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STOCHASTIC CHARACTERISTICS OF DAILY RAINFALL AT PURAJAYA REGION

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ABSTRACT

Aim of this research is to study stochastic characteristics of daily rainfall series. The study was undertaken using 25 years (1977-2001) data of Purajaya region. The series of the daily rainfall data assumed was trend free. The periodic components of daily rainfall series were represented by using 253 harmonic expressions and stochastic components of daily rainfall series were presented using second order autoregressive parameters. Validation of generated daily rainfall series was done by comparing between generated with measured daily rainfall series. For periodic modeling, mean of the correlation coefficient between generated and measured daily rainfall series with the number of the data N is equal to 512 days for 25 years was found to be 0,9576. For periodic and stochastic modeling, mean of the correlation coefficient was found to be 0.9999. Therefore, developed periodic and stochastic model could be used for future prediction of daily rainfall time series.

Keyword: autoregressive model, daily rainfall prediction, Purajaya region, fourier analysis, least squares method.

INTRODUCTION

To design water consuming of irrigation, detailed information about the rainfall with respect to time is required. To provide long sequence record of rainfall data was very difficult, so sometime, to extend the rainfall record, generating the synthetic rainfall record is necessary. Various methods have been used by Engineers and scientists to provide this information. Most the existing methods are either deterministic or probabilistic in nature, Kotegoda, (1980) and Yevjevich (1972). While the former methods do not consider the random effects of various input parameter, the later method employ the concept of probability to the extent that the time based characteristics of rainfalls are ignored. With the ever increasing demand for accuracy of analyzing rainfall data, these methods are no longer sufficient.

The rainfalls are periodic and stochastic in nature, because they are affected by climatological parameter, i.e., periodic and stochastic climate variations are transferred to become periodic and stochastic components of rainfall. Hence the rainfall should be computed considering both the periodic and the stochastic parts of the process. Considering all other factors known or assumed that the rainfall is a function of the stochastic variation of the climate. Hence periodic and stochastic analysis of rainfall series will provide a mathematical model that will account for the periodic and stochastic parts and will also reflect the daily variation of rainfall.

During the past years, some researches that study the periodic and stochastic modeling have been published by Zakaria (1998), Rizalihadi (2002), Bhakar (2006), Zakaria (2008). Rizalihadi (2002) and Bhakar (2006) studied periodic and stochastic modeling for monthly rainfall series, but Zakaria (2008) have studied for daily rainfall series.

Aim of this research is to study stochastic characteristics of daily rainfall series in Purajaya using fast Fourier transform, Fourier analysis, autoregressive model and method of least squares. The model can be used to provide synthetic and reasonably rainfall data for planning the irrigation or water resource projects in the future.

MATERIALS AND METHODS

Study area

The study area comes under the humid region of the subdistrict of West Lampung, Profince of Lampung, Indonesia.

Collection of rainfall data

Daily rainfall data of Purajaya region was collected from Indonesian Meteorological, Climatological and Geophysical Agency, Profince of Lampung. Rainfall data for a period of 25 years (1977-2001) was used in the study.

The mathematical procedure adopted for formulation of a predictive model has been discussed as follows: The principal aim of the analysis was to obtain a reasonable model for estimating the generation process and its parameters by decomposing the original data series into its various components.

Generally a time series can be decomposed into a deterministic or periodic component, which could be formulated in manner that allowed exact prediction of its value, and a stochastic component, which is always present in the data and can not strictly be acounted for as it is made by random effects. The time series X(t), was represented by a decomposition model of the additive type, as folows: (Rizalihadi, 2002; Bhakar, 2006; dan Zakaria, 2008),

$$X_{(t)} = T_{(t)} + P_{(t)} + S_{(t)} \qquad (1)$$

Where

$T_{(t)}$ = trend component
$P_{(t)}$ = periodic component

$S_{(t)}$ = stocastic component
t = 1,2,3,...,N
N = number of observation points

The trend component describes the long smooth movement of the variable lasting over the span of observations, ignoring the short term fluctuations. A hypothesis of no trend was made. So the equation can be presented as an equation as follows:

$$X_{(t)} \approx P_{(t)} + S_{(t)} \qquad (2)$$

Equation (2) is as a approximation equation to model a periodic and stochastic modeling of daily rainfall.

Spectral method

Spectral method is one of the transformation method which widely used in many applications. it can be presented as Fourier transform as follows, (Zakaria, 2003; Zakaria, 2008):

$$P(f_m) = \frac{\Delta t}{2\sqrt{\pi}} \sum_{n=-N/2}^{n=N/2} P(t_n).e^{\frac{-2\pi i}{M}.mn} \qquad (3)$$

Where $P(t_n)$ is a daily rainfall data series in time domain and $P(f_m)$ is a daily rainfall data series in frequency domain. Where the $P(f_m)$ is used in Equation (4) and (5) as an angular frequency (ω_r). The t_n is a series of time that present a length of the rainfall data to N, The f_m is a series of frequencies.

Based on the rainfall frequencies resulted using Equation (3), amplitudes as functions of the rainfall frequencies can be generated. The maximum amplitudes can be obtained from the amplitudes as significant amplitudes. The rainfalls frequencies of significant amplitudes have been used to simulated synthetic daily rainfalls were assumed as significant rainfall frequencies. The significant rainfall frequencies resulted in this study was used to calculate the angular frequencies and obtain the periodic components of Equation (4) or (5).

Periodic components

The periodic component $P(t)$ concerns an oscillating movement which is repetitive over a fixed interval of time (Kottegoda 1980). The existence of $P(f_m)$ was identified by the fourier transformation method. The oscillating shape verifies the presence of $P(f_m)$, with the seasonal period, at the multiples of which peak of estimation can be made by a Fourier Analysis. The frequencies of the spectral method clearly showed the presence of the periodic variations indicating its detection. The periodic component $P(t)$ was expressed in Fourier series as follows (Zakaria, 1998):

$$\hat{P}(t) = S_o + \sum_{r=1}^{r=k} A_r \sin(\omega_r.t) + \sum_{r=1}^{r=k} B_r \cos(\omega_r.t) \qquad (4)$$

Equation (4) could be arranged to be Equation (5) as follows,

$$\hat{P}(t) = \sum_{r=1}^{r=k+1} A_r \sin(\omega_r.t) + \sum_{r=1}^{r=k} B_r \cos(\omega_r.t) \qquad (5)$$

Where

$\hat{P}(t)$ = model of periodic component
$S_o = A_{K+1}$ = mean of daily rainfall (mm)
ω_r = angular frequencies (rad)
t = time (day)
A_r, B_r = Fourier coefficients
k = number of significant harmonics

Stochastic components

The stochastic component was constituted by various random effects, which could not be estimated exactly. In the case of rainfall series from Purajaya region. A stochastic model in the form of autoregressive model was used for the presentation in the time series. This model was applied to the $\hat{S}(t)$ which was treated as a random variable. The deterministic components were removed and the residual was stationary in nature. Mathematically, an autoregressive model of order p can be written as follows:

$$\hat{S}_t = \varepsilon + \sum_{j=1}^{j=k} b_j.S_{t-j} \qquad (6)$$

Equation (6) can be presented as follows:

$$\hat{S}_t = \varepsilon + b_1.S_{t-1} + b_2.S_{t-2} + ... + b_k.S_{t-k} \qquad (7)$$

Where

b_k = autoregressive model parameters
ε = independent random number
j = 1, 2, 3, 4, ..., k
k = number of stochastic order

To generate a number of model parameters and independent random number of the stochastic model, method of least squares was applied.

METHOD OF LEAST SQUARES

Determination of periodic parameters

In curve fitting, as an approximate solution of periodic components $P(t)$, to determine Function $\hat{P}(t)$ of Equation (5), a procedure widely used is method of least squares. From Equation (5) we can calculate sum of squares (Zakaria, 1998) as follows:

Sum of squares =

$$J = \sum_{t=1}^{t=m} \left\{ P(t) - \hat{P}(t) \right\}^2 \qquad (8)$$

Where J depends on A_r B_r, and ω_r. A necessary condition for J to be minimum is

$$\frac{\partial J}{\partial A_r} = \frac{\partial J}{\partial B_r} = 0 \ \ \text{with } r = 1, 2,3,4,5,..., \text{k} \qquad (9)$$

Using method of least squares, we can find equations as follow:

a) mean of daily rainfall,

$$S_o = A_{k+1} \qquad (10)$$

b) amplitude of significant harmonic,

$$C_r = \sqrt{A_r^2 + B_r^2} \qquad (11)$$

c) phase of significant harmonic,

$$\varphi_r = \arctan\left(\frac{B_r}{A_r}\right) \qquad (12)$$

Mean of daily rainfall, amplitudes, and phases of significant harmonics can be substituted into an equation as follows:

$$\hat{P}(t) = S_o + \sum_{r=1}^{r=k} C_r . Cos(\omega_t . t - \varphi_r) \qquad (13)$$

Equation (13) is a harmonic model of daily rainfall where can be found based on daily rainfall series of Purajaya.

Estimation of autoregressive parameters

Using Equation (2) we can find stochastic component of daily rainfall as follows:

$$S_{(t)} \approx X_{(t)} - \hat{P}_{(t)} \qquad (14)$$

Following Equation (8), using Equation (14) and Equation (7) we can calculate sum of squares as follows:

$$\text{Sum of squares} = J = \sum_{t=1}^{t=m}\left\{S(t) - \hat{S}(t)\right\}^2 \qquad (15)$$

Where J depends on   and b_k. A necessary condition for J to be minimum is

$$\frac{\partial J}{\partial} = \frac{\partial J}{\partial b_k} = 0 \ \text{with } k = 1,2,3,4,5,...,p \qquad (16)$$

Using method of least squares, we can find independent random number   and autoregressive model parameters b_k.

RESULTS AND DISCUSSIONS

For testing the statistical characteristics of daily rainfall series, 25 years data (1977-2001) of daily rainfall from station Purajaya was taken. The statistical characteristic of the annual mean and maximum rainfall of daily rainfall series were estimated. Figure-1 shows the daily rainfall time series.
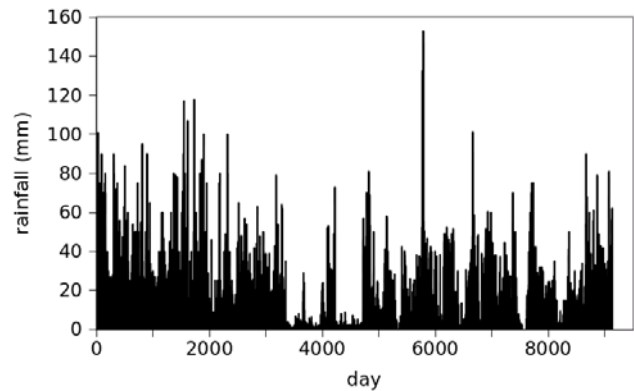
Figure-1. Variation of daily rainfall series for 25 years from Purajaya station.

Based on the analysis, mean annual daily rainfall values vary from 2.00 mm in the year of 1986 to 12.5 mm in the year of 1977. Maximum annual daily rainfall values vary from 35 mm in 1986 to 152.9 mm in the year of 1992. The variation may be attributed towards the natural changes in yearly climate. Annual cumulative rainfall of Purajaya indicates minimum 552.5 mm in the year of 1989 and maximum 4308.9 mm in the year of 1996 with mean annual cumulative rainfall 2553.5 mm.

Figure-1 presented the mean annual daily rainfall values vary from 2 mm in the year of 1986 to 12.5 mm in the year of 1977. Maximum annual daily rainfall values vary from 35 mm in the year of 1986 to 152.9 mm in the year of 1992. For annual cumulative daily rainfall indicate minimum value of 552.5 mm in the year of 1989 and maximum value of 4308.9 mm in the year of 1996 with mean annual cumulative daily rainfall value of 2553.5 mm.

Spectrum of daily rainfall time series can be generated using fast Fourier transform method. For 25 years daily rainfall data, result of the Fourier transformation is presented in Figure-2 as follows:
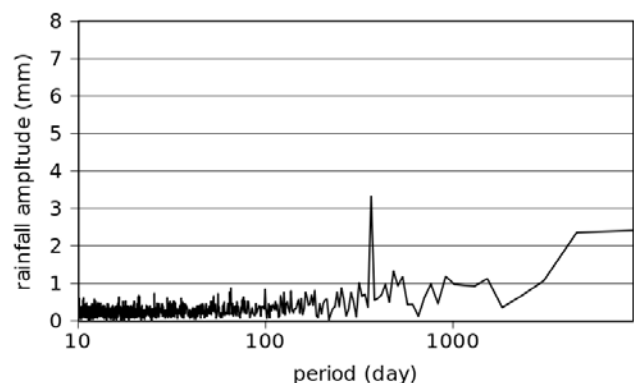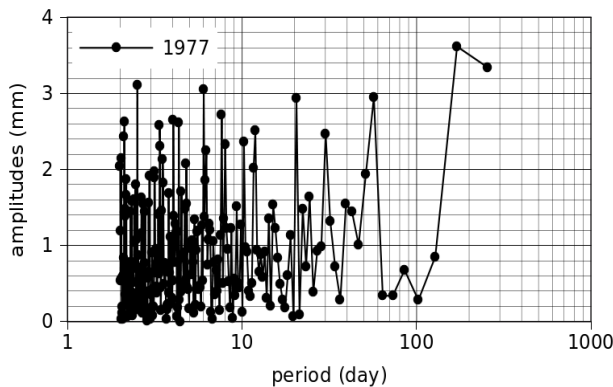

Figure-2. Variation of daily rainfall period series for 25 years from Purajaya station.

Figure-2 shows that the maximum amplitude of daily rainfall is occurred at 3.3255 mm for period of 365.2 days or one year. It indicates that the annual component of periodicity is quite dominant compared with the others.

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The spectrum above is presented in the rainfall amplitudes as a function of periods. Spectrum of daily rainfall data presented in Figure-2 was generated by using the FFT toolbox of the Matlab software.

To confirm the presence of periodic component in daily rainfall series, the Fourier transform method was applied to generate dominant rainfall frequencies. For one year daily rainfall data, 512 days of rainfall data series were used to get the dominant rainfall frequencies. The generated frequencies were obtained using an algorithm which proposed by Cooley and Tukey (1965) where the number of data N to be analyzed is a power of 2, i.e., N = 2 k. Based on the results, spectrum of one year daily rainfall, calculated and measured daily rainfalls for the year of 1977 are presented in Figures 3 and 4 as follows:

Figure-3. Variation of daily rainfall periods in the year of 1977.

Figure-3 presents periods of daily rainfall for the year of 1977, using a number of data, N equal to 512. The data is started at 1st of January for every year. The Figure presents the daily rainfall amplitudes as a function of the daily rainfall periods. The daily rainfall amplitudes vary highly. It indicates that the values of rainfall periodicities also vary highly.
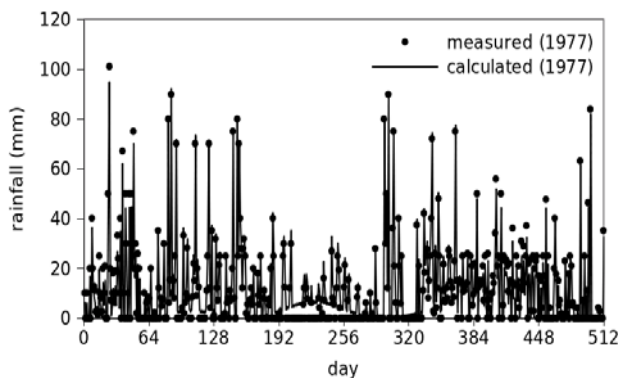
Figure-4. Variation of measured and predicted daily rainfall in the year of 1977. (P)

Figure-4 presents periodic modeling and measured daily rainfall series in the year of 1977 for a number of the data, N is equal to 512. The 253 significant

periods were generated from the spectrum of daily rainfall. The significant frequencies of calculated daily rainfall series presented here were found using the rainfall periods of daily rainfall series presented in the Figures-3. Calculated daily rainfall series presented here are predicted values of the best fitting model. Because of measured daily rainfall series highly varies in time, so calculated daily rainfall time series significantly varies in time.

Using Equation (14), stochastic components of daily rainfalls at Purajaya region have been calculated. Stochastic components of daily rainfall in the year of 1977 is presented in Figure-5 as follows:
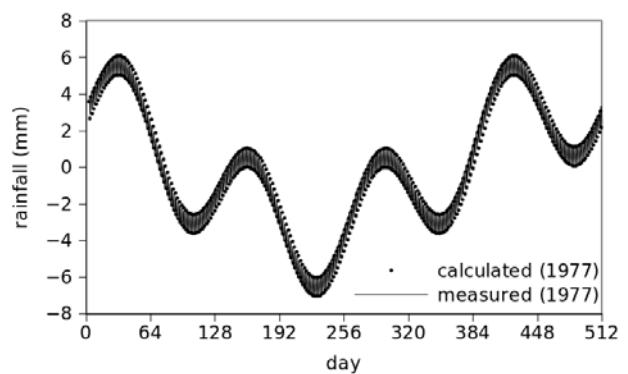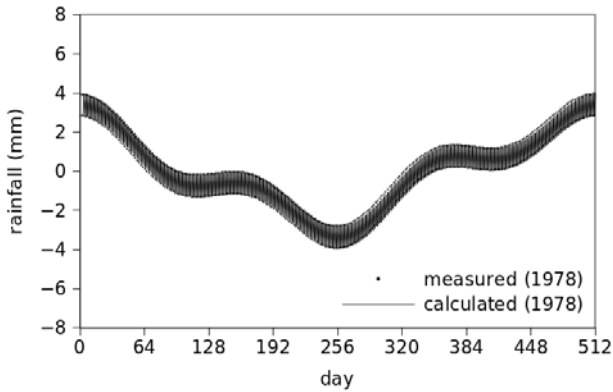

Figure-5. Stochastic components of calculated and measured rainfall in the year of 1977.

Based on an analysis of stochastic components of daily rainfall in the year of 1977 resulted in Figure-5, a periodic and stochastic modeling of daily rainfall for year of 1977 is found such as presented in Figure-6.
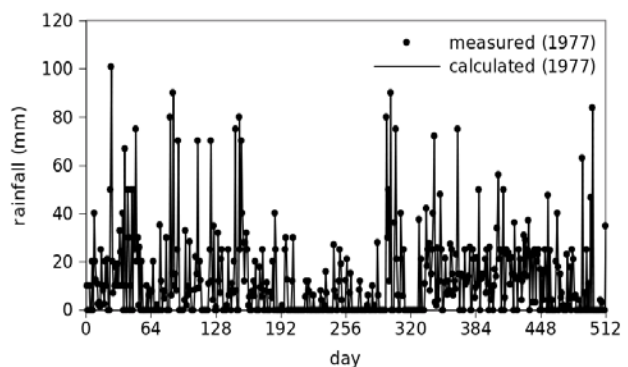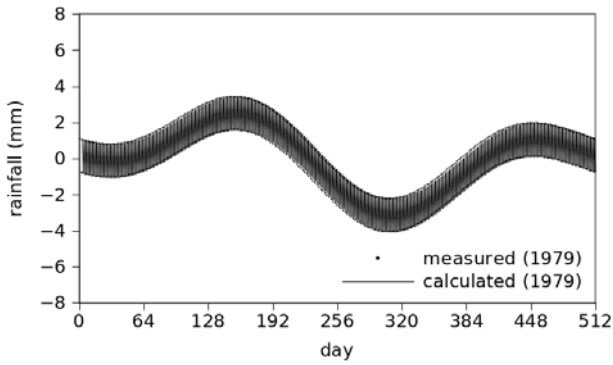

Figure-6. Variation of measured and predicted daily rainfall in the year of 1977.(P+S)
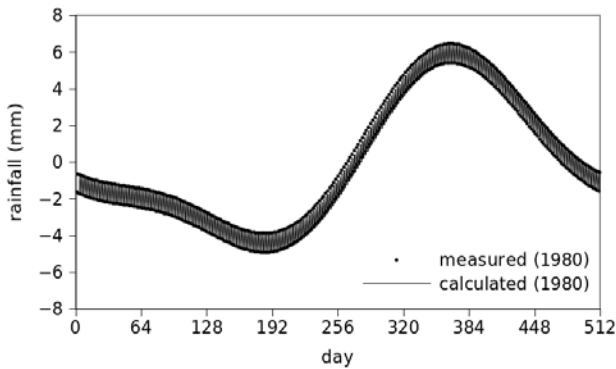
For stochastic components of daily rainfall from the years of 1977 to 2000 are presented in Figure-7 to Figure-29.
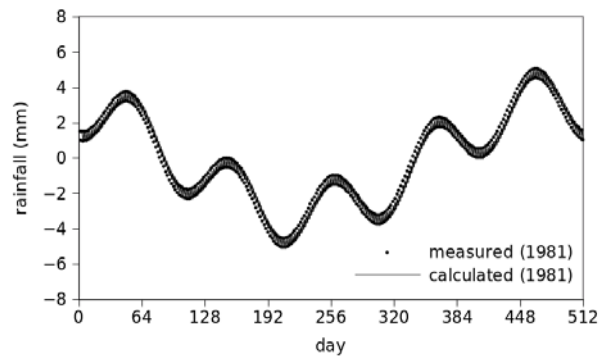
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Figure-7. Stochastic components of measured and calculated rainfall in the year of 1978.
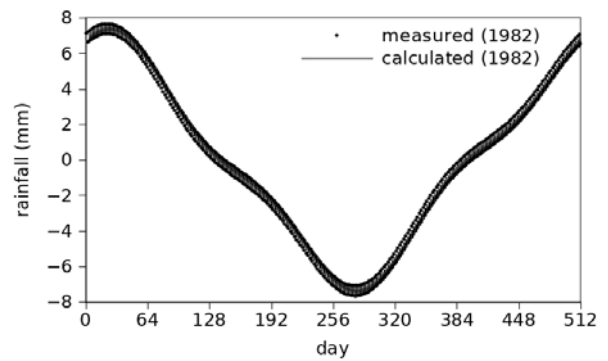

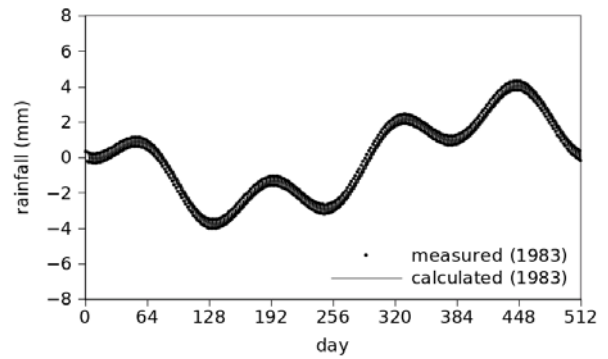**Figure-8.** Stochastic components of daily rainfall in the year of 1979.

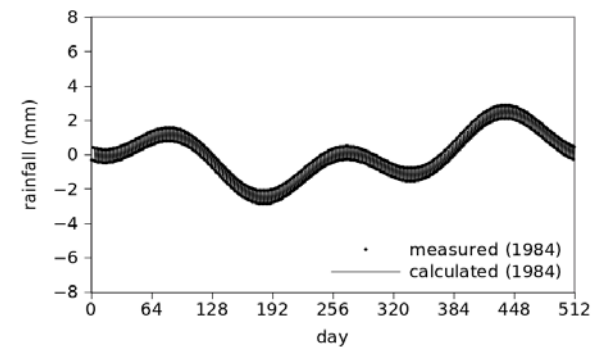**Figure-9.** Stochastic components of measured and calculated rainfall in the year of 1980.

Figure 10. Stochastic components of measured and calculated rainfall in the year of 1981.
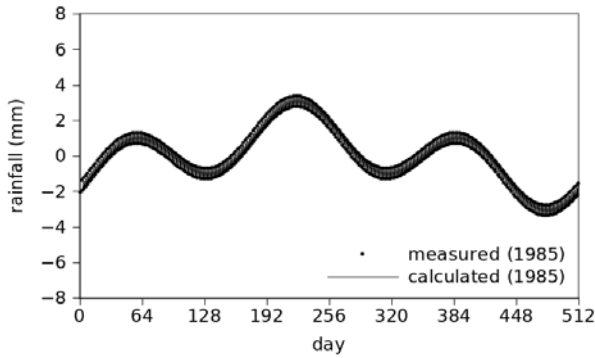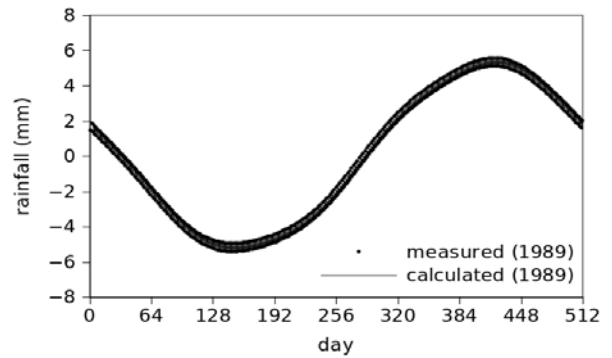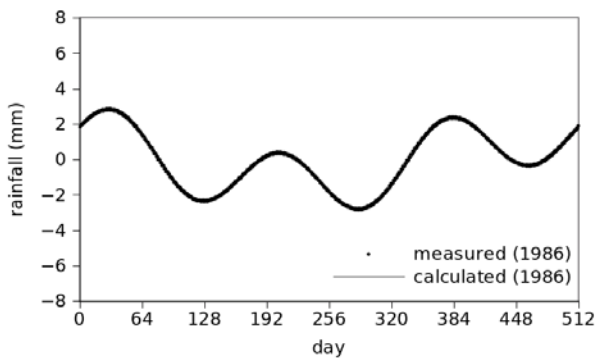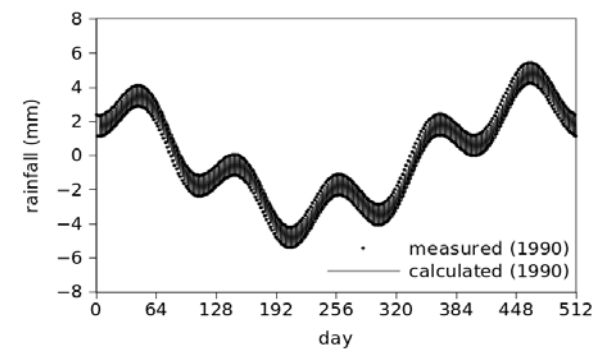

Figure-11. Stochastic components of measured and calculated rainfall in the year of 1982.
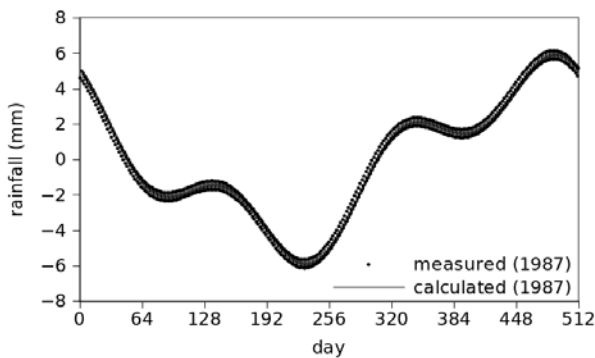

Figure-12. Stochastic components of measured and calculated rainfall in the year of 1983.
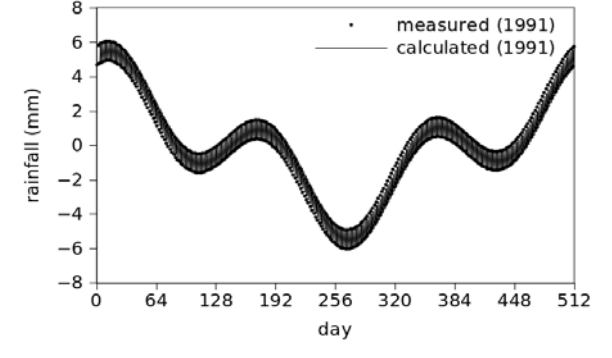

Figure 13. Stochastic components of measured and calculated rainfall in the year of 1984.
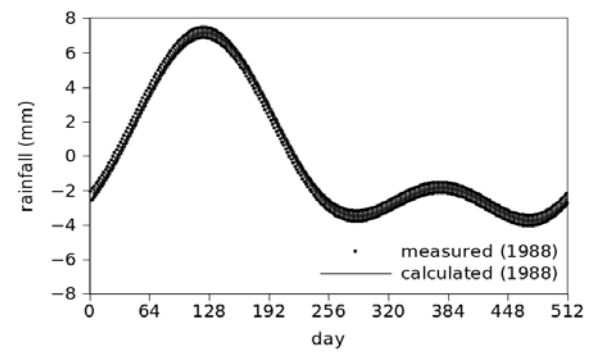
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Figure-14. Stochastic components of measured and calculated rainfall in the year of 1985.

Figure-15. Stochastic components of measured and calculated rainfall in the year of 1986.

Figure-16. Stochastic components of measured and calculated rainfall in the year of 1987.

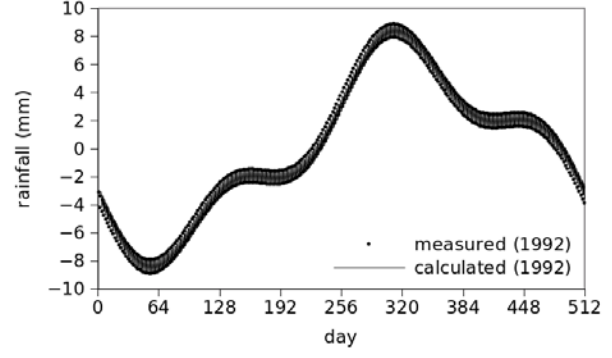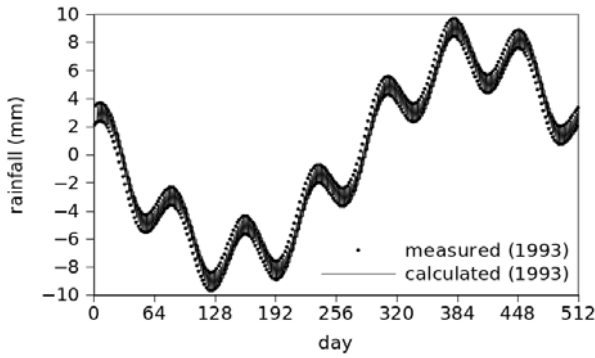**Figure-17.** Stochastic components of measured and calculated rainfall in the year of 1988.

Figure-18. Stochastic components of measured and calculated rainfall in the year of 1989.
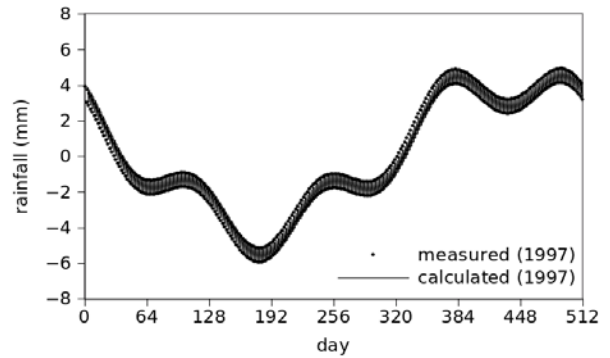

Figure-19. Stochastic components of measured and calculated rainfall in the year of 1990.
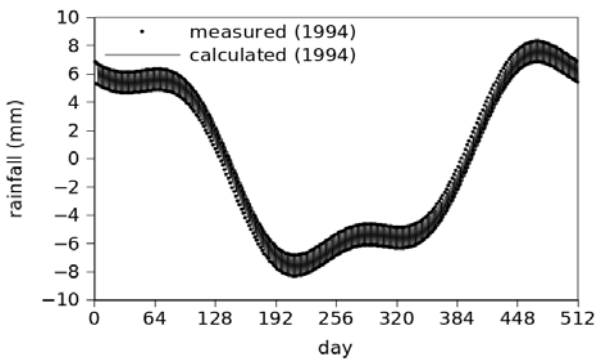

Figure-20. Stochastic components of measured and calculated rainfall in the year of 1991.

Figure-21. Stochastic components of measured and calculated rainfall in the year of 1992.

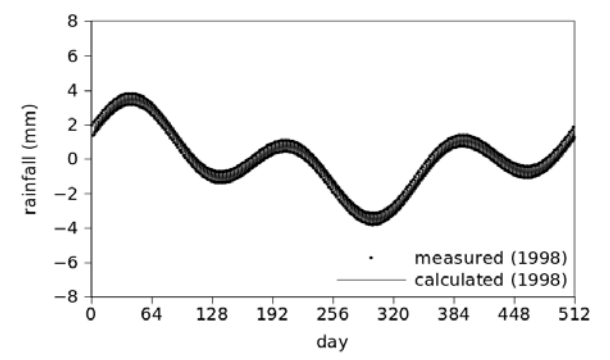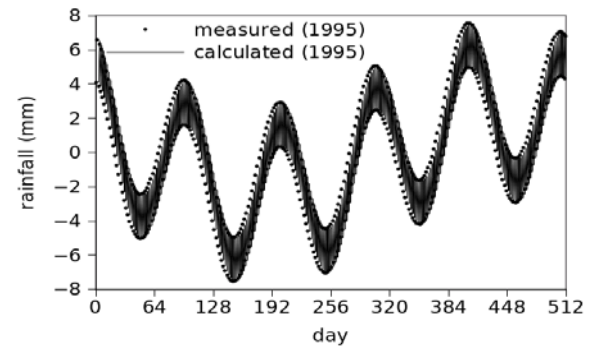## ARPN Journal of Engineering and Applied Sciences

Figure-22. Stochastic components of measured and calculated rainfall in the year of 1993.

Figure-23. Stochastic components of measured and calculated rainfall in the year of 1994.

Figure-24. Stochastic components of measured and calculated rainfall in the year of 1995.

Figure-25. Stochastic components of measured and calculated rainfall in the year of 1996.

Figure-26. Stochastic components of measured and calculated rainfall in the year of 1997.
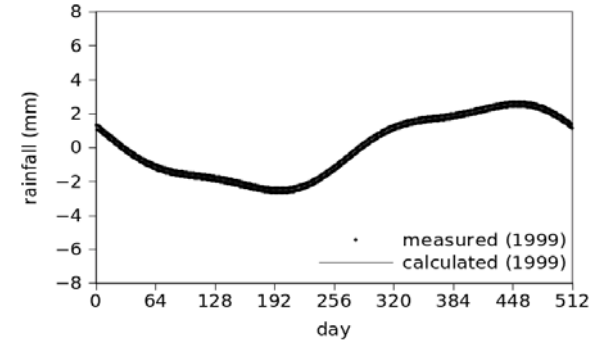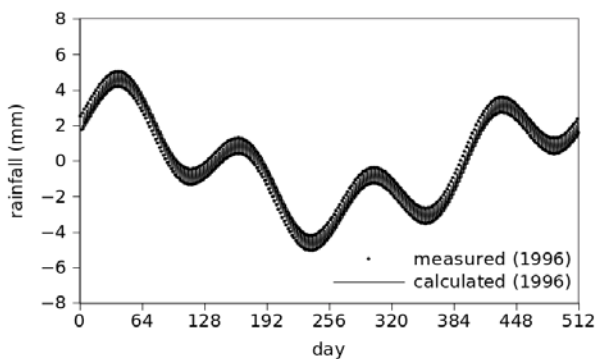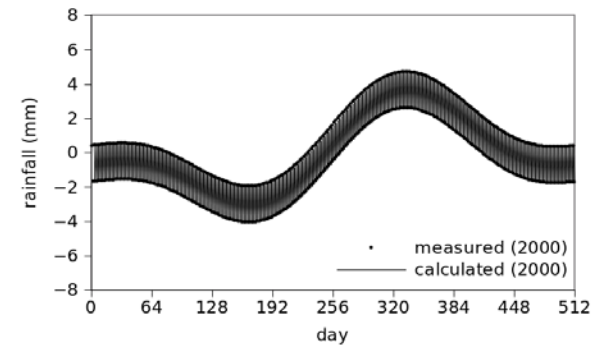

Figure-27. Stochastic components of measured and calculated rainfall in the year of 1998.

Figure-28. Stochastic components of measured and calculated rainfall in the year of 1999.

**Figure-29.** Stochastic components of measured and calculated rainfall in the year of 2000.

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Stochastic components of measured and calculated daily rainfall presented from Figure-7 to Figure-29 show that the shapes of the stochastic components of measured and calculated daily rainfall quite vary for every year. It is indicated that the stochastic characteristics of daily rainfall series significantly vary yearly.

Independent random number (ε) and second order (b_1 and b_2) autoregressive model parameters of the periodic modeling of daily rainfall for 25 years at Purajaya region are presented in Figure-30 as follows:
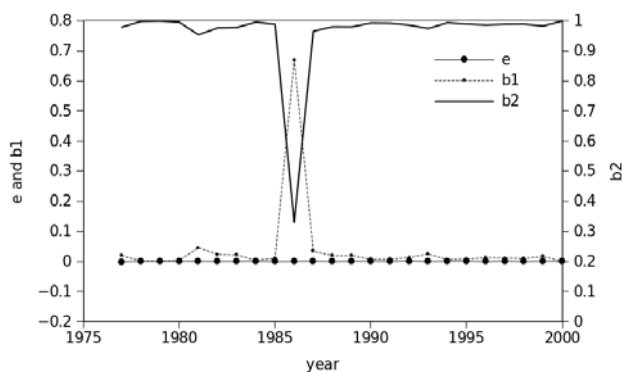
Figure-30. Variation of independent random number (ε) and autoregressive model parameters (b_1, b_2) for 25 years at Purajaya region.

Figure-30 indicates that the parameters of periodic modeling relatively is not quite varies yearly.

A coefficient of correlation R is as the best fitting parameter to measure level of correlation between calculated and measured daily rainfall series data. From the results as presented in Figure-31, it indicates that the coefficients of correlation are varying for every year.
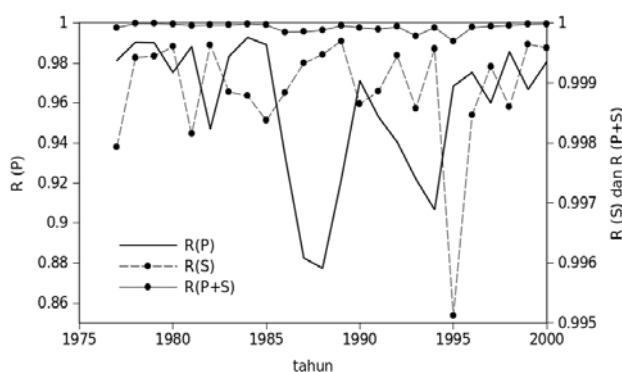
Figure-31. Correlation coefficients of the periodic R (P) stochastic R (S) and periodic + stochastic R (P+S) models.

For the periodic modeling of daily rainfall series, the R values vary from 0.8773 in the year of 1988 up to 0.9928 in the year of 1984 with the mean of correlation coefficient to be 0.9576. The stochastic modeling presents that the R values vary from 0.9951 in the year of 1995 up to 0.9997 in the year of 1989 with the mean of correlation coefficient to be 0.9989. For the periodic and stochastic modeling, the R values vary from 0.9997 in the year of

1995 up to 0.99999 in the year of 1979 with the mean of correlation coefficient to be 0.99993. The periodic modeling indicated that at the years, 1987 and 1988 occurred highly variation of the climate. Also the years before 1985 have least variation of the climate if it is compared with the years after 1985.

CONCLUSIONS

By using fast Fourier transform, autoregressive model, Fourier analysis and method of least squares, calculated daily rainfall series can be produced synthetic rainfall series significantly. Spectrum of daily rainfall series generated by using Fast Fourier Transform can be used to simulated synthetic daily rainfall series accurately.

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