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THE DEVELOPMENT DYNAMIC OF HIV/AIDS IN LAMPUNG USING NONLINEAR DIFFERENTIAL EQUATION MODEL OF SIR (SUSCEPTIBLE, INFECTIOUS, AND RECOVERED)

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Abstract: *This study discusses the dynamics of the development of HIV/AIDS in Lampung using SIR nonlinear differential equation model. The data is used on the number of people of HIV/AIDS and the number of residents in Lampung in 2016-2017 from the Central Bureau of Statistics and Ministry of Health Republic of Indonesia Diseases Prevention Directorate. Stability analysis results based on the eigen values of the Jacobian matrix obtained disease free equilibrium point is* $E(S,I) = (90.909,0)$ *that are semi stable due to the threshold phenomenon with eigen values* $\lambda_1 = 0$ *and* $\lambda_1 = 8{,}93$*. The basic reproductive number of HIV/AIDS in Lampung at 90,313. These results indicate the HIV/AIDS epidemic will cause within a period of up to 100 years into the future.*

INTRODUCTION

There are several health problems in the world that have yet to be resolved. One of the health problems that are now becoming Global Issues is HIV / AIDS. HIV (Human Immunodeficiency Virus) is a family of retroviruses that attacks the human immune system, especially lymphocytes (white blood cells). People who are infected with HIV will sooner or later suffer from AIDS (Acquired Immuno Defiency Syndrome), which is a disease which is a collection of symptoms due to a decreased immune system. HIV / AIDS can spread through body fluids, namely blood, sperm, vaginal fluids, and breast milk infected with HIV. Moreover, babies born to HIV-infected mothers can also become infected with HIV.

HIV / AIDS cases in Indonesia were first reported in Bali in 1981. Since the reporting of this case, the number of people with HIV / AIDS cases in Indonesia has tended to increase every year and has spread to 34 provinces in Indonesia. Lampung Province is in position 20 with 917 AIDS cases reported, in the last year there has been a significant increase and this proportion is expected to continue to increase in the next few years (Ministry of Health, 2019). A mathematical model is a application that can be used to understand disease due to viral infection in an individual population and predict the process of infection progression and the likelihood of reinfection in an individual. To be able to predict the development of the number of patients infected with HIV / AIDS can be modeled using epidemiological mathematical models, namely the nonlinear differential equation model SIR (Susceptible, Infectious, and Recovered). The model was first introduced in 1927 by Kermack and McKendrick [3].

METHODS

1.1 The SIR Epidmic Model

According to [3], explains that the SIR epidemic model consists of three categories, namely: susceptible (S) or individuals who are susceptible to disease, infected (I) or individuals who are infected and can spread the disease to susceptible and recovered individuals (R) or individuals who are assumed to have recovered or have returned to normal immunity so that they are immune to disease. The total number of these individuals is: polynomial functions are functions that have many terms in the independent variable.
The form of the polynomial function equation is as follows:

$$n = s + i + r. \quad (1)$$

SIR models are generally written in the form of ordinary differential equations which are part of the deterministic model, the differential equations obtained in this translation are as follows:

$$\frac{ds}{dt} = -rSI \quad (2)$$

$$\frac{dI}{dt} = -rSI - aI \quad (3)$$

$$\frac{dR}{dt} = aI \quad (4)$$

1.2 Nonlinear Ordinary Differential Equations System

According to [1], a nonlinear ordinary differential equation is a nonlinear ordinary differential equation. Suppose a system of ordinary differential equations is stated as follows:

$$\dot{x} = \frac{dx}{dt} = f(\mathrm{t}, x) \quad (5)$$

with

$$x = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \quad (6)$$

$$f(x,t) = \begin{bmatrix} f1(t, x_1, x_2, \dots, x_n) \\ f2(t, x_1, x_2, \dots, x_n) \\ \vdots \\ fn(t, x_1, x_2, \dots, x_n) \end{bmatrix} \quad (7)$$

1.3 Fixed Point Stability

According to [5], suppose there is a system of linear differential equations $\dot{x} = Ax$ with $A = \begin{bmatrix} k & l \\ m & n \end{bmatrix}$ has a characteristic equation $\lambda^2 - \tau\lambda + \delta = 0$ with $\tau = k + n$ and $= \det(A) = kn - ml$. The eigen values of A are:

$$\lambda_{1,2} = \frac{1}{2}\left(\sqrt{\tau \pm 4\delta}\right) \quad (8)$$

the stability of the point $\bar{x}$ can still be determined by paying attention to the eigenvalues, namely λ_i with
$i = 1,2, \dots$, n obtained from the characteristic equation.

1.4 Basic Reproduction Numbers

According to [2], the basic reproduction number is a number that shows of susceptible individuals who can suffer from a disease caused by an infected individual. The basic reproduction number is denoted by R_0 and expressed as follows:
the Riemann-Liouville fractional derivative is defined as follows:

$$R_0 = \frac{r}{a}N = \frac{r}{a}S_0 \quad (9)$$

Some of the conditions that will arise are:
1. If $R_0 < 1$, the disease will disappear.
2. If $R_0 = 1$, 1, the disease will persist.
3. If $R_0 > 1$, the disease will escalate to become an epidemic.

The data used are secondary data on the number of HIV / AIDS sufferers and the total population in Lampung Province obtained from the Central Statistics Agency (BPS) of Lampung Province and the Ministry of Health of the Republic of Indonesia, Directorate General of Disease Prevention, and using methods literature study, namely studying text books in the Mathematics Department library and the University of Lampung library, journals and internet access that support the research process.

The procedure in the method is as follows:
1. Estimating the parameters of the rate of recovery (infectious individuals to recovered) and the rate of disease transmission (susceptible individuals to infected individuals) in the S (Susceptible), I (Infected) and R (Recovered) subpopulations.
2. Specifies the fixed point of the model.
3. Do an stability analysis using the linearization method.
4. Determine the basic reproduction number (R_0).
5. Plot the S, I, and R subpopulations and take a snapshot of the system phase using Maple 12 Software.

RESULTS AND DISCUSSION

The data used to determine these parameters is data on the number of AIDS sufferers and the number of patients who died in 2016 and 2017.

Table 1. Number of HIV / AIDS sufferers in Lampung 2016-2017

Year	Number of Cases Found		
	HIV	AIDS	Died
2016	381	76	-
2017	580	41	5

from the table above obtained:

$$S(0) = S(2016) = 8.210.315S(t) = S(2017) = 8.209.858$$
$$S(t) = S(2017) = 8.209.858$$
$$R(t) = R(2017) = 5$$

with N stating the total population of Lampung province in 2016, amounting to 8.210.315 people. Then,

$$(t) = S(0)e^{-\frac{R(t)}{\rho}} = S(0)e^{-\frac{R(t)r}{\alpha}}$$
$$8.209.858 = 8.210.315\ e^{-\frac{5r}{\alpha}}$$
$$\frac{r}{\alpha} = 1{,}1\ \times 10^{-5}.$$

Assuming the rate of recovery or death of a person with HIV/AIDS in 10 years is $\frac{1}{10}$, it is obtained $a = 10^{-1} = 0{,}1$ and $r = 1{,}1\ \times 10^{-6}$, means that the rate of recovery from HIV / AIDS infected individuals to recover or die (Recovered) in Lampung based on 2016-2017 data is 0.1 and the rate of disease transmission from susceptible individuals to infected individuals is $1{,}1\ \times 10^{-6}$.

1.5 Fixed Point Determination

Looking for a fixed point from the following system of algebraic equations which is equal to zero in order to be balanced.

$$\frac{dS}{dt} = -rSI = 0 \tag{10}$$
$$\frac{dI}{dt} = rSI - aI = 0 \tag{11}$$
$$\frac{dR}{dt} = aI = 0 \tag{12}$$

Will look for the value of x using Equation (11) as follows:

$$\frac{dI}{dt} = rSI - aI = 0$$
$$rSI = aI$$
$$S = \frac{a}{r} \tag{13}$$

Since the values of S, r, a are not equal to 0, then $I = 0$, then we will look for the fixedpoint value $E(S, I) = E\left(\frac{a}{r}, 0\right) = E\left(\frac{0{,}1}{1{,}1\ \times 10^{-6}}, 0\right) = E(90.909{,}0)$. The fixed point obtained is a diseasefree equilibrium or a condition where there is no spread of an infectious disease in a subpopulation of infected individuals at time t equals zero [4].

1.6 Balance Point Analysis

The stability of each fixed point will be analyzed by the Jacobi matrix as follows:

$$J=\begin{bmatrix}\frac{\partial f_1(S,I,R)}{\partial S} & \frac{\partial f_1(S,I,R)}{\partial I} & \frac{\partial f_1(S,I,R)}{\partial R}\\ \frac{\partial f_2(S,I,R)}{\partial S} & \frac{\partial f_2(S,I,R)}{\partial I} & \frac{\partial f_2(S,I,R)}{\partial R}\\ \frac{\partial f_3(S,I,R)}{\partial S} & \frac{\partial f_3(S,I,R)}{\partial I} & \frac{\partial f_3(S,I,R)}{\partial R}\end{bmatrix}$$

$$J=\begin{bmatrix}0 & -rS & 0\\ 0 & rS-a & 0\\ 0 & a & 0\end{bmatrix}$$

$$\det(\lambda I-J)=0$$

$$(\lambda I-J)=\begin{bmatrix}\lambda & -rS & 0\\ 0 & \lambda-(rS-a) & 0\\ 0 & a & \lambda\end{bmatrix}$$

by expanding the third column it is obtained

$$(\lambda I-J)=\begin{bmatrix}\cdots & \cdots & 0\\ 0 & \lambda-rS-a & \vdots\\ 0 & a & \vdots\end{bmatrix}-\begin{bmatrix}\lambda & -rS & \vdots\\ \cdots & \cdots & 0\\ 0 & a & \vdots\end{bmatrix}+\begin{bmatrix}\lambda & -rS & \vdots\\ 0 & \lambda-(rS-a) & \vdots\\ \cdots & \cdots & \lambda\end{bmatrix}$$

$$|\ \lambda I-J\ |=0\begin{bmatrix}0 & \lambda-(rS-a)\\ 0 & a\end{bmatrix}-0\begin{bmatrix}0 & -rS\\ 0 & a\end{bmatrix}+\lambda\begin{bmatrix}\lambda & -rS\\ 0 & \lambda-(rS-a)\end{bmatrix}$$

$=\lambda^2(\lambda-rS+a)$

obtained eigen values that are valued at $\lambda_1=0$ and $\lambda_2=rS-a$. The eigen value $\lambda_1=0$ indicates a neutral direction of stability along the fixedpoint axis, while for the eigen values $\lambda_2=rS-a$ there are two possible conditions, namely:

1. $\lambda_2=rS-a$ will be positive if $S>\frac{a}{r}$
2. $\lambda_2=rS-a$ will be negative if $S<\frac{a}{r}$.

If the parameter and variable values obtained through data analysis are substituted for $\lambda_2=rS-a$, then the positive eigenvalues will be obtained, namely $\lambda_2=8{,}93$. The stability of this fixed point E will be depicted with a phase portrait for the subpopulations S and I (susceptible-infected) using Maple 12 software. The phase portrait from fixed point E is presented as in the following figure

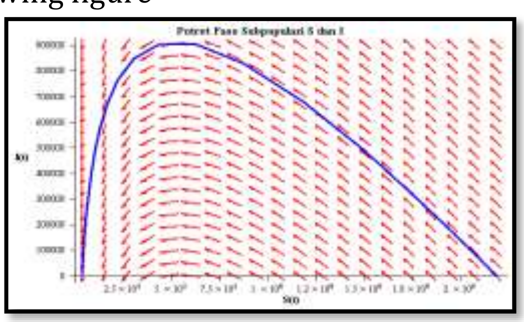

Figure 1. Portrait of the S and I subpopulation phases

1.7 Determination of Basic Reproduction Numbers

The basic reproduction number seen from the equilibrium point of the model, in this case is denoted by R_0. If R_0<1, then the disease will not be epidemic and tends to disappear from the population, but if R_0>1 then the disease will spread in the population

$$R_0 = \frac{r}{a}N = \frac{r}{a}S_0$$

$$R_0 = \frac{1{,}1 \times 10^{-6}}{0{,}1} 8.210.315$$

$$R_0 = 90{,}313.$$

The basic reproduction number obtained is $R_0 = 90{,}313$ the number of individuals who are susceptible to being directly infected by another infected individual if the infected individual enters a population that is entirely still vulnerable. This means that one infected individual (infectious) can transmit the disease to 90 to 91 susceptible individuals in the population on average.

1.8 Plot of Changes in Subpopulation Forms S, I and R Against Time

The plot of changes in the S, I and R subpopulations uses the following parameter and variable values:

$a = 0{,}1$; $r = 0{,}0000011$;$N(0) = N(2016) = 8.210.315$;$S(0) = S(2016) = 8.210.315$;
$S(t) = S(2017) = 8.209.858$; $I(0) = I(2016) = 76$;$I(t) = I(2017) = 41$;$R(0) = R(2016) =$ 0;
$R(t) = R(2017) = 5$. The dynamic model of the spread of HIV / AIDS in Lampung can be assumed as follows:

$$\frac{dS}{dt} = 8.210.315 - 1{,}1 \times 10^{-6} S(t)I(t);$$

$$\frac{dI}{dt} = 1{,}1 \times 10^{-6} S(t)I(t) - 0.1I(t);$$

$$\frac{dR}{dt} = 0.1I(t)$$

by using the DE-plot command in the DE-tools software package Maple 12, the proportion of S, I and R subpopulations in a certain time interval is obtained as follows:

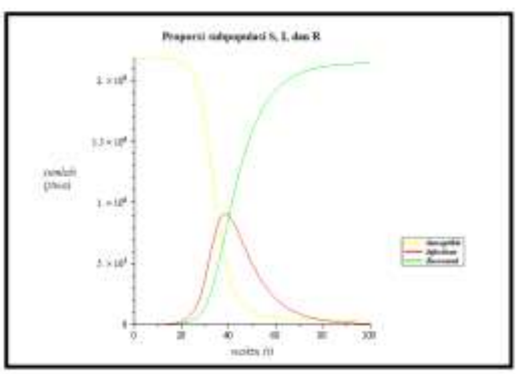

Figure 2. Proportion of S, I and R subpopulations for $0 \leq t \leq 100$

The figure explains that at $t \leq 100$ years, the number of susceptible individuals decreases while the number of infected and recovered individuals increases over time. Then, the number of infected individuals will increase and reach a maximum number within 10-40

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years. After reaching the maximum number of 900,000 people, the number of infected individuals will continue to decrease and this will happen in the next 40-100 years.

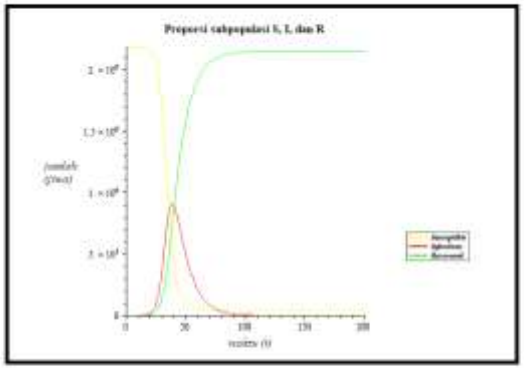

Figure 3. Proportion of S, I and R subpopulations for $0 \leq t \leq 200$

At $t > 100$ years, the number of individuals in the subpopulation susceptible to infection and recovery / death did not change significantly. This condition is called the equilibrium or stable condition of the system. This equilibrium condition is achieved by the system when $E = (S, I) = (90.909{,}0)$. The fixedpoint E that has been obtained is a disease-free fixed point because $I = 0$. In this condition, there is no spread of infectious disease in the population because the number of subpopulations of individuals is infected at time $t = 0$ HIV / AIDS in Lampung will only always be present in the next 100 years. Therefore, HIV / AIDS in Lampung will be endemic during this period.

CONCLUSION

1. A mathematical model on the dynamics of HIV / AIDS development in Lampung using the SIR model obtained the following parameters:

$$\frac{dS}{dt} = 8.210.315 - 1{,}1 \times 10^{-6} S(t)I(t);$$

$$\frac{dI}{dt} = 1{,}1 \times 10^{-6} S(t)I(t) - 0.1I(t);$$

$$\frac{dR}{dt} = 0.1I(t);$$

2. A diseasefree fixed point is obtained, namely $E(S, I) = (90.909{,}0)$ which is semi-stable due to the threshold phenomenon with eigenvalues $\lambda_1 = 0$ and $\lambda_2 = 8{,}93$.
3. It was found that the reproductive number of HIV / AIDS in Lampung was $R_0 = 90{,}313$, which increased to become an epidemic or cause an epidemic within the next 100 years.

REFERENCE

[1] Braun, M. 1983. Differential Equations and Their Applicatons. Springer Verlag, London.
[2] Giesecke, J. 1994. Modern Infectious Disease Epidemiologi. Oxford University, New York.
[3] Murray, J.D. 2002. Mathematical Biology: An Introduction. 3rd Edition. Springer Verlag, New York.
[4] Tjolleng, Amir. 2013. Dinamika Perkembangan HIV/AIDS di Sulawesi Utara Menggunakan Model Persamaan Diferensial Nonlinear SIR (Susceptible, Infectious &

Recovered). Jurnal Ilmiah Sains. 13:4-7.
[5] Tu, P.N.V. 1994. Dynamical System: An Introduction with Applications in Economics and Biology. Springer Verlag, New York.
[6] Meyer, W. J. 1984. *Concepts of Mathematical Modeling,* McGraw-Hill, Inc, New York.
[7] Panvilov, A. 2004. *Qualitative Analysis of Differential Equation*. Utrecht University, Utrecht.
[8] Perko, L. 1991. *Differential Equations and Dynamical System*. Springer-Verlag Berlin Heidelberg, New York.
[9] Wiggins, S. 1990. *Introduction to Applied Nonlinear Dynamical System and Chaos.* Springer Verlag, New York.