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Research Paper

Modeling Vector Error Correction with Exogeneous (VECMX) Variable for Analyzing Nonstationary Variable Energy Used and Gross Domestic Product (GDP)

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Abstract
Analysis of energy used, GDP and population has been carried out in many countries and has become a topic of interest for many researchers and governments. This is because energy used is an important factor for society and industry in a country. In this study, the modeling of the relationship between energy used, GDP and population as an exogenous variable for the cases of Indonesia from 1967-2023 will be discussed. The energy used and GDP data are nonstationary with order one, I(1), and there is cointegration between energy used and GDP. Therefore, the model which will be used is the Vector Error Correction Model with Exogenous variable (VECMX) with population as the exogenous variable. From the results of analysis, the best model is VECMX(3,1) with cointegration rank R=1. Based on this model, the pattern of the relationship among the three variables, Granger-causality between energy used and GDP, exogenous impact on energy used and GDP, and forecasting for the next 10 years will be discussed.

Keywords
Nonstationary, Cointegration, VECMX Model, Ganger-Causality, Forecasting

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1. INTRODUCTION

Energy is an important factor for industry, society and economic growth in any country. Therefore, the availability of energy and resources is expected to increase economic growth in many countries. Energy used has increased rapidly compared to previous times (Pachiyappan et al., 2021). Research involving energy used, GDP and population has been carried out in many countries, which shows that understanding the relationship among those variables are very important for decision makers. Research involving these variables and connecting them with other variables of interest to researchers has been carried out in many countries (Aqeel and Butt, 2001; Bekhet and Yusop, 2009; Binh, 2011; Loi, 2012; Mahadevan and Asafu-Adjaye, 2007; Chontanawat et al., 2008).

The importance to understand the energy used and its implications for a country's economic development. Many studies discuss economic growth models seen from the impact of energy consumption and its associated with many other variables (Chontanawat et al., 2008). Canh (2011) studied economic growth patterns and their relationship with energy (electricity) consumption in Vietnam during the period 1975-2010. Loi (2012) in his study using a granger-causality approach discusses examining the relationship between traders, energy consumption and GDP in Vietnam in the 1986-2006 period. Loi (2012) concluded that there is a cointegration relationship between electricity consumption and GDP. Pachiyappan et al. (2021) in their study using the VECM model and Autoregressive Distribution Lag (ADRL) methods discussed the relationship between energy use, CO_2, GDP and population growth in India in the 1980-2018 period. The results from the cointegration test show that there is a long-term relationship between variables. The results of the Granger causality test show that there is a two-way causality relationship between GDP and energy used, and unidirectional causality between CO_2 and energy used, CO_2 and population growth, CO_2 and GDP, and population growth and energy used.

Hannesson (2009) conducted study to discuss the relationship between energy use and gross national product (GDP) growth in 1950-2004. Hannesson (2009) examines the close relationship between energy use and GDP growth. The results show that the closeness of relations between poor countries and developed countries varies. The analytical method used is linear regression on the rate of energy growth which is used on the rate of GDP growth in 171 countries, oil prices and GDP per capita, and for several poor, middle-rich and rich countries, as well as market economies versus centralized economies, and oil importing countries versus oil exporting countries. The results of his study show that there is a significant positive relationship between GDP growth and energy use in all countries. Mansoor and Sultana (2018) used the Autoregressive Distribution Lag (ARDL) approach in their research to study the relationship between population, CO_2 emissions, energy consumption and economic growth in Pakistan in 1975-2016. The results conclude that population growth and energy use increase CO_2 emissions, while the relationship between CO_2 emissions and GDP is negative in the long term, this conclusion could be due to new technological advances allow a country to achieve the same level of production but with a lower level of CO_2 emissions thereby increasing the indicator air quality in a country.

Ajmi and Inglesi-Lotz (2020) used the VECM model analysis approach and Granger causality to discuss the relationship between economic growth and biomass energy consumption in twenty-six OECD countries in the period 1980-2013, and concluded that there is a one-way relationship between consumption energy and economic growth in OECD countries. Al-Mulali and Sab (2012) conducted a study using panel data from 19 countries from 1980-2008, to discuss the relationship between energy consumption and economic growth, and concluded that there was a positive relationship between economic growth and energy consumption. Bouyghrissi et al. (2021) conducted a study using ARD model analysis and Granger-causality tests to analyze the relationship between renewable energy consumption and economic growth in Morocco in 1990–2014. The results of his research concluded that there is a one-way causal relationship between renewable energy consumption and economic growth in Morocco.

Analysis of the causal relationship between economic growth and energy consumption has been widely discussed in the energy economics literature since the pioneering study of Kraft and Kraft (1978). Akarca and Long (1979) in their study stated that there is a unidirectional Granger causality relationship from energy consumption to employment using US monthly data from 1973-1978. Squalli (2007) states that there are two views in the literature analyzing the relationship between energy consumption and economic growth. The first view states that energy consumption is the main means of achieving economic growth. The influence of energy consumption on economic growth both indirectly and directly, especially in the production process as a complement to labor and capital. In contrast, the second view states that energy consumption should not affect economic growth because it represents too small a proportion of a country's gross domestic product. However, most of the literature so far provides inconsistent results and makes it difficult to understand the causal relationship between economic growth and energy consumption. Studies conducted by Hwang and Gum (1992), Hoa (1993), Ebohon (1996), Cheng and Lai (1997), Yang (2000), Hondroyiannis et al. (2002), Soytas and Sari (2003), Ghali and El-Sakka (2004), and Oh and Lee (2004) concluded that there is a reciprocal interaction between energy consumption and income as proxied by GNP. Meanwhile, studies conducted by Yu and Hwang (1984), Yu and Choi (1985), Erol and Yu (1987), Yu and Jin (1992), Cheng (1996), and Altinay and Karagol (2004) concluded that there was no relationship causality between energy consumption and income and proxied by GNP.

Magazzino (2015) in his study by using time series analysis method obtained empirical evidence of a relationship between energy consumption and GDP in Italy during the period 1970-2009. The stationarity test shows that the two series are non-stationary, or I(1) and there is cointegration between the two variables. In his study, Magazzino (2015) used the VECM model and concluded that energy is a limiting factor in GDP growth in Italy and that energy conservation policies must be formulated and implemented wisely.

The aim of this research is to expand the empirical literature on the causal relationship between energy used and economic growth (GDP) with the exogenous variable being population for the case in Indonesia. The aim of this research is to build the best model of the relationship pattern between energy used and GDP based on the underlying data assumptions. The influence of exogenous variables, population, on energy used and GDP will be discussed in depth. Further analysis of the relationship pattern between energy used and GDP will be carried out. Forecasting for energy used and GDP will be carried out for the next 10 periods.

2. EXPERIMENTAL SECTION

In developing the cointegration concept, Granger (1988) used the stationary nature of two time series. With the first step, a stationary time series data test is carried out, for cointegrated series they must be stationary with the same order, that is, if the time series data is nonstationary with order one I(1) and they become stationary if they are differentiated with first differencing or become I(0), stationary. For the stationary test, the Augmented Dickey-Fuller (ADF) test is used (Dickey and Fuller, 1979, 1981). Phillip and Perron (1988) for the unit root test which was developed to check stationary time series data. Gujarati (2003) built the ADF test using the regression model as follows:

$$\Delta Z_t = \beta_0 + \gamma Z_{t-1} + \sum_{i=1}^{p} \beta_i \Delta Z_{t-i} + \varepsilon_t \quad (1)$$

Where β_0 is intercept, γ is the lagged difference terms, β_i is parameter, and ε_t is vector noises. The null and alternative hypotheses of the ADF test are $H_0 : \gamma = 0$ and $H_a : \gamma < 0$. The test statistic is as follows:

$$t_\gamma = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \quad (2)$$

Reject H_0 if the p-value < 0.05.

2.1 Cross Correlation Matrix

Matrix D, a diagonal matrix of standard deviation $Z_{it}, i = 1, 2, \ldots, m$, is defined as follows:

$$D = \text{diag}\{\sqrt{\Gamma_{11}(0)}, \sqrt{\Gamma_{22}(0)}, \ldots, \sqrt{\Gamma_{mm}(0)}\}. \quad (3)$$

The cross-correlation matrix for lag-0 is defined as follows:

$$\rho_0 = [\rho_{ij}(0)] = D^{-1}\Gamma_0 D^{-1}. \quad (4)$$

where

$$\Gamma_0 = \text{Cov}(Z_t, Z_t) = \mathbb{E}[(Z_t - \mu)(Z_t - \mu)'],$$
$$Z_t = \left[Z_{1t}, Z_{2t}, \ldots, Z_{mt}\right]',$$

and

$$D^{-1} = \text{diag}\{\frac{1}{\sqrt{\Gamma_{11}(0)}}, \frac{1}{\sqrt{\Gamma_{22}(0)}}, \ldots, \frac{1}{\sqrt{\Gamma_{mm}(0)}}\}. \quad (5)$$

Specifically, the (i, j)th element of ρ_0 is

$$\rho_{ij}(0) = \frac{\Gamma_{ij}(0)}{\sqrt{\Gamma_{ii}(0)\Gamma_{jj}(0)}} = \frac{\text{Cov}(Z_{it}, Z_{jt})}{\text{Sd}(Z_{it}) \cdot \text{Sd}(Z_{jt})}, \quad (6)$$

which is the correlation coefficient between Z_{it} and Z_{jt}. Clearly, ρ_0 is a symmetric matrix with all diagonal elements being 1.

A very important topic in multivariate time series analysis is the lag-correlation between series components, which ultimately results in the cross-correlation matrix being used as a measure of the strength of the linear relationship between time series data (Tsay, 2010). The lag-k cross-correlation matrix of Z_t is defined as follows:

$$\rho_k = [\rho_{ij}(k)] = D^{-1}\Gamma_k D^{-1}. \quad (7)$$

where

$$\rho_{ij}(k) = \frac{\Gamma_{ij}(k)}{\sqrt{\Gamma_{ii}(0)\Gamma_{jj}(0)}} = \frac{\text{Cov}(Z_{it}, Z_{j,t-k})}{\text{Sd}(Z_{it}) \cdot \text{Sd}(Z_{jt})}, \quad (8)$$

and

$$\Gamma_k = \begin{bmatrix} \Gamma_{11}(k) & \Gamma_{12}(k) & \cdots & \Gamma_{1m}(k) \\ \Gamma_{21}(k) & \Gamma_{22}(k) & \cdots & \Gamma_{2m}(k) \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{m1}(k) & \Gamma_{m2}(k) & \cdots & \Gamma_{mm}(k) \end{bmatrix}.$$

If $k > 0$, this correlation measures the linear dependence between Z_{it} and $Z_{j,t-k}$, which occurs before time t. Therefore, if $\rho_{ij}(k) \neq \rho_{ji}(k)$ and $i \neq j$, the two coefficients measure different linear relationships between $\{Z_{it}\}$ and $\{Z_{jt}\}$.

2.2 Granger-Causality

In studying multivariate time series, we often want to know whether there are causal effects between the variables being discussed. Specifically, in the VAR(p) model:

$$Z_t = \theta_0 + \Phi_1 Z_{t-1} + \cdots + \Phi_p Z_{t-p} + \varepsilon_t \quad (9)$$

or

$$\Phi_p(B)Z_t = \theta_0 + \varepsilon_t, \quad \Phi_p(B) = I - \Phi_1 B + \cdots + \Phi_p B^p.$$

Without loss of generality, let

$$Z_t = \begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix},$$

so that

$$\begin{bmatrix} \Phi_{11}(B) & \Phi_{12}(B) \\ \Phi_{21}(B) & \Phi_{22}(B) \end{bmatrix} \begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} = \begin{bmatrix} \theta_{01} \\ \theta_{02} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}. \quad (10)$$

If $\Phi_{12}(B) = 0$, then Equation (10) becomes:

$$\Phi_{11}(B)Z_{1t} = \theta_{01} + \varepsilon_{1t}. \quad (11)$$

The future values of Z_{2t} are affected not only by its own past, but also by the past of Z_{1t}, while the future values of Z_{1t} are affected only by its own past. In other words, we say that variable Z_{1t} causes Z_{2t}, but variable Z_{2t} does not cause Z_{1t}. This concept is known as Granger-causality (Granger, 1988). For more discussion about causality and its test, it can be seen in Granger (1988), Pierce and Haugh (1977), Hamilton (1994), and Wei (2019).

2.3 Model VECMX and Cointegration Test

The Vector Error Correction Model with exogenous variables is written as follows:

$$\Delta Z_t = C(t) + \Pi Z_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta Z_{t-i} + \sum_{i=0}^{s} \Theta_{t-i} X_{t-i} + \varepsilon_t, \quad (12)$$

where $C(t)$ is a vector constant, and the cointegrating vector β is sometimes called the long-run parameters (Wei, 2006; Tsay, 2010). Π is a matrix of parameters, Θ_{t-i} is a vector of parameters for exogenous variables, and ε_t is a vector of white noise.

In principle, cointegration testing is to test the rank of Π. Mathematically, the rank of Π is the number of nonzero eigenvalues of Π, which can be obtained if a consistent estimate of Π is available (Tsay, 2010).

The cointegration test with null and alternative hypotheses is as follows:

$$H_0 : \text{Rank}(\Pi) = r \quad \text{versus} \quad H_a : \text{Rank}(\Pi) > r.$$

Johansen (1988) suggested the likelihood ratio (LR) test statistic:

$$LK_{tr}(r) = -(T - p) \sum_{i=r+1}^{k} \ln(1 - \hat{\lambda}_i), \quad (13)$$

where $\hat{\lambda}_i$ should be small for $i > r$. The test is referred to as the trace cointegration test (Tsay, 2010, 2014).

3. RESULT AND DISCUSSION

In this study, data on energy used per capita of the Indonesian population and Gross Domestic Product (GDP) data (www.ourworldata, 2023) from 1967 to 2023 and population data from 1967-2023 as exogenous variables will be used for modeling and explaining variables energy used per capita and Indonesia's GDP. Figure 1 shows energy used per capita and GDP data from 1967 to 2023. Figure 1 shows an upward trend for both energy used data and GDP data, this indicates that the time series energy used per capita and GDP data are not stationary.

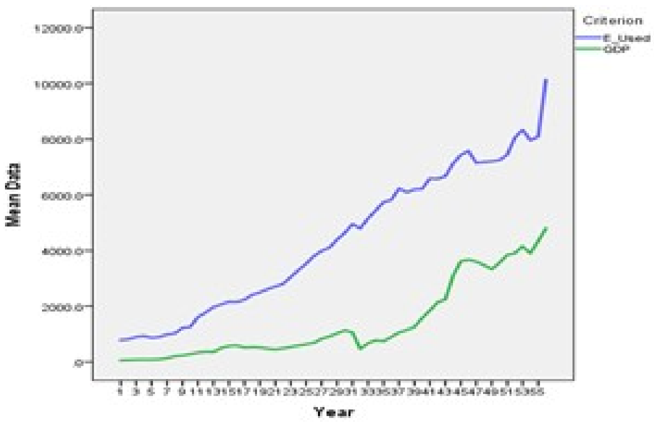


Figure 1. Plot Data Energy Used and GDP from 1967 to 2023

From the results of the unit roots test or Augmented Dickey-Fuller (ADF) test (Table 1), the energy used data and GDP data contain unit roots, with the null hypothesis that the data contains unit roots. The data before differentiating all Tau tests shows that the null hypothesis is not rejected, so there are unit roots. After differencing once (d=1), the ADF test results showed that all tau tests were significant with a p-value < 0.05, so the null hypothesis was rejected, in other words the data became stationary. So energy used data and GDP data are data with Integrated order one, I(1) (Hamilton, 1994; Wei, 2019).

From the results of the cointegration rank test (Table 2) on the VAR(3) model, it shows that there is cointegration between energy used data and GDP data. The results show that from the null hypothesis test Ho:Rank=0 against H1:Rank> 0, Ho is rejected with p-value=0.0056. From the results of the null hypothesis test Ho:Rank=1 against H1:Rank> 1 null hypothesis not rejected with p-value=0.5413. So Ho:Rank=1 is not rejected. So from the results of the cointegration test Rank test using trace under restriction there is a cointegration with the rank cointegration equal to 1.

From the results of the cross correlation analysis of energy used and GDP data, Table 3 and Table 4 show that there is a significant cross correlation up to lag 10 between the energy used data and GDP data. From the results of the cross correlation test with the null hypothesis that the cross correlation is zero, Ho: ρ=0), there is no cross correlation against the alternative hypothesis that the cross correlation is different from zero, Ha: $\rho \neq 0$, there is a cross correlation. Table 4 shows that the cross correlation is significant at significant level α=0.05, the sign +, means that the cross correlation is positive and the null hypothesis Ho: ρ=0 is rejected. Based on the initial conditions of the data, that the data is nonstationary, integrated with order one, I(1), there is cointegration between time series data energy used and GDP, and there is cross correlation between time series data energy used and GDP, then the p-value is suitable for This kind of data is modeled using a multivariate time series analysis approach, Vector Error Correction Model with Exogenous (VECMX) variable population (Hamilton, 1994; Tsay, 2014; Wei, 2019). The VECMX(3,1) model with cointegration rank=1 will be used to analyze the data.

3.1 Model VECMX(3,1) with Rank Cointegration R=1

The VECMX model for the relationship between variable energy used and GDP, and the exogenous variable is population is given as follows:

$$\Delta \begin{bmatrix} E\_Used_t \\ GDP_t \end{bmatrix} = C(t) + \Pi \begin{bmatrix} E\_Used_{t-1} \\ GDP_{t-1} \end{bmatrix} + \sum_{i=1}^{p-1} \Phi_i \Delta \begin{bmatrix} E\_Used_{t-i} \\ GDP_{t-i} \end{bmatrix} + \sum_{i=0}^{s} \Theta_i POP_{t-i} + \varepsilon_t \quad (14)$$

Where E_Used_t is energy used at time t, ε_t is noise, $C(t)$ is a 2×1 constant term, Π is a 2×2 matrix of parameters, Θ_i is a 2×1 vector parameter for the exogenous variable Population, and Φ_i is a 2×2 matrix of parameters. From the results of analysis, the estimate Model (12) is given as follows:

$$\Delta \begin{bmatrix} E\_Used_t \\ GDP_t \end{bmatrix} = \begin{bmatrix} 125.1333 \\ -57.5894 \end{bmatrix} + \begin{bmatrix} -0.3269 & -0.1386 \\ -0.1578 & -0.0676 \end{bmatrix} \begin{bmatrix} E\_Used_{t-1} \\ GDP_{t-1} \end{bmatrix} + \begin{bmatrix} -0.1366 & 0.7518 \\ -0.1407 & 0.3636 \end{bmatrix} \Delta \begin{bmatrix} E\_Used_{t-1} \\ GDP_{t-1} \end{bmatrix} + \begin{bmatrix} -0.5192 & 0.0221 \\ -0.3776 & 0.3069 \end{bmatrix} \Delta \begin{bmatrix} E\_Used_{t-2} \\ GDP_{t-2} \end{bmatrix} + \begin{bmatrix} -539.4683 \\ -253.2819 \end{bmatrix} POP_t + \begin{bmatrix} 556.5503 \\ 262.1121 \end{bmatrix} POP_{t-1} \quad (15)$$

The covariance innovations are given as follows:

$$\text{Cov}(\varepsilon_t) = \begin{bmatrix} 55066.8596 & 8572.2558 \\ 8572.2558 & 27689.2077 \end{bmatrix}$$

Table 1. Dickey-Fuller Unit Root Tests Before and After Differencing First (d=1)

Variable	Before Differencing					After Differencing (d=1)				
	Type	Rho	p-value	Tau	p-value	Type	Rho	p-value	Tau	p-value
E_Used	Zero Mean	1.96	0.9852	3.28	0.9996	Zero Mean	-38.83	<0.0001	-2.43	0.0157
	Single Mean	1.20	0.9903	1.26	0.9981	Single Mean	-234.92	0.0001	-5.01	0.0002
	Trend	-23.36	0.0183	-2.64	0.2643	Trend	-260.10	0.0001	-5.25	0.0004
GDP	Zero Mean	2.75	0.9970	2.50	0.9966	Zero Mean	-27.65	<0.0001	-3.29	0.0014
	Single Mean	1.92	0.9967	1.33	0.9985	Single Mean	-39.90	0.0005	-4.04	0.0025
	Trend	-2.40	0.9554	-0.81	0.9579	Trend	-49.78	<0.0001	-4.60	0.0028

Table 2. Cointegration Rank Test Using Trace Under Restriction

Ho: Rank=r	H1: Rank>r	Eigenvalue	Trace	p-value	Drift in ECM	Drift in Process
0	0	0.3561	26.5472	0.0056	Constant	Constant
1	1	0.0589	3.2156	0.5413		

Table 3. Cross Correlation of Dependent Series

Lag	Variable	E_USED	GDP	Lag	Variable	E_USED	GDP
0	E_USED	1.00000	0.89071	5	E_USED	0.74169	0.72215
	GDP	0.89071	1.00000		GDP	0.55692	0.67210
1	E_USED	0.92804	0.83481	6	E_USED	0.69645	0.69748
	GDP	0.81428	0.92840		GDP	0.49351	0.60533
2	E_USED	0.88481	0.81041	7	E_USED	0.65260	0.67521
	GDP	0.74289	0.86231		GDP	0.43408	0.54258
3	E_USED	0.84400	0.78759	8	E_USED	0.60588	0.65009
	GDP	0.68537	0.80587		GDP	0.37968	0.48479
4	E_USED	0.79431	0.75438	9	E_USED	0.55795	0.62410
	GDP	0.61906	0.73688		GDP	0.31861	0.42072

Table 4. Schematic Representation of Cross Correlations

Variable/Lag	0	1	2	3	4	5	6	7	8	9
E_Used	++	++	++	++	++	++	++	++	++	++
GDP	++	++	++	++	++	++	++	++	++	++

Note: + is > 2*std error, - is < -2*std error, . is between

3.2 Diagnostic Model VECMX(3,1) With Cointegration Rank R=1

From Model (15) and Table 5, the univariate models are as follows:

$$\Delta(E\_Used_t) = 125.1333 - 539.4683 POP_t + 556.5503\, POP_{t-1} - 0.3236\, E\_Used_{t-1} - 0.1386\, GDP_{t-1} - 0.1366\, \Delta(E\_Used_{t-1}) + 0.7518\, \Delta(GDP_{t-1}) - 0.5192\, \Delta(E\_Used_{t-2}) + 0.0221\, \Delta(GDP_{t-2}). \quad (16)$$

$$\Delta(GDP_t) = -57.5894 - 253.2819\, POP_t + 266.1121\, POP_{t-1} - 0.1578\, E\_Used_{t-1} - 0.0676\, GDP_{t-1} - 0.1407\, \Delta(E\_Used_{t-1}) + 0.3636\, \Delta(GDP_{t-1}) - 0.3776\, \Delta(E\_Used_{t-2}) + 0.3069\, \Delta(GDP_{t-2}). \quad (17)$$

Table 6 shows the univariate diagnostics for Model (16) and Model (17), where Model (16) is very significant with p-value = 0.0005 and R^2 = 0.4488, and Model (17) is very significant with p-value = 0.0200 and R^2 = 0.3214.

Table 7 shows the results of the normality test using the Jarque-Bera normality test with the null hypothesis that the residuals have normality distribution. The test results for both residual energy used data and GDP tested significant with each p-value <b 0.0001, which means the null hypothesis is rejected and the residuals distribution is not normally distributed. However, Figures 2 and 3 show that the distribution of residuals for both energy used data and GDP data is close to normal. In the last column of the F test in Table 7 is the test the null hypothesis that the residuals have equal covariance, and the test shows that the null hypothesis is not rejected, therefore the

Table 5. Model Parameter Estimates of VECMX(3,1) with Cointegration Rank=1

Equation	Parameter	Estimate	Standard Error	t-Value	p-Value	Variable
D_E_USED	CONST1	125.13332	660.36731	0.19	0.8506	1
	XL0_1_1	-539.46836	153.08706	-3.52	0.0010	POP(t)
	XL1_1_1	556.55039	155.03833	3.59	0.0008	POP(t-1)
	AR1_1_1	-0.32369	0.09794			E_USED(t-1)
	AR1_1_2	-0.13866	0.04195			GDP(t-1)
	AR2_1_1	-0.13663	0.20670	-0.66	0.5120	D_E_USED(t-1)
	AR2_1_2	0.75184	0.21081	3.57	0.0009	D_GDP(t-1)
	AR3_1_1	-0.51928	0.20784	-2.50	0.0163	D_E_USED(t-2)
	AR3_1_2	0.02215	0.23593	0.09	0.9256	D_GDP(t-2)
D_GDP	CONST2	-57.58941	468.26929	-0.12	0.9027	1
	XL0_2_1	-253.28196	108.55469	-2.33	0.0243	POP(t)
	XL1_2_1	262.11215	109.93835	2.38	0.0215	POP(t-1)
	AR1_2_1	-0.15786	0.06945			E_USED(t-1)
	AR1_2_2	-0.06762	0.02975			GDP(t-1)
	AR2_2_1	-0.14076	0.14657	-0.96	0.3421	D_E_USED(t-1)
	AR2_2_2	0.36364	0.14948	2.43	0.0191	D_GDP(t-1)
	AR3_2_1	-0.37766	0.14738	-2.56	0.0139	D_E_USED(t-2)
	AR3_2_2	0.30692	0.16730	1.83	0.0733	D_GDP(t-2)

Table 6. Univariate Model (16) and Model (17) ANOVA Diagnostics

Variable	R-Square	Standard Deviation	F Value	p-value
E_USED	0.4488	234.66329	4.48	0.0005
GDP	0.3214	166.40074	2.61	0.0200

Table 7. Univariate Model (16) and Model (17) White Noise Diagnostics

Variable	Durbin Watson	Normality		Normality ARCH	
		Chi-Square	p-value	F Value	p-value
E_USED	1.86280	62.30	<0.0001	1.03	0.3148
GDP	1.99715	37.04	<0.0001	1.87	0.1778

Table 8. Univariate Model AR Diagnostics

Variable	AR1		AR2		AR3		AR4	
	F Value	P-value	F Value	p-value	F Value	p-value	F Value	p-value
E_USED	0.76	0.3864	0.46	0.6346	0.42	0.7415	0.75	0.5627
GDP	0.00	0.9987	0.11	0.8979	0.51	0.6763	0.89	0.4777

Table 9. Roots of AR Characteristic Polynomial

Index	Real	Imaginary	Modulus	Radian	Degree
1	1.0000	0.0000	1.0000	0.0000	0.0000
2	0.7764	0.0000	0.7765	0.0000	0.0000
3	0.5221	0.0000	0.5221	0.0000	0.0000
4	0.0297	0.8438	0.8444	1.5355	87.9781
5	0.0297	-0.8438	0.8444	-1.5355	-87.9781
6	-0.5224	0.0000	0.5225	3.1416	180.0000

Table 10. Granger-Causality Wald Test

Test	Variable	Null hypothesis (Ho)	DF	Chi-square	p-value
1	Group 1: Variable E_Used Group 2: Variable GDP	Energy used is affected only by itself , and not affected by GDP.	3	7.80	0.0664
2	Group 1: Variable GDP Group 2: Variable E_Used	GDP is affected only by itself, and not affected by Energy used.	3	11.51	0.0093

Table 11. Simple Impulse Response of Transfer Function by Variable

Variable Response\Impulse	Lead	POP	Variable Response\Impulse	Lead	POP
E_Used	0	-539.4683	GDP	0	-253.2819
	1	110.1049		1	94.9485
	2	508.8844		2	232.3437
	3	31.1632		3	-8.3068
	4	-297.4651		4	-141.3661
	5	6.0250		5	19.5768
	6	248.5994		6	116.3801
	7	37.5989		7	5.1050
	8	-153.9304		8	-73.8396
	9	-19.9475		9	-0.7619
	10	120.9597		10	58.2566

Table 12. Forecast

Var	Obs	Forecast	Std	95% Confidence Limit	
E_Used	57	10097.5921	242.8749	9621.5660	10573.6183
	58	9078.7278	316.1669	8459.0520	9698.4036
	59	9339.0955	335.1089	8682.2941	9995.8969
	60	10499.3800	343.4400	9826.2499	11172.5101
	61	10744.9539	373.6883	10012.5383	11477.3696
	62	10314.8227	399.6654	9531.4928	11098.1525
	63	10470.2108	410.6386	9665.3738	11275.0477
	64	11138.4962	423.1594	10309.1189	11967.8736
	65	11404.4619	443.9841	10534.2690	12274.6548
	66	11272.4364	462.8400	10365.2866	12179.5862
GDP	57	4767.8427	168.9781	4436.6517	5099.0338
	58	4189.7040	273.6241	3653.4105	4725.9975
	59	4428.0422	384.1868	3675.0498	5181.0346
	60	4965.0274	447.3442	4088.2489	5841.8060
	61	5070.6478	496.9956	4096.5542	6044.7414
	62	4891.9698	550.8070	3812.4078	5971.5319
	63	5035.4454	605.3959	3848.8911	6221.9998
	64	5384.0910	647.7873	4114.4511	6653.7309
	65	5519.9078	682.6052	4182.0262	6857.7895
	66	5488.4396	718.6336	4079.9435	6896.9356

equal covariance is fulfilled. Table 8 is the F test to test the AR(1), AR(1,2), AR(1,2,3), and AR(1,2,3,4) residuals models with the null hypothesis that the residuals are uncorrelated. Table 8 shows that all the tests that the null hypothesis are not rejected. Therefore, the residuals are uncorrelated this means that the residuals fulfill the assumption of white noise (Wei, 2006).

From the result of analysis root AR characteristic polynomial (Table 9) shows modulus < 1, this indicates that the VECMX(3,1) with cointegration rank=1 is a stable model

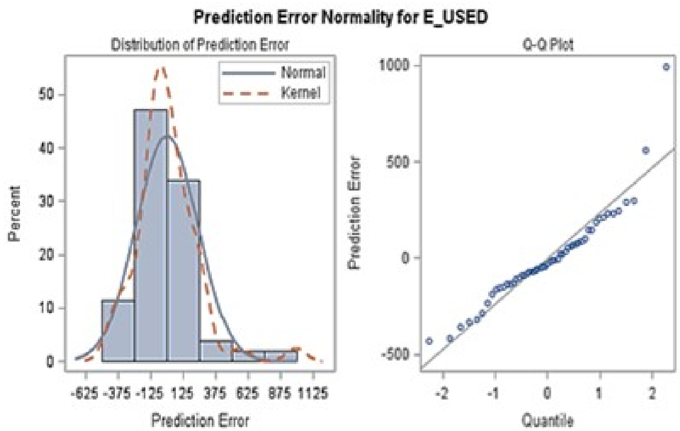


Figure 2. Prediction Error Normality for Data E_Used

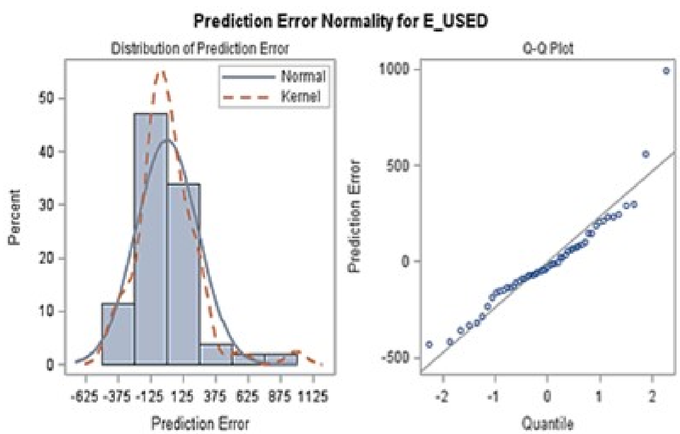


Figure 3. Prediction Error Normality for Data GDP

(Hamilton, 1994; Tsay, 2014).

From the analysis results of the VECMX(3,1) model with cointegration rank=1 and the diagnostic model, it shows that the VECMX(3,1) model with cointegration rank=1 is a reliable model and can be used to further explain the relationship patterns between time series data. Energy used and GDP and the effect of exogenous variable population.

3.3 Discussion

From the results of analysis Model (11) and Table 5, the effect of one variable on other variables can be described as follows:

Figure 4 shows that the variable D(E_Used$_t$) is significantly influenced by POPt, POPt-1, D(GDP$_{t-1}$), and D(E_Used$_{t-1}$) data with the respective p-values being 0.0010, 0.0008, 0.0009, and 0.0163. The variable D(GDP$_t$) is significantly influenced by data POPt, POP$_{t-1}$, D(GDP$_{t-1}$), D(E_Used$_{t-1}$) and D(GDP $_{t-2}$) with respective P-values being 0.0243, 0.0215, 0.0191, 0.0139 and 0.0733. The population at time t (POP$_t$) has a negative influence both on energy used at time t, D(E_Used$_t$), and on GDP at time t, D(GDP$_t$). POP$_{t-1}$ information has a positive influence both on energy used at time t, D(E_Usedt), and on

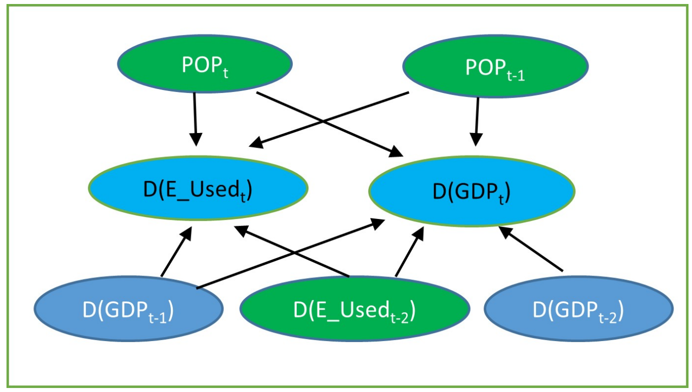


Figure 4. The sign X→Y means That Variable X Has Significant Effect on Variable Y

GDP at time t, D(GDP$_t$). Information D(GDP$_{t-1}$) has a positive influence both on energy used at time t, D(E_Used$_t$), and on GDP at time t, D(GDPt). Information D(E_Used$_{t-2}$) has a negative influence both on energy used at time t, D(E_Used$_t$), and on GDP at time t, D(GDP$_t$). Information D(GDP$_{t-2}$) has a positive influence on D(GDP$_t$).

This result is in accordance with the results of the Granger-Causality Wald test analysis, in test 1 (Table 10) shows that the hypothesis test with the null hypothesis Ho: Energy used is influenced by itself and is not influenced by GDP information; Test 2 (Table 9) shows that the hypothesis test with the null hypothesis Ho: GDP is influenced by itself and is not influenced by Energy used information. Test 1 is only significant at the significance level α=0.10, Test 2 is significant at α=0.01. Therefore, we can conclude that Energy used is not only influenced by past information itself, but is also influenced by past GDP. Likewise, GDP is not only influenced by past information about itself, but is also influenced by past Energy used.

3.4 Effect of Exogenous Variable

We can analyze the influence of population on the development of E_Used and GDP from the results in Table 5 and Figure 4. Table 5 and Figure 4 show that if there is an addition of one unit of population (addition of 1 million population) at time t and time (t-1) has a significant effect on E_Used, where if there is an addition of one unit of population in the t-th year, it affects E_Used$_t$ negatively by -539.4683 with p-value=0.0010, and a change of one unit in the t-1 year affects E_Used in year t is positively 556.5503 with p-value=0.0008. Table 5 and Figure 3 show that if there is an addition of one unit of population (addition of 1 million population) at time t and time (t-1) it has a significant effect on GDP, whereas if there is an addition of one unit of population in year t , affects GDPt negatively by -253.2819 with p-value=0.0243, and a change of one unit in the 1st year (t-1) affects GDP in year t positively by 262.1121 with p-value=0.0215.

Figure 5 and Table 11 show that if there is a shock of

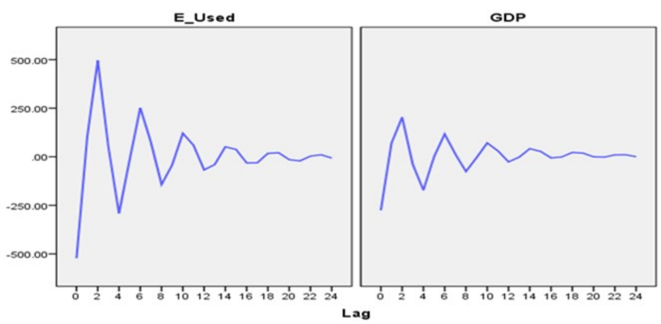


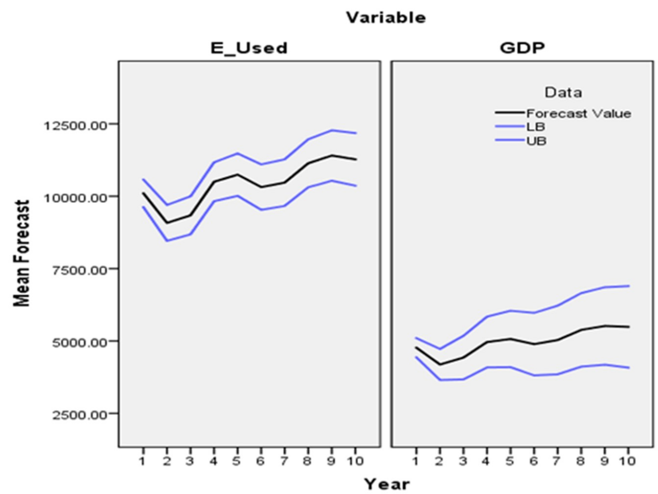
Figure 5. Response of E_Used and GDP to Impulse in POP

one unit (one million population) increase in population, then E_Used and GDP respond fluctuate in positive and negative directions for quite a long time around the next 10 years and then reach equilibrium (Figure 5). If there is a shock of one population unit (an increase of 1 million population), then the energy used (E_Used) responds in year 0 (when the shock occurs) negatively by -539.4683; for the next three years, namely the 1st, 2nd, and 3rd years, E_Used responded positively at 110.1049, 508.8844, and 31.1632 respectively; the 4th year had a negative response of -297.4651; the 5th, 6th and 7th years were positively 6.0250, 248.5994, and 37.5989 respectively; the 8th and 9th years responded negatively respectively -153.9304, -19.9475; in the 10th year the response was positive at 120.9597, and from the 11th year onwards the effect of the population shock on E_Used began to weaken towards equilibrium (Figure 5).

If there is a shock of one unit population (an increase of 1 million population), then GDP responds in year 0 (when the shock occurs) negatively by -253.2819; for the next two years, namely the 1st and 2nd years, GDP responded positively at 94.9485 and 232.3437 respectively; the 3rd and 4th years responded negatively respectively -8.3068, and 141.3661; the 5th, 6th, and 7th years were positively 19.5768, 116.3801, and 5.1050 respectively; the 8th and 9th years responded negatively respectively -73.8396, -0.7619; in the 10th year the response was positive at 58.2566, and from the 11th year onwards the effect of the population shock on GDP began to weaken towards equilibrium (Figure 5).

3.5 Forecasting

In the study model of the relationship between E_Used and GDP with Population (POP) as an exogenous variable using the Vector Error Correction Model with Exogenous variables (VECMX). Forecasting for E_Used uses Model (12) and Forecasting for GDP uses Model (13). Figure 6 (a) shows that Model (12) provides a forecasting for the next 10 years for E_Used. Figure 6 shows that the predicted energy used (E_Used) in the first two years' decreases, and the 3rd to 10th years have an upward trend. Figure 6 (b) also shows that Model (13) provides forecasting for the next 10 years for GDP. Figure 6 (b) shows that the predicted GDP in the first two years' decreases,



(a) (b)

Figure 6. Forecasting Values for (a) E_Used and (b) GDP The Next 10 Years with 95% Confidence Interval

and the 3rd to 10th years have an upward trend. In the next ten years, Indonesia's GDP is predicted to be USD 5596.3928.

4. CONCLUSIONS

Energy used, economic growth (GDP) and population are serious concerns for many countries. Therefore, studying the relationship models between them has become an interesting concern for many researchers in various countries. This research discusses the pattern of relationship between energy used and GDP with the exogenous variable, namely population, for cases in Indonesia from 1967-2023. The best model for the relationship pattern of these three variables is the Vector Error Correction Model with Exogenous (VECMX (3,1)) with cointegration rank R=1. From the results of the granger-causality analysis, there is a reciprocal relationship between energy used and GDP. Population size has a significant effect on energy used and GDP. If there is a shock to the population, the impact will be quite long on energy used and GDP, namely around 10 years and after that the effect will weaken. From the results of the forecasting analysis for the next 10 years, both energy used and GDP have an upward and fluctuating trend.

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