

Analysis Multivariate Time Series Using State Space Model for Forecasting Inflation in Some Sectors of Economy in Indonesia

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Abstract

Many analytical methods can be utilized for multivariate time series modeling. One of the analytical models for modeling time series data with multiple variables is the State Space Model. The data to be analyzed in this study is inflation data from expenditure groups such as processed foods, beverages, cigarettes, and tobacco; and housing inflation for water, electricity, gas, and fuel from January 2001 to December 2021. The aim is to determine the best State Space Model that fits the data for forecasting. In this study, the State Space method will be utilized further with multivariate time series data and represent State Space in Vector Autoregressive (VAR) to determine the relationship between groups of observed variables.

Keywords

Inflation, State Space, Vector Autoregressive, State Vector, Granger Causality, Forecasting

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1. INTRODUCTION

Analysis of time series data that is univariate and multivariate has been widely carried out in the field of economics and other fields of science, such as in studies conducted by (Russel et al., 2020; Russel et al., 2022; Warsono et al., 2019a; Warsono et al., 2019b). The State Space Model is one of the models available for use in analyzing multivariate time series data and was first introduced by (Kalman, 1960). The State Space method is generally utilized for forecasting and modeling multivariate time series that interact dynamically. By considering the autocorrelation among all variables, State Space Models can provide more accurate forecasts than techniques that model each variable separately. The State Space Model offers a standardized methodology for analyzing many problems in time series data (Durbin and Koopman, 2012a).

In the field of economics, Aoki and Havenner (1991) show that for both stationary and non-stationary data, the State Space Model and procedures are recommended. The State Space Model in economics has been widely discussed over the last decade. Books that discuss the State Space Model are provided by (Harvey, 1990; Harvey, 1993; Hamilton, 1994; Kim and Nelson, 1999; Shumway et al., 2000; Durbin, 2004; Gómez,

2016). According to Wei (2006), the State Space Model is an approach to simultaneously model and predict several interrelated time series data variables, where the variables in it have dynamic interactions. The main purpose of the State Space method is to infer relevant results from a series of vectors with the obtained observations.

The State Space method is flexible because it can be represented in univariate and multivariate data. Usually, this State Space method is applied to data with a single variable, which does not require a relationship between the observed variables. Therefore, this study will apply the State Space method with multivariate data and represent State Space in VAR to ascertain the group's relationship with observed variables, where the VAR model is an analytical method that can be applied to explain the relationship between data variables.

When developing State Space Models for time series, Akaike (1975) Akaike (1976) introduced state vectors as canonical variate vectors between data and future observations. Wei (2006) discusses how to use canonical correlation to adjust the State Space Model. According to Durbin and Koopman (2012b), the analysis of time series can be performed effectively using the State Space Models in many fields such as statistics, econometrics, and others.

The goal of this research is to obtain the best State Space Model that fits the multivariate time series data used in this study, namely inflation data from expenditure groups such as processed foods, beverages, cigarettes, and tobacco, as well as housing inflation for water, electricity, gas, and fuel from January 2001 to December 2021. Moreover, the State Space Model is also used to see the behavior of data over the following 12 months of forecasting.

2. EXPERIMENTAL SECTION

2.1 Statistical Modelling

The assumption of stationarity is the first condition of the data to be checked before we analyse time series data. This assumption is very basic in analysis of time series. The stationarity of the data is checked by data plot and Augmented Dickey-Fuller (ADF) test. Vector Autoregressive (VAR) Model According to Tsay (2005) and Tsay (2014) VAR can be applied to multivariate time series data. The following is the Vector Autoregressive model with order p , VAR(p) model: $Z_t = \phi_0 + \sum_{i=1}^p \alpha_i$

Z_t is $n \times 1$ time series vector, ϕ_0 is $n \times 1$ constant vector, ϕ_i $n \times n$ is matrix parameters (for $i > 0$, $\phi_p \neq 0$) and α_i is vector shock with zero mean vector, and variance covariance $\sum a$.

2.2 Granger Causality Test

According to Lütkepohl (2005), to detect short-term relationships between observed variables in the form of reciprocity the Granger Causality Test is used. Suppose we are going to analyze the Granger Causality between variables X and Y and the model for the Granger Causality Test is:

$$X_t = C_1 + \alpha X_{t-1} + \alpha_2 X_{t-2} + \dots + \alpha X_{t-p} + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + u_t$$

The null hypothesis, according to the assumption of Ordinary Least Squares (OLS), is as follows: $H_0: \beta_1 = \beta_2 = \dots = \beta_p = 0$ (Y is not Granger Causality of X) with the alternative H_1 at least one of $\beta_p \neq 0$ (Y is Granger Causality of X). The test statistic:

$$F_{Test} = \frac{(RSS_0 - RSS_1)/P}{RSS_1/(T - 2P - 1)}$$

RSS_0 denotes the sum of residual squares when the null hypothesis H_0 is true., RSS_1 is the residuals sum of squares full model, T is total observations, and p is number of parameters related to variable Y. Reject H_0 if $F\text{-Test} > F_{(\alpha, p, T-2p-1)}$ or if p-value < 0.05 (Hamilton, 1994).

2.3 State Space Model

The State Space Model is a model approach and predicts simultaneously several interconnected time series data, where the variables have dynamic interactions (Wei, 2006). The representation of the State Space Model is as follows:

$$Y_{t+1} = AY_t + GX_{t+1}$$

and the output equation is:

$$Z_t = HY_t$$

Y_t is $k \times I$ state vector, A is a transition matrix $k \times k$, G is $k \times n$ input matrix, X_t is $n \times I$ vector input, Z_t is $m \times I$ output vector, and H is $m \times k$ matrix observation (Wei, 2006).

2.4 Canonical Correlation Analysis

Canonical correlation analysis is a statistical analysis that is applied to see the relationship between a group of dependent variables and a group of independent variables. According to (Wei, 2006), state vectors are uniquely determined through canonical correlation analysis between a series of current and previous observations and a series of future and current observations. For a discussion of canonical correlation, see (Wei, 2006; Tsay, 2005).

2.5 Forecasting

The Kalman filter is the most common approach for prediction in the State Space model, The Kalman filter can handle changes to model parameters and variances. According to Welch et al. (2001) at the forecasting stage, the estimated value is generated for the current state and the covariance value is used as initial predictive information for the next step. The Kalman filter is a recursive updating procedure that begins with a prediction of the initial state and then revises that prediction by adding corrections to the initial prediction. The basic recursive formula is used to update the mean and covariance matrices (Lai and Bukkapatanam, 2013). Forecast, when a new observation becomes available, should be used and applied to update the state vector and hence update the forecast. For this purpose, the Kalman Filter method is available, which is a recursive procedure used to conclude the state vector Y_t (Wei, 2006).

3. RESULT AND DISCUSSION

The data used are the inflation data from the expenditure groups, namely inflation data for processed foods, beverages, cigarettes, and tobacco (INF01), and housing inflation, water, electricity, gas and fuel (INF02) from January 2001 to December 2021 sourced from the Ministry of Trade (Kementerian Perdagangan, 2022, <https://satudata.kemendag.go.id/data-informasi/perdagangan-dalam-negeri/inflasi-2020>). Table 1 and Figure 1 show that INF01 fluctuates around the number 0.5527 and does not show a trend in the data; INF02 fluctuates around 0.46167 and there is no indication of a trend in the data. From the behaviour of the data in Figures 1 (a) and (b) which indicates that there is no trend, it is able to be concluded that the INF01 and INF02 data are stationary. The INF01 and INF02 data are stationary based on the behavior of the data in Figures 1 (a) and (b), which shows that there is no trend.

Non-stationary data can be formally checked using the ADF test with the null hypothesis is that the data are non-stationary. Table 2 shows that the average value is zero and single with a p-value $< .0001$ which is smaller than the level of significance = 0.05 for all data variables indicating that the data is stationary.

Table 1. Summary Statistics

Variable	Type	N	Mean	Standard Deviation	Min	Max
INF01	Dependent	252	0.55270	0.49922	-0.86000	3.21000
INF02	Dependent	252	0.46167	0.61447	-0.45000	7.40000

Table 2. The ADF Test for Processed Foods, Beverages, Cigarettes and Tobacco Inflation Data (INF01) and Housing, Water, Electricity, Gas and Fuel Inflation Data (INF02)

Variable	Type	Rho	p-value	Tau	p-value
INF01	Zero Mean	-43.99	<.0001	-4.57	<.0001
	Single Mean	-137.48	0.0001	-8.13	<.0001
	Trend	-169.68	0.0001	-8.90	<.0001
INF02	Zero Mean	-60.12	<.0001	-5.49	<.0001
	Single Mean	-125.49	0.0001	-7.86	<.0001
	Trend	-191.93	0.0001	-9.73	<.0001

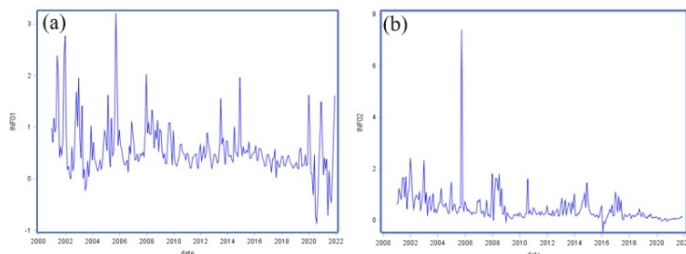


Figure 1. The Plot of Inflation Data by Expenditure Group: (a) Inflation of Processed Foods, Beverages, Cigarettes and Tobacco (INF01), (b) Inflation of Housing, Water, Electricity, Gas and Fuel (INF02).

To get the best VAR(p) model, the step that must be done is to ascertain the optimal order (length of the lag) p. To ascertain the length of the lag (p), the Akaike’s Information Criterion Corrected (AICC) is used. The optimal order of p selected based on the minimum value of AICC is VAR (2) (Table 3 shows that the values of AICC for models VAR(1), VAR(2) and VAR(3) are very closed). In the parameter representation scheme in the VAR(2) model, all parameters for INF01 and INF02 have significant parameters on AR1 which are indicated by the signs – (minus), and + (plus) (Table 4).

3.1 Model Vector Autoregressive (VAR (p))

The estimate Model VAR(2) is as follows:

$$\begin{bmatrix} INF01_t \\ INF02_t \end{bmatrix} = \begin{bmatrix} 0.28216 \\ 0.18711 \end{bmatrix} + \begin{bmatrix} 0.52741 & 0.03460 \\ 0.28932 & 0.10681 \end{bmatrix} \begin{bmatrix} INF01_{t-1} \\ INF02_{t-1} \end{bmatrix} + \begin{bmatrix} -0.07792 & 0.01404 \\ 0.00737 & 0.13245 \end{bmatrix} \begin{bmatrix} INF01_{t-2} \\ INF02_{t-2} \end{bmatrix}$$

Covariance of Innovation

$$\Sigma = \begin{bmatrix} 0.18626 & 0.12228 \\ 0.12228 & 0.33139 \end{bmatrix}$$

3.2 Impulse Response

Figure 2(a) showed that if there is a shock in INF01 one unit (or one unit changes in INF01), in the first month (lag1) INF01 and INF02 give a response of 0.52741 and 0.28932, respectively. In the second month (lag2) the response of INF01 to a change of one unit in INF01 was 0.21026, while the response of INF02 was 0.19087. In the third month (lag3) the response of INF01 to a change of one unit in INF01 was 0.08046, while the response of INF02 was 0.12343. In the fourth month (lag4) the responses of INF01 and INF02 to changes in the impact of one unit on INF01 were 0.03301 and 0.06329, respectively. The response effect lasted for six months in normal conditions. From Figure 2(b), if there is a shock in INF02 one unit, in the first month (lag1) INF01 and INF02 give a response of 0.03460 and 0.10681, respectively. In the second month (lag2) the response of INF01 to a change of one unit in INF01 was 0.03599, while the response of INF02 was 0.15387. In the third month (lag3) the response of INF01 to a change of one unit in INF01 was 0.02311, while the response of INF02 was 0.04125. In the fourth month (lag4) the responses of INF01 and INF02 to changes in the impact of one unit on INF01 were 0.01297 and 0.03174, respectively. The response effect lasted for six months in normal conditions.

3.3 Granger Causality Test

From Table 6, Test 2 showed a significant value (p-value <0.05), but Test 1 did not show a significant value (p value > 0.05). Test 2 shows that the p-value = 0.0066 is smaller than the 0.05 level of significance. Thus the null hypothesis (H=0), namely INF02 is influenced by itself and is not influenced by INF01, thus the hypothesis is rejected, which means that future INF02 data is

Table 3. Akaike’s Information Criteria AICC for Model VAR(1), VAR(2), and VAR(3)

VAR(0)	VAR (1)	VAR (2)	VAR (3)
-2.74	-3.02	-3.02	-3.01

Table 4. Schematic Representation of Parameter Estimates VAR(1)

Variable/Lag	C	AR1	AR2
INF01	+	+	..
INF02	+	+	..

+ is > 2*std error,- is < -2*std error,. is between,* is N/A

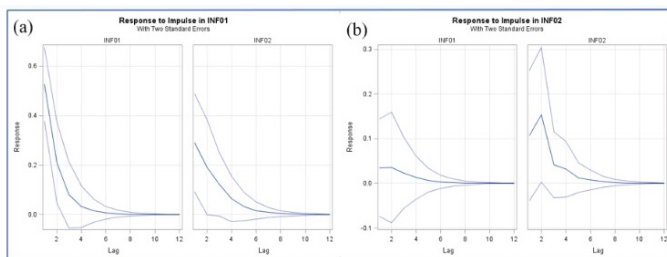


Figure 2. Impulse Response for (a) INF01 and (b) INF02

Based on Table 6., Granger Causality Can be Depicted as Follows:

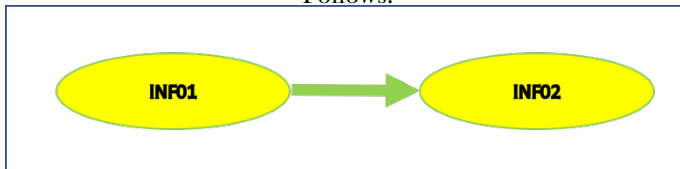


Figure 3. Granger Causality Model Based on the Results of Table 6

influenced by past and future INF01 data and is influenced by past INF02 data.

3.4 Canonical Correlation Analysis

This study found the length of the lag order (p) for the best Vector Autoregressive model, namely for p = 2. So the model chosen was VAR (2). Now to build a state space model, the first step to do is to select a state vector. The process of selecting a state vector follows the method given by Durbin and Koopman (2012b) by using canonical correlation. The state vector is determined by the Information Criterion (IC) value, where the IC value is negative <0, then the minimum canonical correlation (ρ_{min}) is taken as zero Wei (2006) otherwise, it is considered greater than zero. From Table 7, for the first step, we will consider the set of state vectors $INF01_t, INF02_t, INF01_{t+1|t}$ and for this set of state vectors, the value of IC is negative (-2.63729) therefore $INF01_{t+1|t}$ is excluded from the state vector. In the second step, we consider the set of state

vectors $INF01_t, INF02_t, INF02_{t+1|t}$, and for this set of state vectors, the IC value is positive (1.520112). Therefore, the variable $INF02_{t+1|t}$ is entered into the state vector. In the second step, we also consider the set of state vectors $INF01_t, INF02_t, INF02_{t+1|t}, INF02_{t+2|t}$, and for this set of state vectors, the IC value is negative (-4.47121). Therefore, the variable $INF02_{t+1|t}$ is excluded from the state vector.

Based on canonical correlation analysis the components of the state vector are as follows:

$$Y_t = \begin{bmatrix} INF01_t \\ INF02_t \\ INF02_{t+1|t} \end{bmatrix}$$

3.5 Model State Space

Based on the parameter estimates in Table 6, then the State Space Model is as follows: $Y_{t+1}=AY_t+GX_{t+1}$

Where:

$$Y_{t+1} = \begin{bmatrix} INF01_{t+1|t} \\ INF02_{t+1|t} \\ INF02_{t+2|t} \end{bmatrix} Y_t = \begin{bmatrix} INF01_{t|t} \\ INF02_{t|t} \\ INF02_{t+1|t} \end{bmatrix}$$

$$A = \begin{bmatrix} 0.480325 & 0.040279 & 0 \\ 0 & 0 & 1 \\ -0.05574 & 0.025091 & 0.847525 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 & 1 \\ 0.284043 & 0.066229 & 1 \end{bmatrix} X = \begin{bmatrix} vt + 1 \\ \delta t + 1 \end{bmatrix}$$

$$VAR(X) = VAR \begin{bmatrix} vt + 1 \\ \delta t + 1 \end{bmatrix} \begin{bmatrix} 0.184444 & 0.116403 \\ 0.116403 & 0.312132 \end{bmatrix}$$

3.6 Forecasting

In this study, the forecasting is performed using the Kalman filter technique and is based on State Space Models. Data for

Table 5. Estimation Parameter of Model VAR(2)

Equation	Parameter	Estimate	Standard Error	tValue	p-value	Variable
INF01	CONST1	0.28216	0.04429	6.37	0.0001	1
	AR1_1_1	0.52741	0.07404	7.12	0.0001	INF01(t-1)
	AR1_1_2	0.03460	0.05467	0.63	0.5274	INF02(t-1)
	AR2_1_1	-0.07792	0.07439	-1.05	0.2960	INF01(t-2)
	AR2_1_2	0.01404	0.05472	0.26	0.7977	INF02(t-2)
INF02	CONST2	0.18711	0.05908	3.17	0.0017	1
	AR1_2_1	0.28932	0.09876	2.93	0.0037	INF01(t-1)
	AR1_2_2	0.10681	0.07292	1.46	0.1443	INF02(t-1)
	AR2_2_1	0.00737	0.09923	0.07	0.9408	INF01(t-2)
	AR2_2_2	0.13245	0.07299	1.81	0.0708	INF02(t-2)

Table 6. Granger Causality Test

Test and Group Variables	DF	Chi-Squares	p-value
Test 1: Group 1: INF01	2	0.50	0.7782
Test 1: Group 2: INF02			
Test 2: Group 1: INF02	2	10.03	0.0066
Test 2: Group 2: INF01			

Table 7. Analysis Canonical Correlation

State Vector	Canonical Correlation	IC	Chi-Square	DF
INF01 _t , INF02 _t , INF01 _{t+1 t}	1, 1, 0.145106	-2.63729	5.320152	4
INF01 _t , INF02 _t , INF02 _{t+1 t}	1, 1, 0.192545	1.520112	9.444555	4
INF01 _t , INF02 _t , INF02 _{t+1 t} , INF02 _{t+2 t}	1, 1, 0.245818, 0.077771	-4.47121	1.519691	3

Table 8. Parameter Estimate of the State Space Model

Parameter	Estimate	Standard Error	tValue
F(1,1)	0.480325	0.065723	7.31**
F(1,2)	0.040279	0.053397	0.75
F(3,1)	-0.05574	0.064247	-0.87
F(3,2)	0.025091	0.072659	0.35
F(3,3)	0.847525	0.129645	6.54**
G(3,1)	0.284043	0.087226	3.26**
G(3,2)	0.066229	0.071324	0.93

Note : ** significant at alpha=0.01

Table 9. Forecasting For INF01 and INF02 For the Next 12 Months

Month	INF01	Std	INF02	Std
Jan 2022	1.04598	0.42947	0.56891	0.55869
Feb 2022	0.79395	0.48167	0.48455	0.57685
Mar 2022	0.66950	0.49418	0.45626	0.58705
Apr 2022	0.60858	0.49749	0.44421	0.59307
May 2022	0.57884	0.49845	0.44022	0.59699
Jun 2022	0.56439	0.49875	0.43994	0.59971
July 2022	0.55744	0.49886	0.44126	0.60165
Aug 2022	0.55415	0.49891	0.44317	0.60308
Sept 2022	0.55265	0.49894	0.44522	0.60414
Oct 2022	0.55201	0.49895	0.60414	0.60494
Nov 2022	0.55179	0.49896	0.44898	0.60553
Dec 2022	0.55175	0.49897	0.45059	0.60598

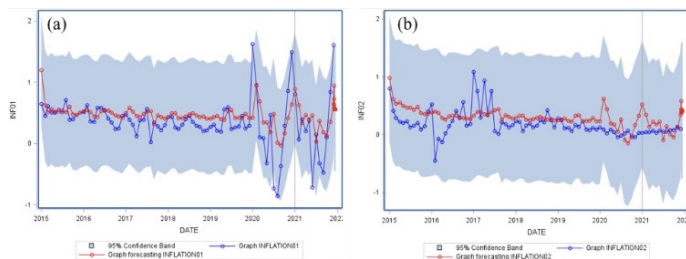


Figure 4. Forecasting for (a) INF01 and (b) INF02 For the Following 12 Months

the next 12 months period are forecasted using the State Space Model. The results given in Table 7 show that for INF01 data, the following 12 months' forecasting is flat (Table 9 and Figure 4) in the range between 0.55175 to 1.04598 and the standard deviation is in the range between 0.42947 and 0.49897; for INF02 data, the following 12 months forecasting is flat (Table 9 and Figure 4) in the range between 0.43994 and 0.56891 and the standard deviation in the range between 0.55869 and 0.60598.

4. CONCLUSION

Based on the AICC value, the VAR(2) model is selected as the best VAR model, while based on the VAR(2) model, Granger Causality concludes that INF01 is Granger Causality to INF02, while INF02 is not Granger Causality to INF01. According to the Granger Causality analysis results, INF02 is not Granger Causality to INF01, which means that INF01 is only influenced by itself and not by INF02, and it is known from the parameters estimation and test of the VAR(2) model that INF01 was significantly influenced by INF01 information one month earlier. INF01 is Granger Causality to INF02, which means that INF02 is influenced not only by itself but also by past information of INF01, and the results of the parameters estimation and test of the VAR(2) model show that INF02 is significantly influenced by INF01's information one month

earlier and INF02's information two months earlier. From the results of forecasting using the State Space approach, it can be concluded that for the next 12 months of forecasting, for the INF01 data, inflation tends to decrease from 1.05498 in January 2022 and 0.55175 in December 2022; Meanwhile, the forecasting values for INF02 data for the next 12 months tend to be stable and fluctuates around 0.44000.

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REFERENCES

Akaike, H. (1975). Markovian Representation of Stochastic Processes by Canonical Variables. *SIAM Journal on Control*, **13**(1); 162–173

Akaike, H. (1976). Canonical Correlation Analysis of Time Series and the Use Of An Information Criterion. *System Identification: Advances and Case Studies*, **126**; 27–96

Aoki, M. and A. Havenner (1991). State Space Modeling of Multiple Time Series. *Econometric Reviews*, **10**(1); 1–59

Durbin, J. (2004). Introduction to State Space Time Series Analysis. *State Space and Unobserved Component Models: Theory and Applications*, **4**(5); 3–25

Durbin, J. and S. J. Koopman (2012a). *Time Series Analysis By State Space Methods*, volume 38. OUP Oxford

Durbin, J. and S. J. Koopman (2012b). *Time Series Analysis by State Space Methods. (2nd ed.)*, volume 38. OUP Oxford

Gómez, V. (2016). *Multivariate Time Series with Linear State Space Structure*. Springer

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press

Harvey, A. C. (1990). *Forecasting, Structural Time Series Models and The Kalman Filter*. Cambridge university press

Harvey, A. C. (1993). *Time Series Models (2nd ed.)*. Cambridge university press

- Kalman, R. E. (1960). A New Approach To Linear Filtering And Prediction Problems. *Journal of Basic Engineering*, **82**(1); 35–45
- Kim, C. J. and C. R. Nelson (1999). State-space Models With Regime Switching: Classical and Gibbs sampling Approaches With Applications. *MIT Press Books*, **1**(3); 31–35
- Lai, T. L. and V. Bukkapatanam (2013). Adaptive Filtering, Nonlinear State-space Models, and Applications In Finance and Econometrics. *State-Space Models*, **3**(4); 3–22
- Lütkepohl, H. (2005). *New Introduction Tto Multiple Time Series Analysis*. Springer Science & Business Media
- Russel, E., F. S. D. Kesumah, A. Rialdi, and M. Usman (2020). Dynamic Modeling and Forecasting Stock Price Data by Applying AR-GARCH Model. *TEST Engineering and Management*, **82**(8); 6829–6842
- Russel, E., Wamiliana, Nairobi, Warsono, M. Usman, and J. I. Daoud (2022). Dynamic Modeling and Forecasting Data Energy Used and Carbon Dioxide (CO₂). *Science and Technology Indonesia*, **7**(2); 228–237
- Shumway, R. H., D. S. Stoffer, and D. S. Stoffer (2000). *Time Series Analysis and Its Applications*, volume 3. Springer
- Tsay, R. S. (2005). *Analysis of Financial Time Series*. John Wiley and Sons, Inc Publication, New York
- Tsay, R. S. (2014). *Multivariate Time Series Analysis*. John Wiley and Sons, Inc Publication, New York
- Warsono, W., E. Russel, W. Wamiliana, W. Widiarti, and M. Usman (2019a). Modeling and Forecasting By the Vector Autoregressive Moving Average Model For Export of Coal and Oil Data (Case Study From Indonesia Over the Years 2002-2017). *International Journal of Energy Economics and Policy*, **9**(4); 240–247
- Warsono, W., E. Russel, W. Wamiliana, W. Widiarti, and M. Usman (2019b). Vector Autoregressive With Exogenous Variable Model and Its Application In Modeling and Forecasting Energy Data: Case Study Of Ptba and Hrum Energy. *International Journal of Energy Economics and Policy*, **9**(2); 390–398
- Wei, W. W. S. (2006). *Time Series Analysis: Univariate and Multivariate Methods. (2nd ed.)*. New York: Pearson Education.
- Welch, G., G. Bishop, L. Vicci, S. Brumback, K. Keller, and D. Colucci (2001). High-performance Wide-area Optical Tracking: The Hiball Tracking System. *Presence*, **10**(1); 1–21