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Study Velocities and Dispersion Effects of Wave Propagation over Mangrove Models using 2-D Non-Linear and Compressional Wave Equations

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Abstract

Most researches to modeling wave propagation in shallow water are using two class of waves, non linear shallow water wave equation as gravity wave equation, and simple wave equation as compressional wave equation. In this research, wave propagation over mangrove models was conducted. The aim of this research were to study velocities and dispersion effects of wave propagation where propagate toward the mangrove models. The first order accuracy of the explicit finite-difference method was used to approximate the gravity wave equation and the second order accuracy of the explicit finite-difference method. It was used to approximate the compressional wave equation. Variety of water depths, 5 m up to 100 m are used to simulate the wave propagation. Results of non linear wave propagation also are compared with a result of compressional wave propagation. The mangrove models are employed for different water depths and different wave equations. Results of this research present that if the water depths increase from 5 m up to 100 m, the wave velocity will be increases significantly. For the same values of water depth, equal to 5 m, compressional waves propagate faster than gravity waves. For deeper water depths, the dispersion effects are less than for shallow water depths.

KEYWORDS: velocities, dispersion effects, mangrove models, gravity and compresional waves

1. Introduction

Wave propagation theory was used by scientists and engineers, in particular civil engineers in developing models for wave propagation. Usually engineers involve the theory in developing numerical models in coastal environments, of particular importance in civil engineering, while scientists develop the numerical models which are of importance in geophysical exploration. In this paper, algorithms of 2-D non linear shallow water and compressional wave propagations have been developed to simulate free surface water wave propagations in shallow water.

Surface water wave propagation researches have been conducted by Zakaria [3], Nelamani [4] and Wamsly [5]. In study conducted by Zakaria [3], research has been done by using physical modeling or experimental research. Physical breakwater was modeled to study wave run-up, reflection, dissipation and dispersion effects. Wave propagation across a submerged breakwater has been modeled to investigate transmission or dispersion effects [4]. Artificial reefs have been produced by one of several reef ball foundation which having research in reef restoration and coastal protection [5]. A numerical simulation of 2-D non linier shallow water wave propagation has been introduced by Kowalik [6] to study one dimensional case of tsunami run up. And more discussion about 2-D non linear shallow water wave propagation has been presented in [7], where they studied dispersion problems of two-dimensional wave run up. In research done by Mihardja [2], a simple compressional wave equation has been used to simulate propagation of tidal waves. The compressional wave equation also has been used by Zakaria [3] to study edge and grid dispersion problems numerically using lower to higher order accuracies. One of differences in numerical modeling using both equations, non linear and simple wave equations are the wave speeds or the wave velocities. So these are being problems in simulating real wave propagations.

This research is intended to study wave velocities for a variety of water depths and dispersion effects of mangroves which is modeled to dissipate wave

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propagation over them numerically. Mangroves can be used as natural or green breakwater to protect coastal zone from wave attacks.

2. Methodology

Research methodology used in this study is only using a numerical method. Using the method, cost of the research is more less if it is compared with physical modeling and field modeling researches.

In this research, a wave equation used to model 2-D non linear shallow water wave propagation over the mangrove models is as introduced in [6, 7] and a wave equation used to model 2-D compressional wave propagation is as introduced by Zakaria [3].

For modeling of the 2-D non linear shallow water wave propagation, set of equations of motion and continuity is usually used as follow,

a. Equation of motions,

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial \eta}{\partial x}-r.u.f \qquad (1)$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+u\frac{\partial v}{\partial x}=-g\frac{\partial \eta}{\partial y}-r.v.f \qquad (2)$$

b. Equation of continuity,

$$\frac{\partial \eta}{\partial t}=-\frac{\partial (Du)}{\partial x}-\frac{\partial (Dv)}{\partial y} \qquad (3)$$

For modeling of 2-D compressional wave propagation is used an equation as follows,

$$\frac{\partial^2 \eta}{\partial t^2}=c^2\left\{\frac{\partial^2 \eta}{\partial x^2}+\frac{\partial^2 \eta}{\partial y^2}\right\} \qquad (4)$$

Where:

$f=\frac{\sqrt{u^2+v^2}}{D}$

η = elevation of water surface (m)
c = wave speed (m/s)
$c = \sqrt{g.h}$
r = friction coefficient = 0.025
h = water depth (m)
u = wave velocity in x direction (m/s)
v = wave velocity in y direction (m/s)
g = gravitational acceleration (m/s^2)
D = total water depth (m)
$D = h + \eta$
Δt = time step = 0,01 second
$\Delta x = \Delta y$ = grid spaces = 1 meter

A solution of the 2-D non linear shallow water wave equation used in this research is explicit finite-difference method. Using this method, the equations could be approximated by using the first order accuracies as follow,

$$\frac{\partial u}{\partial t}=\frac{u_{i,j}^{k+1}-u_{i,j}^{k}}{\Delta t} \qquad (5)$$

$$u\frac{\partial u}{\partial x}=u_{i,j}^{k}\left(\frac{u_{i+1,j}^{k}-u_{i,j}^{k}}{\Delta x}\right) \qquad (6)$$

$$v\frac{\partial u}{\partial y}=v_{i,j}^{k}\left(\frac{u_{i,j+1}^{k}-u_{i,j}^{k}}{\Delta y}\right) \qquad (7)$$

$$g\frac{\partial \eta}{\partial x}=g\left(\frac{\eta_{i+1,j}^{k}-\eta_{i,j}^{k}}{\Delta x}\right) \qquad (8)$$

$$r.u.f=r.u_{i,j}^{k}.\frac{\sqrt{\left(u_{i,j}^{k}\right)^2+\left(v_{i,j}^{k}\right)^2}}{D_{i,j}^{k}} \qquad (9)$$

$$\frac{\partial v}{\partial t}=\frac{v_{i,j}^{k+1}-v_{i,j}^{k}}{\Delta t} \qquad (10)$$

$$v\frac{\partial v}{\partial y}=v_{i,j}^{k}\left(\frac{v_{i,j+1}^{k}-v_{i,j}^{k}}{\Delta y}\right) \qquad (11)$$

$$u\frac{\partial v}{\partial x}=u_{i,j}^{k}\left(\frac{v_{i+1,j}^{k}-v_{i,j}^{k}}{\Delta x}\right) \qquad (12)$$

$$g\frac{\partial \eta}{\partial y}=g\left(\frac{\eta_{i,j+1}^{k}-\eta_{i,j}^{k}}{\Delta y}\right) \qquad (13)$$

$$r.v.f=r.v_{i,j}^{k}.\frac{\sqrt{\left(u_{i,j}^{k}\right)^2+\left(v_{i,j}^{k}\right)^2}}{D_{i,j}^{k}} \qquad (14)$$

$$D_{i,j}^{k}=h_{i,j}+\eta_{i,j}^{k} \qquad (15)$$

$$\frac{\partial \eta}{\partial t}=\frac{\eta_{i,j}^{k+1}-\eta_{i,j}^{k}}{\Delta t} \qquad (16)$$

$$\frac{\partial(Du)}{\partial x}=D_{i,j}^{k}.\left(\frac{u_{i,j}^{k}-u_{i-1,j}^{k}}{\Delta x}\right)+u_{i,j}^{k}.\left(\frac{D_{i,j}^{k}-D_{i-1,j}^{k}}{\Delta x}\right) \quad (17)$$

$$\frac{\partial(Dv)}{\partial y}=D_{i,j}^{k}.\left(\frac{v_{i,j}^{k}-v_{i,j-1}^{k}}{\Delta y}\right)+v_{i,j}^{k}.\left(\frac{D_{i,j}^{k}-D_{i-1,j}^{k}}{\Delta y}\right) \quad (18)$$

A solution of the 2-D compressional or hyperbolic wave equation used in this research is explicit finite-difference method. Using this method, the equation could be approximated as follows,

$$\frac{\partial^2\eta}{\partial t^2}=\frac{\eta_{i,j}^{n-1}-2.\eta_{i,j}^{n}+\eta_{i,j}^{n+1}}{\Delta t^2} \quad (19)$$

$$\frac{\partial^2\eta}{\partial x^2}=\frac{\eta_{i-1,j}^{n}-2.\eta_{i,j}^{n}+\eta_{i+1,j}^{n}}{\Delta x^2} \quad (20)$$

$$\frac{\partial^2\eta}{\partial y^2}=\frac{\eta_{i,j-1}^{n}-2.\eta_{i,j}^{n}+\eta_{i,j+1}^{n}}{\Delta y^2} \quad (21)$$

Using Equations (19), (20) and (21) and obtained wave velocity as $c=\sqrt{g.h}$ [2], so we can rearrange a solution for the above equation as follows,

$$\frac{\eta_{i,j}^{n-1}-2.\eta_{i,j}^{n}+\eta_{i,j}^{n+1}}{\Delta t^2}=g.h.\left[\frac{\eta_{i-1,j}^{n}-2.\eta_{i,j}^{n}+\eta_{i+1,j}^{n}}{\Delta x^2}+\frac{\eta_{i,j-1}^{n}-2.\eta_{i,j}^{n}+\eta_{i,j+1}^{n}}{\Delta y^2}\right] \quad (22)$$

Equation (22) is an approximation solution for explicit finite-difference of the 2-D hyperbolic wave equation.

Modeled surface wave propagation is restricted by boundaries, as physically it is not real. The boundaries usually mentioned as nonphysical boundaries or open boundaries. To simulate the waves in other to be able to propagate pass the nonphysical boundaries, mathematical equation is applied on the boundaries to minimize or reduce nonphysical reflection from computational array boundaries, a number of techniques have been developed, having advantages and disadvantages.

In this research, a boundary condition method which usually used in modeling of wave propagation is transparent boundary condition method. The boundary condition method is needed to reduce waves propagate through to over nonphysical boundaries. From the boundaries, reflected waves are not allowed. Equation used as open boundaries is as introduced by Reynolds in [1] as follows,

$$\frac{\partial\eta}{\partial t}+c\frac{\partial\eta}{\partial x}=0 \quad (22)$$

Using Equation (22) above, nonphysical reflections from the boundaries are possible to be reduced. Using hyperbolic wave equation and Reynolds transparent boundary condition method, surface wave propagation could be modeled. For non linear shallow water wave propagation, the boundary condition above can be used by change wave speed (*c*) to gravitational acceleration (*g*). The same way can be done for the velocities (*u* and *v*). Using different values of the wave velocities, mangroves could be modeled. How far dispersion and dissipation effects caused the mangroves could be investigated.

3. Environmental set-up

In the numerical modeling, as a source is point source. To model a surface wave source, a source model of Ricker wavelet is applied as used in [2]. In this research, to model wave propagation is using environmental setup as presented in Fig. 1.

The mangroves are located at the center of the boundary or close to the point source. in the center of the mangroves location is opened. It is intended as a way for the waves to propagate over and pass the mangroves model (see Fig. 1)

Grid schemes used to model mangroves effects are such as presented in Fig. 2. Where at the positions of rectangular grid points, velocities (*c*) are set to zero, because of at the positions, waves are not allowed to propagate or equal to zero.

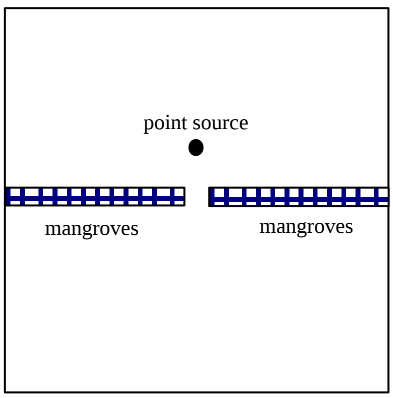

Fig. 1. Environment setup of wave propagation

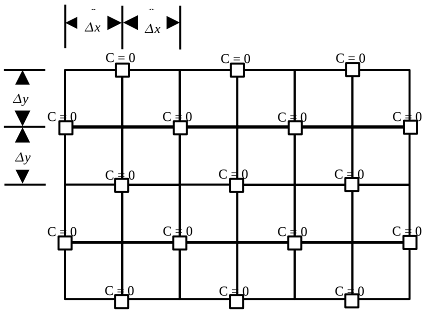

Fig. 2. Grid scheme for the mangroves model.

4. Results and Discussion

Results of this research presented here are using scenario from Fig. 1 and 2. Results of using the non linear shallow water wave equation are presented in Fig. 3, Fig. 4, Fig. 5, Fig. 6, and Fig. 7, and result of using the compressional wave equation is presented in Fig. 8 as follow,

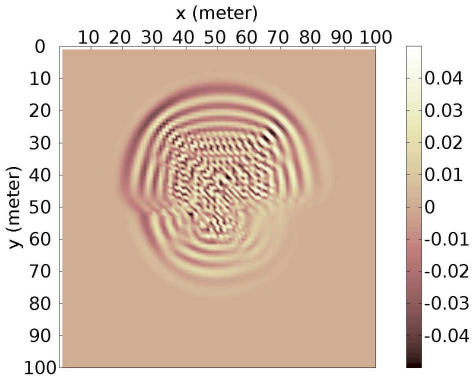

Fig. 3. Snapshot of wave propagation for h=5 m and at t = 10 sec (non linear wave equation)

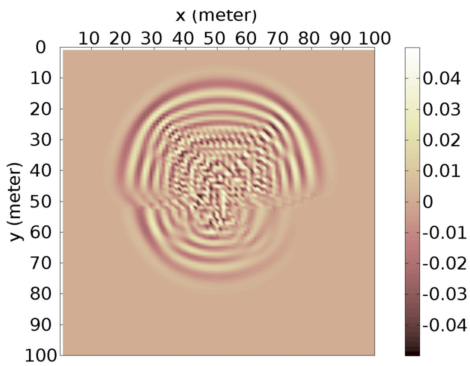

Fig. 4. Snapshot of wave propagation for h=10 m and at t = 7.5 sec (non linear wave equation)

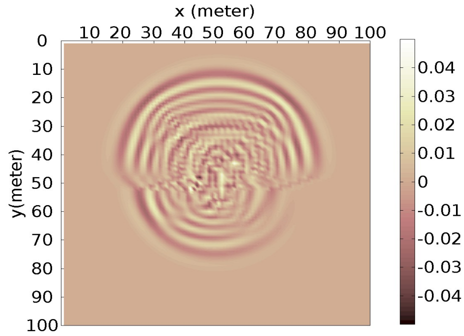

Fig. 5. Snapshot of wave propagation for h=20 m and at t = 5.5 sec (non linear wave equation)

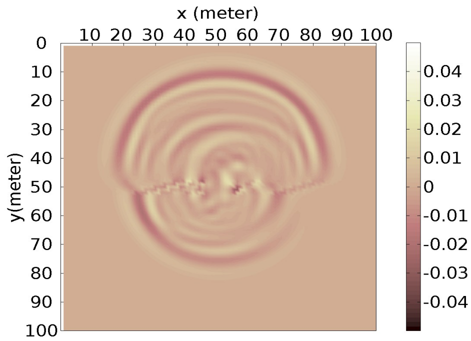

Fig. 6. Snapshot of wave propagation for h=50 m and at t = 3.75 sec (non linear wave equation)

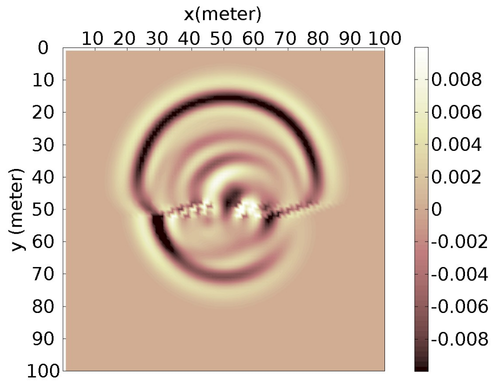

Fig. 7. Snapshot of wave propagation for h=100 m and at t = 2.5 sec (non linear wave equation)

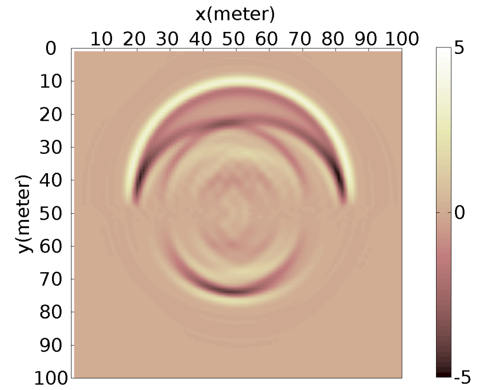

Fig. 8. Snapshot of wave propagation for h=5 m and at t = 5.25 sec (compressional wave equation)

To generate and simulate wave propagation such as presented above, 101 times 101 grid points are used. The grid spaces Δx equal to Δy used in this model are 1 meter. The depth (*h*) which used in the numerical domain are varies of 5 m, 10 m, 20 m, 50 m, and 100 m. The time step Δt in this model is 0.01 second. So numerically, the model is stable, because Δt is less than $\Delta x / \sqrt{(gh)}$.

The first order accuracy finite difference method and leap frog or staggered grid scheme are used to approximate the first order derivatives of the non-linear shallow water wave equation. The second order accuracy finite difference method and normal grid scheme are be used to approximate the second order derivatives of the compressional wave equation.

Snapshot results of using a variety of water depth for non-linear shallow water wave propagation can be compared each other's. The snapshots present in Fig. 3, Fig. 4, Fig. 5, Fig. 6 and Fig. 7, compared with snapshot result of using compressional wave equation such as presented in Fig. 8.

In Fig. 3, snapshot of wave propagation is using the water depth (h) of 5 meter. The waves need time (t) is 10 second to propagate at the position. In Fig. 4, for using the water depth (h) of 10 m, the waves propagate in 7.5 second to reach the same position. In Fig. 5, for using the water depth of 20 m, the wave propagates in 5.5 second. In Fig. 6, for using the water depth of 50 m, the wave propagates in 3.75 second. In Fig. 7, for using the water depth of 100 m, the waves propagate in 2.5 second. In Fig. 8, snapshot of wave propagation is using the compressional wave equation for the water depth of 5 m. The wave propagates in 2.5 second to reach the same position as the snapshots of Fig. 3, Fig. 4, Fig. 5, Fig. 6, and Fig. 7

It concluded that in the numerical modelling of wave propagation, using the non-linier shallow water wave equation, the wave velocities (*u* and *v*) would be increase if the waves propagate from shallow water to deep water.

In the shallow water, the non-linear or gravity wave propagations present more dispersion effects than the wave propagations in the deep water. The waves produce the dispersion effects since the first propagation time. Beside problems of the dispersion effects, the model also gets instability condition.

If the results of non-linier shallow water wave propagation modelling compared with the result of compresional wave propagation modelling, result of the compresional wave modelling presents less grid dispersion effects than results of the gravity wave propagation modelling.

For the same position and water depth of the first propagation wave, the compressional wave needs 5.25 second to reach the position, but the gravity wave needs 10 second to reach the position. It means that the gravity waves propagate slower than the compresional waves. In the reality, to model or simulate real wave propagation, the correction factors should be involved in order to produce realistic wave propagation. The waves of the compresional wave modelling produce higher amplitudes than the waves of the non-linear wave modelling. The dispersion effects of the mangroves model presented in the results of the non-linear wave equation is proving clearly. For the result of the compresional wave equation doesn't present dispersion effects clearly.

4. Conclusions

Based on the results of this research concluded that for the same water depth, the wave velocities and the wave amplitudes produced by the non linear wave modelling are slower and smaller than the wave velocities and the wave amplitudes produced by the compresional wave modelling.

Mangroves can be modelled numerically to study and to investigate dispersion effects. For the purpose, non-linear shallow water wave equation is better than the compresional wave equation. Moreover, the disadvantage of using the non-linear wave equation, the numerical instability problems more increase.

References

[1] A. C. Reynold, Boundary conditions for the numerical solution of wave propagation problems, Geophysics 43(6), pp 1099-1110, 1978.

[2] D. K. Mihardja, S. Hadi, Model matematika numerik untuk simulasi gerak pasang-surut, Pusat Penelitian dan Pengembangan Oceanografi LIPI, Jakarta, 1989.

[3] Ahmad Zakaria, Numerical modelling of wave propagation using higher order finite-difference

formulas, Thesis (Ph.D), Curtin University of Technology, Perth, W.A., 2003.

[4] S. Nelamani and B.V. Sumalatha, Wave reflection, run-up, run-down and pressure on seawalls defended by an offshore breakwater, J. Indian Inst. Sci, 86, pp 15-31, 2006.

[5] Wamsly, Hans Hanson, and Nicholas C. Kraus, Wave transmission at detached breakwaters for shoreline response modelling, U.S. Army Corps of Engineers, ERDC/CHL CHETN-II-45, March 2002.

[6] Z. Kowalik and T. S. Murty, Numerical simulation of Two-Dimensional Tsunami Runup, Marine Geodesy, Vol 16, pp 87-100, 1993.

[7] J. Horrillo, Z. Kowalik and Y. Shigihara, Wave dispersion study in the Indian Ocean-Tsunami of Desember 26, 2004, Marine Geodesy, Vol 29, pp 149-166, 2006.