = 0$$

$$f_n(x_1, x_2, \cdots, x_n) = 0$$

The solution to this system of nonlinear equations can be done using graphical, substitution, or elimination methods.

- **Gauss-Seidel method**

The Gauss-Seidel method is an iterative method in solving a system of linear equations. This method can also be used in solving nonlinear equations with modifications from the original method. Solving a system of nonlinear equations using this method by determining x_1^{k+1} based on value x_n^k (Ripai, 2012). The general equation of the Gauss-Seidel method is as follows:

$$x_1^{k+1} = f_1(x_1^k, x_2^k, x_3^k, \cdots, x_n^k)$$

$$x_2^{k+1} = f_1(x_1^k, x_2^k, x_3^k, \cdots, x_n^k)$$

$$\vdots$$

$$x_n^{k+1} = f_1(x_1^k, x_2^k, x_3^k, \cdots, x_n^k)$$

The equation for calculating the error is as follows:

$$error = \max\left(\left|x_1^{k+1} - x_1^k\right|, \left|x_2^{k+1} - x_2^k\right|, \cdots, \left|x_n^{k+1} - x_n^k\right|\right)$$

The Gauss-Seidel method algorithm for solving nonlinear problems includes the following:

1. Define each variable $(x_1, x_2, x_3, \cdots, x_n)$
2. Determine the initial guess value for each variable. $(x_1, x_2, x_3, \cdots, x_n)$
3. Determine the initial approach value x_n^0
4. Each literacy calculates x using the existing equation where x_n^{k+1} is the latest value of x_n
5. Calculate the error for all variables, if the error $(<)$ tolerance, literacy stops

- **Newton-Raphson method**

The Newton-Raphson method is used to solve nonlinear equations using a single starting point approach and considering the gradient. This method approximates the graph of f(x) with an appropriate tangent line. To obtain the value x_0 as an initial guess, the root of $f(x)$ located and x_1 is the point of intersection between the x-axis and the tangent line to the curve f(x) at the point x_0. If it is the angle between the tangent and the x-axis then:

$$x_1 = x_0 \frac{f(x_0)}{f'(x_0)}$$

$$x_2 = x_1 \frac{f(x_1)}{f'(x_1)}$$

$$\vdots$$

$$x_{n+1} = x_0 \frac{f(x_n)}{f'(x_n)}, \qquad n = 0,1,2, \ldots$$

If two successive root approximations produce nearly the same value, the iteration is stopped (Purcell & Varberg, 1984). The Newton-Raphson method algorithm includes the following:

1. Defensive $f(x)$ and $f'(x)$
2. Determine the maximum literacy value (n) and error tolerance (e)
3. Determine the initial approach x_0
4. Calculate the value $f(x)$ and $f'(x)$
5. Literacy $i = 1$ s/d n or u $f(x_i) > e$, calculate the value x
6. x_i The final result obtained is the root of the equation obtained

- **Broyden Method**

The Bryoden method is a numerical method designed to develop the Newton method in solving nonlinear equation systems (Ramli et al., 2010). In addition, the Broyden method includes an extension of the Secant method for more than one variable. The Broyden method algorithm includes:

1. Define the function $f(x)$
2. Determine the initial guess x_0, Jacobian approximation, error tolerance (ε)
3. Counting $\in = -\frac{f(x_n)}{J_n}$
4. Counting $x_{n+1} = x_n + \in$ and $f(x)$
5. Calculate the Jacobian update using the formula:
$$J_n = J_{n-1} + \frac{f(x_{n+1}) - f(x_n) - J_n \times (x_{n+1} - x_n)}{x_{n+1} - x_n}$$
6. The iteration stops if $\in$ $(<)$ tolerance
7. The root of the equation is the value x_n last obtained

METHODOLOGY

This study uses a literature study method by tracing various sources relevant to the topic discussed. The focus of this study is to compare the Gauss-Seidel Method, the Newton-Raphson Method, and the Broyden Method in solving nonlinear equation systems. The research process begins with identifying problems and collecting literature related to numerical methods in solving nonlinear equation systems. Furthermore, an analysis and comparison of the three methods are carried out based on criteria such as the number of iterations, computation time, error rate, and convergence to the solution. The results of this study are expected to determine the most effective method in solving the nonlinear equation system studied.

RESULT AND DISCUSSION

This study uses 3 case problems of nonlinear equation systems which will be solved using 3 numerical methods with the help of MATLAB software. This study uses 3 case problems of nonlinear equation systems which will be solved using 3 numerical methods with the help of MATLAB software.

Case 1

The first case in this study will use a nonlinear equation system similar to Utami et al. (2013). The nonlinear equation system to be solved is as follows:

$$f_1(x,y,z) = 2x^2 + y - z^2 - 10 = 0$$
$$f_2(x,y,z) = 3x^2 + 6y - z^2 - 25 = 0$$
$$f_3(x,y,z) = x^2 - 5y + 6z^2 - 4 = 0$$

1. Solution of Nonlinear Equation System Case 1 Using Gauss-Seidel Method

The initial step in solving a system of non-linear equations in case 1 equations using the Gauss-Seidel method is to determine the initial guess for each variable x,y,z obtained:

$$x = \sqrt{\frac{10 - y - z^2}{2}}$$
$$y = \frac{25 - 3x^2 - z^2}{6}$$
$$z = \sqrt{\frac{4 - 5y - x^2}{6}}$$

Next, the Matlab application is used to produce the x, y, and z values, resulting in the x, y, and z values presented in Table 1.

Table 1. Solution of Case 1 Gauss-Seidel Method

Iter	x	y	z	Error	Time
1	2.23607	1.83333	1.16667	1.2360680000	0.000340
2	2.18263	2.01157	1.24459	0.1782407000	0.000096
3	2.18374	2.04047	1.25391	0.0289030300	0.000087
4	2.18309	2.04576	1.25585	0.0052855830	0.000079
5	2.18305	2.04668	1.25616	0.0009153072	0.000007
6	2.18303	2.04684	1.25622	0.0001623991	0.000005
7	2.18303	2.04686	1.25623	0.0000285014	0.000002
8	2.18303	2.04687	1.25623	0.0000050265	0.000003
9	2.18303	2.04687	1.25623	0.0000008845	0.000004

In Table 1, a convergent solution is obtained at the 9th iteration, with a computational completion time of 0.002646 seconds and an error of 0.0000008845 or 8.845×10^{-7}. In addition, the values obtained from each variable are: $x = 2.18303, y = 2.04687, z = 1.25623$.

2. Solution of Nonlinear Equation System Case 1 Using Newton-Raphson Method

The initial step in solving case 1 using the Newton-Raphson method is to find the derivative of each function, as follows:

$$\frac{df}{dx} = 4x \quad \frac{df}{dy} = 1 \quad \frac{df}{dz} = -2z$$
$$\frac{df}{dx} = 6x \quad \frac{df}{dy} = 6 \quad \frac{df}{dz} = -2z$$
$$\frac{df}{dx} = 2x \quad \frac{df}{dy} = -5 \quad \frac{df}{dz} = 12z$$

Next, the Matlab application is used to produce the x, y, and z values, resulting in the x, y, and z values presented in Table 2.

Table 2. Solution of Case 1 Newton-Raphson Method

Iter	x	y	z	Error	Time
1	2.88281	2.04687	1.28906	2.1735890000	0.000617
2	2.26796	2.04687	1.25665	0.6157016000	0.000022
3	2.18462	2.04687	1.25623	0.0833440700	0.000007
4	2.18303	2.04687	1.25623	0.0015897630	0.000005
5	2.18303	2.04687	1.25623	0.0000005788	0.000005

In Table 2, a convergent solution is obtained at the 5th iteration, with a computational completion time of 0.001605 seconds and an error of 0.0000005788 or 5.788×10^{-7}. In addition, the values obtained from each variable are: $x = 2.18303, y = 2.04687, z = 1.25623$.

3. Solution of Nonlinear Equation System Case 1 Using Broyden Method

The initial step in solving case 1 using the Broyden method is to define f(x). Next, the Matlab application is used to generate the values of x, y, and z, resulting in the values of x, y, and z presented in Table 3.

Table 3. Case 1 Solution Broyden Method

Iter	x	y	z	Error	Time
1	9.00000	18.0000	3.00000	18.894440000	0.001026
2	0.92285	2.09654	0.94309	17.955260000	0.000399
3	2.16306	1.46854	7.47697	6.6801340000	0.000604
4	2.93081	2.13148	0.68730	6.8500560000	0.000096
5	1.01809	2.04687	1.13560	1.9726850000	0.000995
6	1.96913	2.04595	0.99683	0.9649094000	0.000019
7	2.24357	2.04660	1.03750	0.2774353000	0.000007
8	2.18321	2.04608	1.09281	0.0818697700	0.000008
9	2.18605	2.04644	1.16934	0.0765800300	0.000010
10	2.18281	2.04691	1.27010	0.1008141000	0.000006
11	2.18299	2.04687	1.25601	0.0140932000	0.000007
12	2.18303	2.04687	1.25619	0.0001852216	0.000011
13	2.18303	2.04687	1.25625	0.0000601371	0.000006
14	2.18303	2.04687	1.25623	0.0000175029	0.000007

In Table 3, we get a convergent solution in the 14th iteration, with a computational completion time of 0.004799 seconds and an error of 0.0000175029 or 1.75×10^{-5} In addition, the values obtained from each variable are: $x = 2.18303, y = 2.04687, z = 1.25623$.

Case 2

The second case in this study will use a nonlinear equation system similar to Devitriani et al. (2019). The nonlinear equation system to be solved is as follows:

$$f_1(x,y,z) = 15x + y^2 - 4z - 13 = 0$$
$$f_2(x,y,z) = x^2 + 10y - e^{-z} - 4 = 0$$
$$f_3(y,z) = y^3 - 25z + 22 = 0$$

1. Solution of Nonlinear Equation System Case 2 Using Gauss-Seidel Method

The initial step in solving the system of non-linear equations in case equation 2 using the Gauss-Seidel method is to determine the initial guess for each variable x,y,z, obtained:

$$x = \frac{13 - y^2 + 4z}{15}$$

$$y = \frac{11 - x^2 - e^{-z}}{10}$$

$$z = \sqrt{\frac{y^3 + 22}{25}}$$

Next, the Matlab application is used to produce the x, y, and z values, resulting in the x, y, and z values presented in Table 4.

Table 4. Solution of Case 2 Gauss-Seidel Method

Iter	x	y	z	Galat	Waktu
1	1.06666	1.02301	0.92280	0.0771747600	0.000308
2	1.04298	1.03095	0.92383	0.0236832600	0.000089
3	1.04216	1.03109	0.92384	0.0008200668	0.000074
4	1.04215	1.03109	0.92384	0.0000135567	0.000070
5	1.04215	1.03109	0.92384	0.0000002236	0.000007

In Table 4, we get a convergent solution in the 5th iteration, with a computational completion time of 0.002397 seconds and an error of 0.0000002236 atau 2.236×10^{-7}. In addition, the values obtained from each variable are: $x = 1.04215, y = 1.03109, z = 0.92384$.

2. Solution of Nonlinear Equation System Case 2 Using Newton-Raphson Method

The initial step in solving case 1 using the Newton-Raphson method is to find the derivative of each function, as follows:

$$\frac{df}{dx} = 15 \quad \frac{df}{dy} = 2y \quad \frac{df}{dz} = -4$$
$$\frac{df}{dx} = 2x \quad \frac{df}{dy} = 10 \quad \frac{df}{dz} = -e^{-z}$$
$$\frac{df}{dx} = 0 \quad \frac{df}{dy} = 3y \quad \frac{df}{dz} = 25$$

Next, the Matlab application is used to produce the x, y, and z values, resulting in the x, y, and z values presented in Table 5.

Table 5. Solution of Case 2 Newton-Raphson Method Table

Iter	x	y	z	Galat	Waktu
1	1.04217	1.03115	0.92374	0.09254939000	0.000665
2	1.04215	1.03109	0.92384	0.00013076210	0.000275
3	1.04215	1.03109	0.92384	0.00000000006	0.000147

In Table 5, a convergent solution is obtained in the 3rd iteration, with a computational completion time of 0.001653 seconds and an error of 0.00000000006 or 6×10^{-11}. In addition, the values obtained from each variable are: $x = 1.04215, y = 1.03109, z = 0.92384$.

3. Solution of Nonlinear Equation System Case 2 Using Broyden Method

The initial step in solving case 2 using the Broyden method is to define f(x). Next, the Matlab application is used to generate the values of x, y, and z, resulting in the values of x, y, and z presented in Table 3.

Table 6. Case 2 Solution Broyden Method

Iter	x	y	z	Error	Time
1	2.00000	1.36788	3.00000	2.2661280000	0.003317
2	2.40822	1.76172	0.00323	3.0499860000	0.022627
3	-0.76352	0.03543	2.14996	4.2010110000	0.000841
4	1.76669	-0.61855	0.88757	2.9022940000	0.000109
5	0.09267	2.64182	1.10825	3.6716560000	0.001341
6	1.03664	1.00558	1.17937	1.8903490000	0.000021
7	0.95822	1.00356	0.98747	0.2073103000	0.000006
8	1.02678	1.08749	0.93138	0.1220300000	0.000006
9	1.03713	1.03749	0.91565	0.0534227500	0.000005
10	1.04171	1.03237	0.92365	0.0105411600	0.000005
11	1.04216	1.03104	0.92385	0.0001414472	0.000005
12	1.04215	1.03109	0.92384	0.0000475652	0.000006
13	1.04215	1.03109	0.92384	0.0000005522	0.000005

In Table 6, we get a convergent solution at the 13th iteration, with a computational completion time of 0.031049 seconds and an error of 0.0000601371 or 5.522×10^{-7}. In addition, the values obtained from each variable are $x = 1.04215, y = 1.03109, z = 0.92384$.

Case 3

The second case in this study will use a nonlinear equation system similar to Azmi et al. (2019). The nonlinear equation system to be solved is as follows:

$$f_1(x,y,z) = x + \cos(xy) - z^2 - 1.1 = 0$$
$$f_2(x,y,z) = x^2 + 10y - e^{xy} + 0.8 = 0$$
$$f_3(x,y,z) = xz + y^2 - z - 0.3 = 0$$

1. Solution of Nonlinear Equation System Case 3 Using Gauss-Seidel Method

The initial step in solving the system of non-linear equations in case equation 3 using the Gauss-Seidel method is to determine the initial guess for each variable x,y,z, obtained:

$$x = 1.1 - \cos(xy) + z^2$$
$$y = \frac{x^2 + 0.8 - e^{xy}}{10}$$
$$z = xz + y^2 - 0.3$$

Next, the Matlab application is used to produce the x, y, and z values, resulting in the x, y, and z values presented in Table 7.

Table 7. Solution of Case 2 Gauss-Seidel Method

Iter	x	y	z	Error	Time
1	0.26000	-0.01298	-0.40383	0.060002000	0.000340
2	0.26309	-0.01273	-0.40608	0.003084232	0.000614
3	0.26491	-0.01264	-0.40741	0.001820734	0.000780
4	0.26599	-0.01258	-0.40821	0.001084515	0.000939
5	0.26664	-0.01255	-0.40869	0.000649587	0.001618
6	0.26703	-0.01253	-0.40897	0.000390365	0.001986
7	0.26726	-0.01252	-0.40914	0.000235050	0.002036
8	0.26741	-0.01251	-0.40925	0.000141699	0.002062
9	0.26749	-0.01251	-0.40937	0.000085483	0.002090
10	0.26754	-0.01250	-0.40935	0.000051592	0.002114
11	0.26758	-0.01250	-0.40937	0.000031146	0.002139
12	0.26759	-0.01250	-0.40939	0.000018805	0.002165
13	0.26761	-0.01250	-0.40939	0.000011355	0.002189
14	0.26761	-0.01250	-0.40940	0.000006857	0.002213
15	0.26762	-0.01250	-0.40940	0.000004141	0.002238
16	0.26762	-0.01250	-0.40940	0.000002500	0.002274
17	0.26762	-0.01250	-0.40940	0.000001510	0.002302
18	0.26762	-0.01250	-0.40941	0.000000912	0.002327

In Table 7, we get a convergent solution at the 18th iteration, by completing the computation in 0.002872 seconds and an error of 0.000000912 atau 9.12×10^{-7}. In addition, the values obtained from each variable are $x = 0.26762, y = -0.01250, z = -0.40941$.

2. Solution of Nonlinear Equation System Case 3 Using Newton-Raphson Method

The initial step in solving case 3 using the Newton-Raphson method is to find the derivative of each function, as follows:

$$\frac{df}{dx} = 15 \quad \frac{df}{dy} = 2y \quad \frac{df}{dz} = -4$$

$$\frac{df}{dx} = 2x \quad \frac{df}{dy} = 10 \quad \frac{df}{dz} = -e^{-z}$$

$$\frac{df}{dx} = 0 \quad \frac{df}{dy} = 3y \quad \frac{df}{dz} = 25$$

Next, the Matlab application is used to produce the x, y, and z values, resulting in the x, y, and z values presented in Table 8.

Table 8. Solution of Case 3 Newton-Raphson Method

Iter	x	y	z	Galat	Waktu
1	0.26658	-0.01274	-0.40926	0.03478173	0.000630
2	0.26775	-0.01249	-0.40948	0.00121312	0.000023
3	0.26762	-0.01250	-0.40941	0.00015258	0.000008
4	0.26762	-0.01250	-0.40941	0.00000860	0.000006
5	0.26762	-0.01250	-0.40941	0.00000047	0.000005

In Table 8, we get a convergent solution at the 5th iteration, with a computational completion time of 0.001675 seconds and an error of 0.00000047 atau 4.7×10^{-7}. In addition, the values obtained from each variable are $x = 0.26762, y = -0.01250, z = -0.40941$.

3. Solution of Nonlinear Equation System Case 3 Using Broyden Method

The initial step in solving case 3 using the Broyden method is to define f(x). Next, the Matlab application is used to generate the values of x, y, and z, resulting in the values of x, y, and z presented in Table 3.

Table 9. Case 3 Solution Broyden Method

Iter	x	y	z	Error	Time
1	0.26000	-0.00299	-0.38010	0.0452185	0.001120
2	0.16636	0.61012	-0.26721	0.6304127	0.000301
3	0.22986	-0.03971	-0.33349	0.6562823	0.000756
4	0.19077	-0.04518	-0.27246	0.0726840	0.000106
5	0.26485	-0.01563	-0.44866	0.1934076	0.001109
6	0.24285	-0.01142	-0.35456	0.0967295	0.000019
7	0.25870	-0.01226	-0.39068	0.0394527	0.000007
8	0.26612	-0.01266	-0.40923	0.0199842	0.000006
9	0.26607	-0.01251	-0.40693	0.0023066	0.000006

In Table 9, we get a convergent solution in the 13th iteration, with a computational completion time of 0.005176 seconds and an error of 0.0000150 or 1.5×10^{-5} . In addition, the values obtained from each variable are
est n$x = 0.26762, y = -0.01250, z = -0.40941$

Based on the output results produced from each method in solving the nonlinear equation system, the following conclusions can be drawn:

1. In case 1, it can be seen that the Newton-Raphson method has the small umber of iterations, computation time, and error. This shows that the Newton-Raphson method has a higher solution accuracy and converges faster than the other two methods.
2. In case 2, it can be seen that the Newton-Raphson method has the smallest number of iterations, computation time, and error. This shows that the Newton-Raphson method has a higher solution accuracy and converges faster than the other two methods.
3. In case 3, it can be seen that the Newton-Raphson method has the smallest number of iterations, computation time, and error. This shows that the Newton-Raphson method has a higher solution accuracy and converges faster than the other two methods.

CONCLUSIONS AND RECOMMENDATIONS

Based on the results and discussion in comparing the three methods for solving nonlinear equation systems, the comparison is made based on error accuracy, computation time, and the number of iterations required by each method.

Table 10. Iiteration Results and Errors in Cases 1-3 with Gauss-Seidel, Newton-Raphson, and Broyden Methods

Case	Gauss-Seidel			Newton-Raphson			Broyden		
	Error	iter	Time	Error	Iter	Time	Error	Iter	Time
Ke-1	8.8×10^{-7}	9	0.003	5.8×10^{-7}	5	0.02	1.7×10^{-5}	14	0.005
Ke-2	2.2×10^{-7}	5	0.003	6.0×10^{-11}	3	0.02	5.5×10^{-7}	13	0.031
Ke-3	9.1×10^{-7}	18	0.003	4.7×10^{-7}	5	0.02	1.5×10^{-5}	13	0.006
$\bar{x}$	6.7×10^{-7}	10.7	0.003	3.8×10^{-7}	4.3	0.02	8.6×10^{-6}	13.3	0.014

As seen in Table 10, the Newton-Raphson method has a smaller iteration value, error, and computation time in each case compared to the Gauss-Seidel Method and the Broyden Method. Of the three cases, the average Newton-Raphson converges at iteration 4 in an average computation time of 0.003 seconds, with an average error of 3.8×10^{-7}. This shows that the Newton-Raphson method has higher solution accuracy, faster convergence, and is more effective and efficient compared to the Gauss-Seidel Method and the Broyden Method.

FURTHER STUDY

Further research is recommended to apply the method to real cases, such as optimizing energy systems, electrical systems and others.

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