HISTORY OF MANUSCRIPT PUBLICATION

(International Journal of Mathematics and Mathematical Sciences)

On the Locating Chromatic Number of Certain Barbell Graphs

Asmiati[1]

I.Ketut Sadha Gunce Yana[1]

Lyra Yulianti[2]

[1]Mathematics Department, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No. 1, Bandar lampung, Indonesia.

[2]Mathematics Department, Faculty of Mathematics and Natural Sciences, Andalas University, Kampus UNAND Limau Manis, Padang, Indonesia.

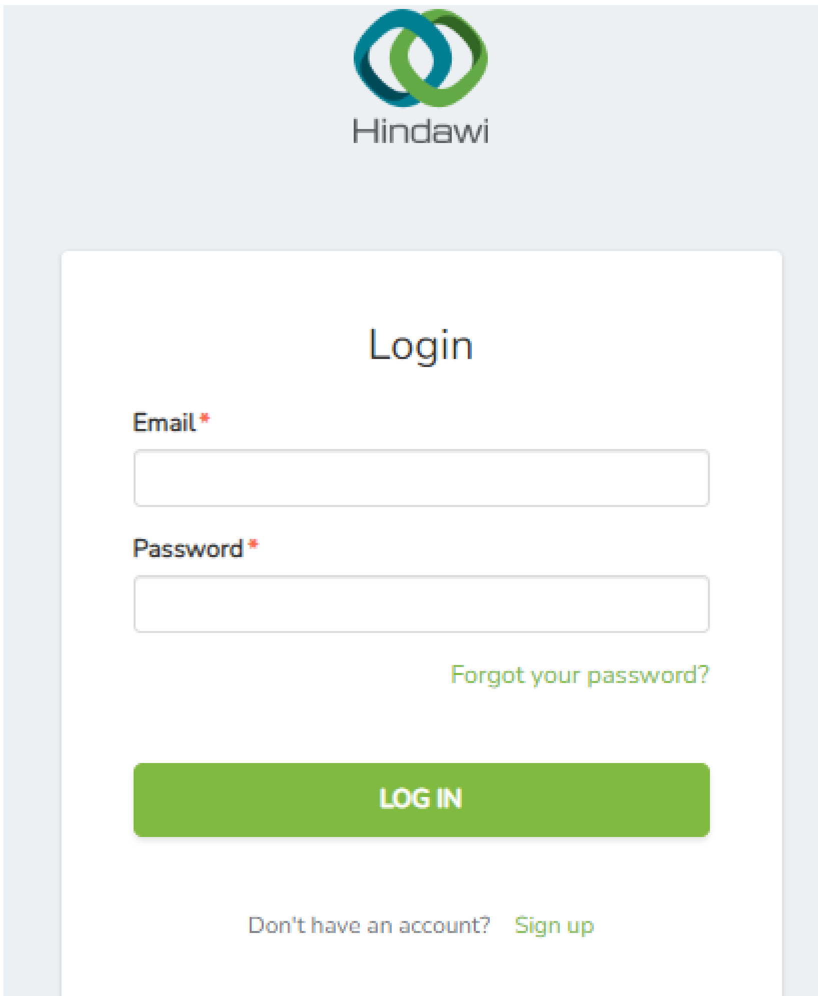
Received : 27 Maret 2018

Published : 6 Agustus 2018

DAFTAR ISI

I. PAPER SUBMISSION

27 MARET 2018



1I. ACKNOWLEDGING RECEIPT

27 Maret 2018

5327504: Acknowledging Receipt

Yahoo/Email M...

International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hind   Sel, 27 Mar 2018 jam 09.11

Kepada: asmiati308@yahoo.com

Cc: ahmed.khaled@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

The Research Article titled "ON THE LOCATING CHROMATIC NUMBER OF SOME BARBELL GRAPHS," by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti has been received and assigned the number 5327504.

All authors will receive a copy of all the correspondences regarding this manuscript.

Thank you for submitting your work to International Journal of Mathematics and Mathematical Sciences.

Best regards,

--
****
Ahmed Khaled
Editorial Office
Hindawi
http://www.hindawi.com
****

https://mail.yahoo.com/d/search/keyword=hindawi/messages/ACmH8QoAAAS3WrmoXwVDqLGz7Og?.intl=id&.lang=id-ID&.partner=none&.src=fp

III. AUTHOR'S FEEDBACK NEEDED
1 April 2018

5327504: Authors' Feedback Needed — Yahoo/Email M...

ahmed.khaled@hindawi.com — Min, 1 Apr 2018 jam 20.18
Kepada: asmiati308@yahoo.com
Cc: sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

This is regarding manuscript 5327504 titled "ON THE LOCATING CHROMATIC NUMBER OF SOME BARBELL GRAPHS" submitted to International Journal of Mathematics and Mathematical Sciences. While checking your manuscript, we had comments regarding the following points:

Please note that the reference list should include a diversity of sources that support the scholarly content of the manuscript. Concentrating on the author's own work could be misinterpreted as an attempt to increase citations to that author's work. To avoid such a misinterpretation, please update the reference list to include more diverse citations and reduce citations to the work of [cited author].

If particular citations to this author's work are important, please keep them and explain why they are needed. All articles in the reference list must be cited in the text and articles should not be only cited in passing; either cite and discuss the article or remove it from the reference list.

Please update the reference list and the manuscript text accordingly and send me the updated PDF file and Word file of the manuscript as an email attachment, and I will process this update on our MTS on your behalf.

We look forward to your quick response.

Hindawi encourages all authors to share the data underlying the findings of their manuscripts. Data sharing allows researchers to verify the results of an article, replicate the analysis, and conduct secondary analyses. Accordingly, please include a "Data Availability" statement for the data used in your manuscript. This statement should describe how readers can access the data supporting the conclusions of the study, and clearly outline the reasons why unavailable data cannot be released.

We look forward to hearing from you.

Best regards,

Ahmed Khaled
Editorial Office
Hindawi
http://www.hindawi.com

https://mail.yahoo.com/d/search/keyword=hindawi/messages/ACmH8QoAAAAwWsDcKw7sUEyre_o?.intl=id&.lang=id-ID&.partner=none&.src=fp

IV. AUTHORS' DATA NEEDED

1 April 2018

Kembali Arsipkan Pindahkan Hapus Spam

5327504: Authors' Data Needed

Yahoo/Email M...

ahmed.khaled@hindawi.com

Kepada: asmiati308@yahoo.com

Min, 1 Apr 2018 jam 20.18

Dear Dr. Asmiati,

This is regarding manuscript 5327504 titled "ON THE LOCATING CHROMATIC NUMBER OF SOME BARBELL GRAPHS" submitted to International Journal of Mathematics and Mathematical Sciences. While checking the data of the authors for verification, we had the following comment(s):

We were unable to find any academic history for I Ketut Sanda Gunce Yana, and Lyra Yulianti.

In order to proceed with the review process of manuscript 5327504, please provide us with institutional email addresses for I Ketut Sanda Gunce Yana, and Lyra Yulianti, along with their institutional web pages, and a list of their previous publications.

We look forward to hearing from you.

Best regards,

Ahmed Khaled
Editorial Office
Hindawi
http://www.hindawi.com

https://mail.yahoo.com/d/search/keyword=hindawi/messages/ACmH8QoAAATRWsDcLgt9aNIb-ko?.intl=id&.lang=id-ID&.partner=none&.src=fp

Kembali   Arsipkan   Pindahkan   Hapus   Spam

Urgent: 5327504: Authors' Feedback Needed

Yahoo/Email M...

Ahmed Khaled <ahmed.khaled@hindawi.com>
Kepada: asmiati308@yahoo.com
Cc: sikesaguya412@gmail.com, lyrayulianti@gmail.com

Rab, 4 Apr 2018 jam 14.36

Dear Dr. Asmiati,

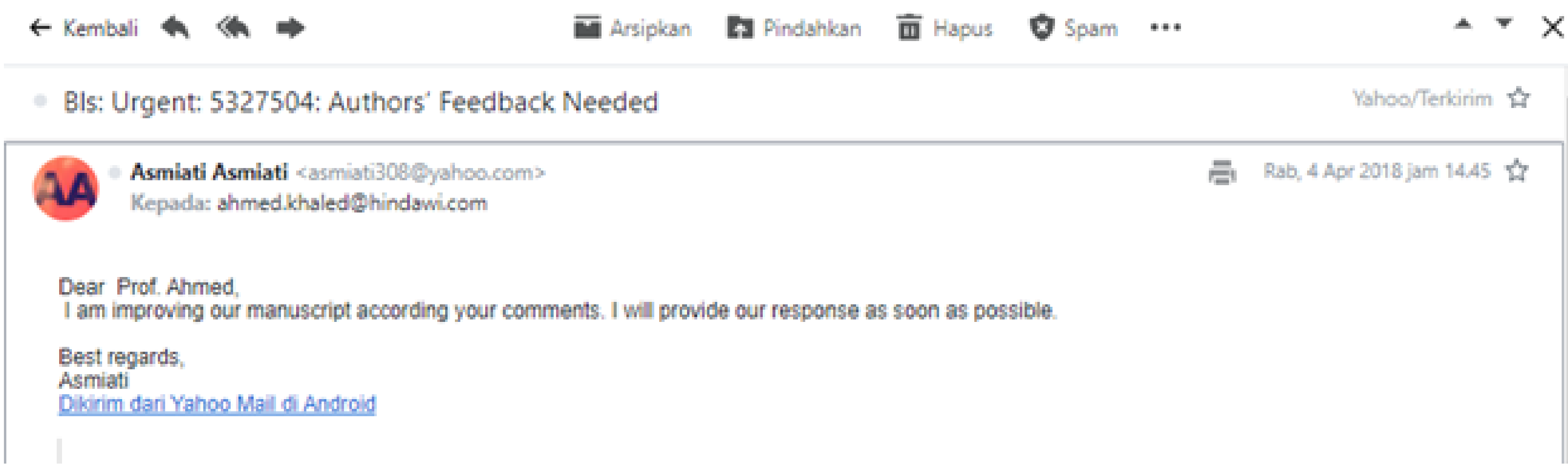
Please confirm the receipt of my previous email, and provide your response as soon as possible.

Your prompt response is needed in order to avoid any further delay in the review process.

Best regards,

Ahmed

--
****
Ahmed Khaled
Editorial Office
Hindawi
http://www.hindawi.com
****



← Kembali   Arsipkan   Pindahkan   Hapus   Spam   •••

Bls: Urgent: 5327504: Authors' Feedback Needed   Yahoo/Terkirim

Asmiati Asmiati <asmiati308@yahoo.com>   Rab, 4 Apr 2018 jam 14.45

Kepada: ahmed.khaled@hindawi.com

Dear Prof. Ahmed,
I am improving our manuscript according your comments. I will provide our response as soon as possible.

Best regards,
Asmiati
Dikirim dari Yahoo Mail di Android

Response For Hindawi

Yahoo/Terkirim

Asmiati Asmiati <asmiati308@yahoo.com>

Kepada: ahmed.khaled@hindawi.com

Kam, 5 Apr 2018 jam 08.33

Dear Prof. Ahmed Khaled
Editorial Office Hindawi,

Hereby I attach some files related to our manuscript. Thank you very much for your attention.

Best regards,
Asmiati

Unduh semua lampiran sebagai file zip

Respon For ... .pdf
114.6kB

Paper Barb... .docx
207.3kB

Paper Barbe... .pdf
538.9kB

REVISI DATA AUTHOR

Dear Prof. Ahmed Khaled

Editorial Office Hindawi,

We have fixed manuscript according to your comments:

1. The institutional email address for Lyra is lyra@sci.unand.ac.id, and the institutional web is : http://matematika.fmipa.unand.ac.id

   List of Publications:

   a) Asmiati, Wamiliana, Devriyadi, **Lyra Yulianti,** On Some Petersen Graphs Having Locating Chromatic Number Four Or Five, *Far East Journal of Mathematical Sciences* 102 (4) : 769 – 778 (2017)
   b) **Lyra Yulianti**, Nirmala Santi, Admi Nazra, Ramsey Minimal Graphs for $2K_2$ versus $2C_n$, *Applied Mathematical Sciences* 9 (85): 4211 – 4217 (2015)
   c) Kristiana Wijaya, **Lyra Yulianti**, Edy Tri Baskoro, Hilda Assiyatun, Djoko Suprijanto, All Ramsey $(2K_2,C_4)$-Minimal Graphs, *Journal of Algorithms and Computation* 46 : 9 – 25 (2015).
   d) Syafrizal Sy, Gema Histamedika, **Lyra Yulianti** , The Rainbow Connection of Fan and Sun, *Applied Mathematical Sciences* 7 (64): 3155 – 3159 (2013).
   e) **Lyra Yulianti**, The asymptotic distribution of the number of 3-star factors in random d-regular graphs, *Discrete Mathematics, Algorithms and Applications* 3(2) : 203 – 222 (2011)
   f) Edy Tri Baskoro, **Lyra Yulianti**, Ramsey minimal graphs for $2K_2$ versus P_n, *Advances and Applications of Discrete Mathematics* 8(2) : 83 – 90 (2011)
   g) **Lyra Yulianti**, Hilda Assiyatun, Saladin Uttunggadewa, Edy Tri Baskoro, On Ramsey $(K_{1,2},P_4)$-minimal graphs, *Far East Journal of Mathematical Sciences* 40(1) : 23 – 36 (2010)
   h) Tomas Vetrik, **Lyra Yulianti**, Edy Tri Baskoro, On Ramsey $(K_{1,2},C_4)$-minimal graphs, *Discussiones Mathematicae Graph Theory* 30(4) : 637 – 649 (2010)
   i) Tomas Vetrik, Edy Tri Baskoro, **Lyra Yulianti**, A Note on Ramsey $(K_{1,2},C_4)$-minimal Graphs of diameter 2, *Proceeding of the International Conference 70 years of FCE STU Bratislava*, pp 1 – 4 (2008)
   j) Edy Tri Baskoro, **Lyra Yulianti**, Hilda Assiyatun, Ramsey $(K_{1,2},C_4)$-minimal graphs, *Journal of Combinatorial Mathematics and Combinatorial Computing* 65: 79 – 90 (2008)

2. I Ketut Sadha Gunce Yana is my student in Mathematics Department, Lampung University, and he does not have the institutional email address and does not have the previous publications yet.
3. I have reduced my paper in the reference list, but there are three important references that I keep because those papers conducted as the references of my previous research.

Thank you very much for your kindest attention,

Regards,

Asmiati

REVISI 1 MENAMBAHKAN DATA AUTHOR DAN REFERENCES

ON THE LOCATING CHROMATIC NUMBER OF SOME BARBELL GRAPHS

Asmiati[1], I Ketut Sadha Gunce Yana[2], Lyra Yulianti[3]
[1]Mathematics Departement, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No.1 Bandar Lampung, Indonesia.
[2] Student of Mathematics Departement, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No.1 Bandar Lampung, Indonesia.
[3] Mathematics Departement, Faculty of Mathematics and Natural Sciences, Andalas University, Kampus UNAND Limau Manis, Padang 25163, Indonesia.
Email: asmiati308@yahoo.com, asmiati.1976@fmipa.unila.ac.id, sikesaguya412@gmail.com, lyra@sci.unand.com

Abstract. The locating chromatic number of a graph is the minimal color required so that it qualifies for a locating coloring. In this paper we will discuss about the locating chromatic number of barbell graph; where both of them contain a complete graph K_n or Petersen graph $P_{n,1}$ for $n \geq 3$.

Keyword: locating chromatic number, barbell graph, complete graph, Petersen graph.

1. Introduction

The partition dimension was introduced by Chartrand et al. [5] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [11], the optimization of threat detecting sensors [10], chemical data classification [8]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [6]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Consider $G=(V,E)$ as the given connected graph and c as the proper coloring of G using k colors $1,2,\ldots,k$ for some positive integer k. We denote $\Pi=\{C_1,C_2,...,C_k\}$ as the partition of $V(G)$, where C_i is the color class, i.e the set of vertices that given the i-th color, for $i\in[1,k]$. For an arbitrary vertex $v \in V(G)$, the color code $c_\Pi(v)$ is defined as the ordered k-tuple

$$c_\pi(v) = (d(v,C_1), d(v,C_2), \ldots, d(v,C_k)),$$

where $d(v,C_i)=\min\{d(v,x) \mid x\in C_i\}$ for $i\in[1,k]$. If for every two vertices $u,v \in V(G)$, their color codes are different, $c_\pi(u) \neq c_\pi(v)$, then c is defined as the locating coloring of G using k colors. The locating chromatic number of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem about the locating chromatic number of a graph, proven by Chartrand et al. [6]. The neighborhood of vertex s in a connected graph G, denoted by $N(s)$, is the set of vertices adjacent to s.

Theorem 1.1 [6] *Let c be a locating coloring in a connected graph G. If s and t are distinct vertices of G such that $d(s,u) = d(t,u)$ for all $u \in V(G) - \{s,t\}$, then $c(s) \neq c(t)$. In particular, if s and t are non-adjacent vertices of G such that $N(s) = N(t)$, then $c(s) \neq c(t)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph *G*.

Corollary 1.1 [6] *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \geq k + 1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on some certain graph classes. Chartrand et al. [7] has successed in constructing tree on *n* vertices, $n \geq 5$ with locating chromatic numbers varying from *3* to *n*, except for $(n - 1)$. Then Behtoei and Omoomi [4] have obtained the locating chromatic number of the Kneser graph. Recently, Asmiati et al.[1] obtained the locating chromatic number of Petersen Graph, $P_{n,1}$, for $n \geq 3$.

There are some recent results for some special cases of trees as follows. Asmiati et al. [3] has successed in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and Asmiati et al. [2] for firecracker graphs. Next, Des Wellyyanti et al.[9] determined the locating chromatic number for complete n-ary tree.

The following definition of Petersen graph is taken from [1]. Let $\{u_1, u_2, \ldots, u_n\}$ be the set of vertices in the outer cycle and $\{v_1, v_2, \ldots, v_n\}$ be the set of vertices in the inner cycle, for $n \geq 3$. From the definition, we have that the Petersen graph, denoted by $P_{n,k}$, for $n \geq 3$ and $1 \leq k \leq \left\lfloor \frac{n-1}{2} \right\rfloor$, has $2n$ vertices and *3n* edges.

Theorem 1.2 and Theorem 1.3 gave the locating chromatic numbers for complete graph and Petersen graph.

Theorem 1.2 [7]
For $n \geq 2$, the locating chromatic number of complete graph K_n is n.

Theorem 1.3 [1]
The locating chromatic number of Petersen Graph $P_{n,1}$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.

The barbell graph is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number of barbell graph $B_{m,n}$ for $m,n \geq 3$, where *G* and *H* are two copies of complete graph on *m* and *n* vertices, K_m and K_n, respectively. If $m = n$, we denote the barbell graph by $B_{n,n}$. Secondly, we obtain the locating chromatic number of barbell graph $B_{P_{n,1}}$ for $n \geq 3$, where *G* and *H* are two copies of Petersen graphs $P_{n,1}$.

2. Results and Discussion

Theorem 2.1

The locating chromatic number of Barbell Graph $B_{n,n}$ is $n+1$, for $n \geq 3$.

Proof:

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains the complete graph K_n, then by Theorem 1.2, we have $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is the locating coloring using n colors. It is clear that there are two vertices have the same color codes, a contrary. Thus, we have that $\chi_L(B_{n,n}) \geq n+1$.

Next, we construct the upper bound of the locating chromatic number for barbell graph $B_{n,n}$. The set of vertices of the first complete graph is denoted by $V(K_n^1) = \{u_i;\ i \in [1,n]\}$, whereas the set of vertices of the second complete graph is denoted by $V(K_n^2) = \{v_i;\ i \in [1,n]\}$.

Let c be a coloring on $B_{n,n}$ using $n+1$ colors. We assign the following colors of $V(B_{n,n})$:

$$c(u_i) = i \qquad ; 1 \leq i \leq n$$

$$c(v_i) = \begin{cases} i & , 2 \leq i \leq n-1; \\ n & , i = 1; \\ n+1 & , \text{otherwise.} \end{cases}$$

By using this coloring, we obtain the color codes of $V(B_{n,n})$ as follows.

$$c_\Pi(u_i) = \begin{cases} 0 & , \text{(i)th} - \text{component for } 1 \leq i \leq n; \\ 2 & , \text{(n + 1)th} - \text{component for } 1 \leq i \leq n-1; \\ 1 & , \text{otherwise.} \end{cases}$$

$$c_\Pi(v_i) = \begin{cases} 0 & , \text{(i)th} - \text{component for } 2 \leq i \leq n-1, \text{ or} \\ & \ \text{(n)th} - \text{component for } i = 1, \text{ or} \\ & \ (n+1) - \text{component for } i = n; \\ 3 & , \text{(1)st} - \text{component for } 1 \leq i \leq n-1; \\ 2 & , \text{(1)st} - \text{component for } i = n; \\ 1 & , \text{otherwise.} \end{cases}$$

Since all vertices on $V(B_{n,n})$ have distinct color codes, then c is a locating coloring. Thus, $\chi_L(B_{n,n}) \leq n+1$. □

The following figure is a minimum locating coloring of barbell graph $B_{6,6}$.

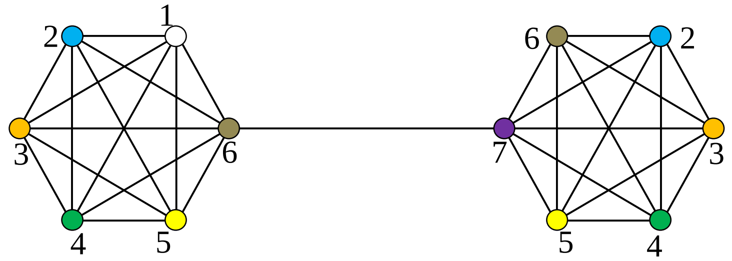


Figure 1. A minimum locating coloring of barbell graph $B_{6,6}$

The following Corollary 2.2 is the direct consequence of Theorem 2.1.

Corollary 2.2
For $n, m \geq 3$ and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is

$$\chi_L(B_{m,n}) = max\ \{n, m\}.$$

Theorem 2.3
For $n \geq 3$, the locating chromatic number of barbell graph $B_{P_{n,1}}$ is

$$\chi_L(B_{P_{n,1}}) = \begin{cases} 4, for\ odd\ n \\ 5, for\ even\ n \end{cases}.$$

Proof. To prove this theorem, we consider two cases as follows.

Case 1. $\chi_L(B_{P_{n,1}}) = 4$, for odd n.

Since the barbell graph $B_{P_{n,1}}$ contains Petersen Graph $P_{n,1}$ for odd n, then by Theorem 1.3, we have $\chi_L(B_{P_{n,1}}) \geq 4$.

Next, we determine the upper bound of the locating chromatic number of $B_{P_{n,1}}$. For odd n, let $\{u_i, u_{n+i}\ ;\ i \in [1, n]\}$ be the set of vertices of the first Petersen Graph and $\{w_i, w_{n+i}\ ;\ i \in [1, n]\}$ be the set of vertices of the second Petersen Graph.

Let c be a coloring of $V(B_{P_{n,1}})$ using 4 colors, defined as follows:

$$c(u_i) = \begin{cases} 1 & , i = 1; \\ 3 & , \text{for even } i,\ i \geq 2; \\ 4 & , \text{for odd } i,\ i \geq 3. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2 & , i = 1; \\ 3 & , for \text{ odd } i \geq 3; \\ 4 & , \text{for even } i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1 & , \text{odd } i < n-1; \\ 2 & , \text{even } i \leq n-1; \\ 3 & , i = \text{n}. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1 & , \text{even } i \leq n-1; \\ 2 & , \text{odd } i < n-1; \\ 4 & , i = n. \end{cases}$$

The color codes of $V(B_{P_{n,1}})$ for odd n are:

$$c_\Pi(u_i) = \begin{cases} i & , (2)\text{nd} - \text{component for } i \leq \frac{n+1}{2}; \\ i-1 & , (1)\text{st} - \text{component for } i \leq \frac{n+1}{2}; \\ n-i+1 & , (1)\text{st} - \text{component for } i > \frac{n+1}{2}. \\ \\ n-i+2 & , (2)\text{nd} - \text{component } i > \frac{n+1}{2}; \\ 0 & , (3)\text{th} - \text{component for even } i \geq 2; \\ & \ \ (4)\text{th} - \text{component for odd } i > 2; \\ 1 & , \text{ otherwise.} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i & , (1)\text{st} - \text{component for } i \leq \frac{n+1}{2}; \\ i-1 & , (2)\text{nd} - \text{component for } i \leq \frac{n+1}{2}; \\ n-i+1 & , (2)\text{nd} - \text{component for } i > \frac{n+1}{2}. \\ \\ n-i+2 & , (1)\text{st} - \text{component for } i > \frac{n+1}{2}; \\ 0 & , (4)\text{th} - \text{component for even } \geq 2; \\ & \ \ (3)\text{th} - \text{component for odd } i \geq 2; \\ 1 & , \text{ otherwise.} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i & , (3)\text{th} - \text{component for } i \leq \frac{n-1}{2}; \\ i+1 & , (4)\text{th} - \text{component for } i \leq \frac{n-1}{2}; \\ n-i & , (3)\text{th} - \text{component for } i \geq \frac{n+1}{2}. \\ \\ n-i+1 & , (4)\text{th} - \text{component for } i \geq \frac{n+1}{2}; \\ 0 & , (2)\text{nd} - \text{component for even } i \leq n-1; \\ & \ \ (1)\text{st} - \text{component for odd } i \leq n-1; \\ 1 & , \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_{n+i}) = \begin{cases} i & , (4)\text{th} - \text{component for } i \leq \frac{n-1}{2}; \\ i+1 & , (3)\text{th} - \text{component for } i \leq \frac{n-1}{2}; \\ n-i & , (4)\text{th} - \text{component for } i \geq \frac{n+1}{2}; \\ n-i+1 & , (3)\text{th} - \text{component for } i \geq \frac{n+1}{2}; \\ 0 & , (1)\text{th} - \text{component for even } i \leq n-1; \\ & (2)\text{th} - \text{component for odd } i \leq n-1; \\ 1 & ; \text{otherwise.} \end{cases}$$

Since all vertices on $V(B_{P_{n,1}})$ have distinct color codes, then c is a locating coloring. As the result, we have that $\chi_L(B_{P_{n,1}}) \leq 4$.

Case 2. $\chi_L(B_{P_{n,1}}) = 5$, for even n.
Since the barbell graph $B_{P_{n,1}}$ contains Petersen Graph $P_{n,1}$ for even n, then by Theorem 1.3, we have $\chi_L(B_{P_{n,1}}) \geq 5$.

Next, we determine the upper bound of the locating chromatic number of $B_{P_{n,1}}$ for even n. Let c be a coloring of $B_{P_{n,1}}$ using 5 colors as follows:

$$c(u_i) = \begin{cases} 1 & , i = 1; \\ 3 & , \text{even } 2 \leq i \leq n-1; \\ 4 & , \text{odd } 2 < i \leq n-1; \\ 5 & , i = n. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2 & , i = 1; \\ 3 & , \text{odd } i > 2; \\ 4 & , \text{even } i \geq 2; \end{cases}$$

$$c(w_i) = \begin{cases} 1 & , \text{odd } i \leq n-2; \\ 2 & , \text{even } i \leq n-2. \\ 3 & , i = n-1; \\ 4 & , i = n. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1 & , \text{even } i \leq n-1; \\ 2 & , \text{odd } i \leq n-1; \\ 5 & , i = n. \end{cases}$$

The color codes of $V(B_{P_{n,1}})$ for even n are:

$$c_\Pi(u_i) = \begin{cases} i & ,\ (2)\text{nd}, (5)\text{th} - \text{component for } i \le \frac{n}{2}; \\ i-1 & ,\ (1)\text{st} - \text{component for } i \le \frac{n}{2}; \\ n-i & ,\ (5)\text{th} - \text{component for } i > \frac{n}{2}; \\ n-i+1 & ,\ (1)\text{st} - \text{component for } i > \frac{n}{2}; \\ n-i+2 & ,\ (2)\text{nd} - \text{component for } i > \frac{n}{2}; \\ 0 & ,\ (3)\text{th} - \text{component for even } 2 \le i \le n-1; \\ & \ \ (4)\text{th} - \text{component for odd } 2 < i \le n-1; \\ 2 & ,\ (4)\text{th} - \text{component for } i = 1; \\ & \ \ (3)\text{th} - \text{component for } i = n; \\ 1 & ,\ \text{otherwise.} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i & ,\ (1)\text{st} - \text{component for } i \le \frac{n}{2}; \\ i-1 & ,\ (2)\text{nd} - \text{component for } i \le \frac{n}{2}; \\ i+1 & ,\ (5)\text{th} - \text{component for } i \le \frac{n}{2}; \\ n-i+1 & ,\ (2)\text{nd and } (5) - \text{components for } i > \frac{n}{2}; \\ & \\ n-i+2 & ,\ (1)\text{th} - \text{component for } i > \frac{n}{2}; \\ 0 & ,\ (3)\text{th} - \text{component for odd } 2 \le i \le n; \\ & \ \ (4)\text{th} - \text{component for even } 2 \le i \le n; \\ 2 & ,\ (3)\text{th} - \text{component for } i = 1; \\ 1 & ,\ \text{otherwise.} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i & ,\ (4)\text{th} - \text{component for } i \le \frac{n}{2}; \\ i+1 & ,\ (5)\text{th} - \text{component for } i \le \frac{n}{2}; \\ & \ \ (3)\text{th} - \text{component for } i \le \left(\frac{n}{2}\right) - 1; \\ n-i & ,\ (4)\text{th} - \text{component for } i > \frac{n}{2}; \\ n-i+1 & ,\ (5)\text{th} - \text{component for } i > \frac{n}{2}. \\ & \\ n-i-1 & ,\ (3)\text{th} - \text{component for } \frac{n}{2} \le i \le n-1; \\ 0 & ,\ (1)\text{st} - \text{component for odd } i \le n-2; \\ & \ \ (2)\text{nd} - \text{component for odd } i \le n-2; \\ 2 & ,\ (1)\text{st} - \text{component for } i = n-1; \\ & \ \ (2)\text{nd} - \text{component for } i = n; \\ 1 & ,\ \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_{n+i}) = \begin{cases} i & \text{, (5)th} - \text{component for } i \le \frac{n}{2}; \\ i+1 & \text{, (4)th} - \text{component for } i \le \frac{n}{2}; \\ i+2 & \text{, (3)th} - \text{component for } i \le \left(\frac{n}{2}\right) - 1; \\ n-i & \text{, (3)th} - \text{component for } \frac{n}{2} \le i \le n-1; \\ & \quad \text{(5)th} - \text{component for } i > \frac{n}{2}; \\ n-i+1 & \text{, (4)th} - \text{component for } i > \frac{n}{2}; \\ 0 & \text{, (1)th} - \text{component for even } i \le n-1; \\ & \quad \text{(2)th} - \text{component for odd } i \le n-1; \\ 2 & \text{, (1)st and (3)th} - \text{component for } i = n; \\ 1 & \text{, otherwise.} \end{cases}$$

Since all vertices have distinct color codes on $V(B_{P_{n,1}})$ for even n, then c is a locating coloring. Thus, we have that $\chi_L(B_{P_{n,1}}) \le 5$. □

The following figure is a minimum locating coloring of barbell graph $B_{P_{5,1}}$.

Figure 2. A minimum locating coloring of $B_{P_{5,1}}$

3. Acknowledgement

We are thankful to DRPM Dikti for the Fundamental Grant 2018.

References

[1] Asmiati, Wamiliana, Devriyadi, and Yulianti, L., On Some Petersen Graphs Having Locating-Chromatic Number Four or Five, *Far East Journal of Mathematical Sciences*, vol. 102, no. 4, pp. 769-778, 2017.

[2] Asmiati, Assiyatun, H., Baskoro, E.T., Suprijanto, D., Simanjuntak, R., Uttunggadewa, S., Locating-chromatic number of firecracker graphs, *Far East Journal of Mathematical Sciences*, 63(1), pp. 11-23, 2012.

[3] Asmiati, Assiyatun, H., and Baskoro, E.T., Locating-Chromatic Number of Amalgamation of Stars, *ITB J. Sci*., vol. 43, no. 1, pp. 1-8, 2011.

[4] Behtoei, A., and Omoomi, B., On the locating chromatic number Kneser Graphs, *Discrete Applied Mathematics*, 159, pp. 2214-2221, 2011.

[5] Chartrand, G., Salehi, E., and Zhang, P., On the partition dimension of graph. *Congr. Numer*., 130, pp. 157-168, 1998.

[6] Chartrand, G., Erwin, D., Henning, M.A., Slater, P.J., and Zhang, P., The Locating-Chromatic Number of a Graph, *Bull. Inst. Combin. Appl*., vol. 36, pp. 89-101, 2002.

[7] Chartrand, G., Erwin, D., Henning, M.A., Slater, P.J., Zhang, P. Graph of order n with locating-chromatic number $n-1$, *Discrete Math*., 269, pp. 65-79, 2003.

[8] Chartrand, G., Zhang, P., The theory and applications of resolvability in graphs: a survey. *Congr. Numer*., 160, pp. 47-68, 2003.

[9] Des Wellyanti, Baskoro, E.T., Simanjuntak, R., Uttunggadewa, S., On Locating chromatic number of complete n-ary tree. *ACKE Int. J. Graphs Comb*., 10(3), pp. 309-319, 2013.

[10] Johnson, M.A., Structure-activity maps for visualizing the graph variables arising in drug design. *J. Biopharm. Statist*. **3**, pp. 203-236, 1993.

[11] Saenpholphat, V., Zhang, P., Conditional resolvability: a survey, *Internat. J. Math. Math. Sci*. **38**., pp. 1997-2017, 2004.

V. MAJOR REVISION REQUIRED

8 Juni 2018

5327504: Major Revision Required

Yahoo/Email M...

Dalibor Froncek <ijmms@hindawi.com>

Jum, 8 Jun 2018 jam 15.15

Kepada: asmiati308@yahoo.com

Cc: dfroncek@d.umn.edu, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

Following the review of Research Article titled "On The Locating Chromatic Number Of Some Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti, I recommend that it should be revised taking into account the changes requested by the reviewer(s). Since the requested changes are major, the revised manuscript will undergo a second round of review by the same reviewer(s). Please login to the Manuscript Tracking System to read the submitted review report(s) and submit the revised version of your manuscript no later than Friday, July 06, 2018.

To submit the revised version of your manuscript, please access "Author Activities" in your account and upload the PDF file of your revised manuscript. Also, please submit your replies to the comments of the reviewer(s) as an additional PDF file.

Best regards,

Dalibor Froncek
dfroncek@d.umn.edu

https://mail.yahoo.com/d/search/keyword=hindawi/messages/AO3sOIJMfjJ5WyBW9ALeYN4CrxE?.intl=id&.lang=id-ID&.partner=none&.src=fp

PENILAIAN REVIEWER 1 (REVISI MAYOR)

REFEREE'S REPORT

on the paper 5327504

Title : On the locating chromatic number of some barbell graphs

Authors: Asmiati, I Ketut Sadha Gunce Yana and Lyra Yulianti

The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In the present paper the authors investigate the locating chromatic number for two families of barbell graphs.

The topic is actual and the results are interesting. Due to the fact that no general theorem for determining the locating chromatic number of graphs is known, it make sense to investigate the locating chromatic number for families of graphs.

The present version of the paper is not prepared carefully and contains several incorrectness and formal mistakes.
Therefore I do not recommend the publication of the paper as it is. A revised version of the paper prepared by the comments below can be accepted for publication.

Comments:

Page 1, title: write "certain" instead "some"

Page 1: Rewrite Abstract with using the definition on locating coloring.

Page 2, after Corollary 1.1: Complete information of the paper [Baskoro, E.T., Asmiati, Characterizing all trees with locating-chromatic number 3, Electronic Journal of Graph Theory and Applications 1(2) (2013), pp. 109-117.], where are characterized all trees with locating-chromatic number 3.

Page 2, Petersen graph: The Petersen graph contains only 10 vertices and 15 edges. You want to consider the generalized Petersen graph $P(n, m)$ with $2n$ vertices and $3n$ edges which was introduced in [Watkins, M.E., A theorem on Tait colorings with an application to the generalized Petersen graphs, J. Combin. Theory 6 (1969), pp. 152-164.]

Page 2, Theorem 1.3: complete "generalized" before "Petersen"

Page 2, line -4: after $m, n \geq 3$ write "where G and H are complete graphs on m and n vertices, respectively."

Page 3, Proof of Theorem 2.1 start as follows: Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1} \{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

Page 3, in the proof of Theorem 2.1 and also in the proof of Theorem 2.3: use "i^{th}" instead "$(i)th$"

Page 4, Corollary 2.2: "$\max\{n, m\}$" should be "$\max\{n, m\} + 1$"

Page 5, line 1 and line 5: "$i < n - 1$" change for "$i \leq n - 2$"

Page 5, line 13: "$i > 2$" change for "$i \geq 3$"

Page 5, line -2 and on page 6, lines 6 and 16: "$i \leq n - 1$" change for "$i \leq n - 2$"

Page 7, line 16: write "$3 \leq i \leq n - 1$" instead "$2 \leq i \leq n$"

Page 7, line -5: "$i \leq n - 2$" change for "$i \leq n - 3$"

Page 7, line -4: write "for even $i \leq n - 2$" instead "for odd $i \leq n - 2$"

Page 8, line 7: "$i \leq n - 1$" change for "$i \leq n - 2$"

REFEREE'S REPORT

on the revised version of the paper 5327504.v2

Title : On the locating chromatic number of certain barbell graphs

Authors: Asmiati, I Ketut Sadha Gunce Yana and Lyra Yulianti

Again the revised version of the paper is not prepared carefully and the authors did not accept all suggestions and recommendations given in the referee's report. Therefore I do not recommend the publication of the paper as it is. A revised version of the paper prepared by the comments below can be accepted for publication.

Comments:

Page 1, Abstract rewrite by the following way: The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion.
In this paper we investigate the locating chromatic number for two families of barbell graphs.

Page 1, lines from -1 to -6 and on page 2 lines from 1 up to 7 - rewrite by the following way: Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \to \{1, 2, \ldots, k\}$ where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$ where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$ where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.
The following theorem is a basic theorem proved by Chartrand et al. in [8]. The neighborhood of vertex s in a connected graph G, denoted by $N(s)$, is the set of vertices adjacent to s.

Page 2, the text after Corollary 1.1 until Theorem 1.2. rewrite by the following way: There are some interesting results related to the determination of the

locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand, et al. in [8] have determined all graphs of order n with locating chromatic number n, namely a complete multipartite graphs of n vertices. Moreover, Chartrand et al. [9] have succeeded in constructing trees on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [6] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [1] obtained the locating chromatic number of the generalized Petersen graph $P(n,1)$ for $n \geq 3$. Baskoro and Asmiati [5] have characterized all trees with locating-chromatic number 3. In [Syofyan, D.K., Baskoro, E.T., Assiyatun, H., Trees with Certain Locating-Chromatic Number, J. Math. Fund. Sci. 48(1) (2016), pp. 39-47] were characterized all trees of order n with locating chromatic number $n-t$, for any integers n and t, where $n > t+3$ and $2 \leq t < \frac{n}{2}$. Asmiati et al. in [4] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [2] for firecracker graphs. Next, Wellyyanti et al. [11] determined the locating chromatic number for complete n-ary trees.
The generalized Petersen graph $P(n,m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [14]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.
Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Page 2 and several times later: The generalized Petersen graph defined by Watkins has notation $P(n,m)$. Therefore change "$P_{n,1}$" for "$P(n,1)$" or use notation $D_n = P_n \Box P_2$ as for prism.

Page 3, line 13: write "of the generalized Petersen graph $P(n,1)$" instead of "of generalized Petersen graphs $P_{n,1}$"

Page 3, Theorem 2.1. rewrite as follows: Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.
Theorem 2.1. Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n+1$.

Page 3, lines -10 and -11: The sentence "Next, suppose that ..." replace by "Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction."

Page 3, lines -2, -3 and -4: The labeling $c(v_i)$ and also all other labelings write by the following way

$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases}$$

Page 4 lines from -1 to -4 and on page 5 lines from 1 to 5 replace as follows:
Proof Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_iu_{i+1}, u_{n+i}u_{n+i+1}, w_iw_{i+1}, w_{n+i}w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_nu_1, u_{2n}u_{n+1}, w_nw_1, w_{2n}w_{n+1}\} \cup \{u_iu_{n+i}, w_iw_{n+i} : 1 \leq i \leq n\} \cup \{u_nw_n\}$.
Let us distinguish two cases.
Case 1, n odd. According to Theorem 1.3 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

Page 6, lines from -8 to -12 rewrite by the following way:
Case 2, n even. In view of the lower bound from Theorem 1.3 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \to \{1, 2, \dots, 5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n \geq 4$, we describe the locating coloring as follows:

Page 8, on the line 7 change "even" for "odd" and on the line 8 change "odd" for "even". It means

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component, } i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component, } i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component, } i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 4^{th} \text{ component, } i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component, } i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component, } \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component, } i \text{ odd, } i \leq n-3 \\ & \text{for } 2^{nd} \text{ component, } i \text{ even, } i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component, } i = n-1 \\ & \text{for } 2^{nd} \text{ component, } i = n \\ 1, & \text{otherwise.} \end{cases}$$

Page 9: insert the reference
Syofyan, D.K., Baskoro, E.T., Assiyatun, H., Trees with certain locating-chromatic number, J. Math. Fund. Sci. 48(1) (2016), pp. 39-47.

Kembali Arsipkan Pindahkan Hapus Spam

Bls: 5327504: Major Revision Required

Yahoo/Terkirim

Asmiati Asmiati <asmiati308@yahoo.com>
Kepada: ahmed.khaled@hindawi.com

Rab, 13 Jun 2018 jam 06.27

Dear Prof. Dalibor Froncek,

Thank you very much for your information. I will revise our paper soon.

Best regards,
Asmiati

Dikirim dari Yahoo Mail di Android

VI. REVISED VERSION RECEIVED

FOR MAYOR REVISON

26 Juni 2018

5327504: Revised Version Received

Yahoo/Email M...

International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hind

Sel, 26 Jun 2018 jam 13.00

Kepada: asmiati308@yahoo.com

Cc: ahmed.khaled@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

The revised version of Research Article 5327504 titled "On The Locating Chromatic Number Of Some Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti has been received. The editor assigned to handle the review process of your manuscript will inform you as soon as a decision is reached.

Thank you for submitting your work to International Journal of Mathematics and Mathematical Sciences.

Best regards,

--
****
Ahmed Khaled
Editorial Office
Hindawi
http://www.hindawi.com
****

https://mail.yahoo.com/d/search/keyword=hindawi/messages/ALOBz7FVaEaRWzHWYgf3ELiO4_s?.intl=id&.lang=id-ID&.partner=none&.src=fp

TANGGAPAN AUTHOR KE REVIEWER 1 (REVISI MAYOR)

Response to Referee's Report on the paper 5327504

We are thankful for the referee's comments. We have revised the manuscript based on suggestions in referee's report, except for Corollary 2.2. The statement in the corollary is correct, that for case $n, m \geq 3$ and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is $max\ \{n, m\}$. The following figure is a counter example for the case.

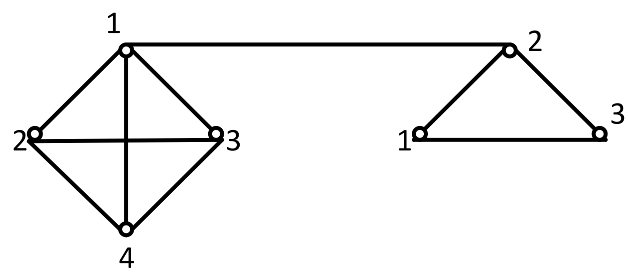


Figure 1. A minimum locating coloring of barbell graph $B_{4,3}$

Let G be a connected graph and c a proper coloring of G. For $i = 1, 2, \ldots, k$ define the color class C_i as the set of vertices receiving color i. The color code $c_\Pi(v)$ of a vertex v in is the ordered k-tuple $(d(v, C_1), \ldots, d(v, C_k))$ where $(d(v, C_1)$ is the distance of v to C_i. If all distinct vertices of G have distinct color codes, then c is called a locating-coloring of G . The locating-chromatic number of graph G, denoted by $\chi_L(G)$ is the smallest k such that G has a locating coloring with k colors. Let $\{u_1, u_2, \ldots, u_n\}$ be some vertices on the outer cycle and $\{v_1, v_2, \ldots, v_n\}$ be some vertices on the inner cycle, for $n \geq 3$. The Petersen graph, denoted by $P_{n,k}$, $n \geq 3$, $1 \leq k \leq \lfloor \frac{n-1}{2} \rfloor$, $1 \leq i \leq n$ is a graph that has $2n$ vertices $\{u_i\} \cup \{v_i\}$, and edges $\{u_i u_{i+1}\}$, $\{v_i v_{i+k}\}$, and $\{u_i v_i\}$. We determined that the locating chromatic number of Petersen Graphs $P_{n,1}$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$. In this paper, we discuss the locating-chromatic number for certain operation of s Petersen Graphs $P_{n,1}$.

Response to Referees Report on the paper 5327504

We are thankful for the referees comments. We have revised the manuscript based on suggestions in referees report.

Page 1, abstract replaced by : The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In this paper we investigate the locating chromatic number for two families of barbell graphs.

Page 1, from 1 to 6 and on page 2 lines from 1 up to 7, replaced by : Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \rightarrow \{1, 2, \ldots, k\}$ where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$ where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$ where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [6]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Page 2, the text after Corollary 1.1 until Theorem 1.2., replaced by: There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand, et al. in [6] have determined all graphs of order n with locating chromatic number n, namely a complete multipartite graph of n vertices. Moreover, Chartrand et al. [7] have succeeded in constructing tree on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n - 1)$. Then Behtoei and Omoomi [5] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [3] obtained the locating chromatic number of the generalized Petersen graph $P(n, 1)$ for $n \geq 3$. Baskoro and Asmiati [4] have characterized all trees with locating-chromatic number 3. In [12] were characterized all trees of order n with locating chromatic number $n - t$,

for any integers n and t, where $n > t+3$ and $2 \leq t < \frac{n}{2}$. Asmiati et al. in [1] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [2] for firecracker graphs. Next, Wellyyanti et al. [14] determined the locating chromatic number for complete n-ary trees.
The generalized Petersen graph $P(n,m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [13]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Page 2 and several times later: Generalized Petersen graph $P_{n.1}$ is replaced by $P(n,1)$.

Page 3, Theorem 2.1. written by :Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.
Theorem 2.1 Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n+1$. .

Page 3, lines -10 and -11, replaced by:Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n+1$.

Page 3, lines -2, -3 and -4, replaced by: The labeling $c(v_i)$ and also all other labelings write by the following way

$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases}$$

Page 4 lines from -1 to -4 and on page 5 lines from 1 to 5, replaced by : Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_i u_{i+1}, u_{n+i} u_{n+i+1}, w_i w_{i+1}, w_{n+i} w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1, u_{2n} u_{n+1}, w_n w_1, w_{2n} w_{n+1}\} \cup \{u_i u_{n+i}, w_i w_{n+i} : 1 \leq i \leq n\} \cup \{u_n w_n\}$.

Let us distinguish two cases.

Case 1, n odd. According to Theorem 1.3 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

Page 6, lines from -8 to -12, replaced by : *Case 2, n even.* In view of the lower bound from Theorem 1.3 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \to \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n \geq 4$, we describe the locating coloring in the following way:

Page 8, on the line 7, replaced by :

$$c_{\Pi}(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, \ i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component}, \ i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component}, \ i \leq \frac{n}{2}-1 \\ n-i, & \text{for } 4^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component}, \ \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component}, \ i \text{ odd}, \ i \leq n-3 \\ & \text{for } 2^{nd} \text{ component}, \ i \text{ even}, \ i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n-1 \\ & \text{for } 2^{nd} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases}$$

Page 9: we have revised references.

References

[1] Asmiati, Assiyatun, H., and Baskoro, E.T., Locating-chromatic number of amalgamation of stars, ITB J. Sci. 43(1) (2011), pp. 1-8.

[2] Asmiati, Assiyatun, H., Baskoro, E.T., Suprijanto, D., Simanjuntak, R., Uttunggadewa, S., Locating-chromatic number of firecracker graphs, Far East Journal of Mathematical Sciences, 63(1) (2012), pp. 11-23.

[3] Asmiati, Wamiliana, Devriyadi, and Yulianti, L., On Some Petersen graphs having locating-chromatic number four or five, Far East Journal of Mathematical Sciences 102(4) (2017), pp. 769-778.

[4] Baskoro, E.T., Asmiati, Characterizing all trees with locating-chromatic number 3, Electronic Journal of Graph Theory and Applications 1(2) (2013), pp. 109-117.

[5] Behtoei, A., and Omoomi, B., On the locating chromatic number of Kneser graphs, Discrete Applied Mathematics 159 (2011), pp. 2214-2221.

[6] Chartrand, G., Erwin, D., Henning, M.A., Slater, P.J., and Zhang, P., The locating-chromatic number of a graph, Bull. Inst. Combin. Appl. 36 (2002), pp. 89-101.

[7] Chartrand, G., Erwin, D., Henning, M.A., Slater, P.J., Zhang, P. Graph of order n with locating-chromatic number $n-1$, Discrete Math. 269 (2003), pp. 65-79.

[8] Chartrand, G., Salehi, E., and Zhang, P., On the partition dimension of graph, Congr. Numer. 130 (1998), pp. 157-168.

[9] Chartrand, G., Zhang, P., The theory and applications of resolvability in graphs: a survey, Congr. Numer. 160 (2003), pp. 47-68.

[10] Johnson, M.A., Structure-activity maps for visualizing the graph variables arising in drug design, J. Biopharm. Statist. 3 (1993), pp. 203-236.

[11] Saenpholphat, V., Zhang, P., Conditional resolvability: a survey, Internat. J. Math. Sci. 38 (2004), pp. 1997-2017.

[12] Syofyan, D.K., Baskoro, E.T., Assiyatun, H., Trees with Certain Locating-Chromatic Number, J. Math. Fund. Sci. 48(1) (2016), pp. 39-47.

[13] Watkins, M.E., A theorem on Tait colorings with an application to the generalized Petersen graphs, J. Combin. Theory 6 (1969), pp. 152-164.

[14] Wellyanti, D., Baskoro, E.T., Simanjuntak, R., Uttunggadewa, S., On locating chromatic number of complete n-ary tree, ACKE Int. J. Graphs Comb. 10(3) (2013), pp. 309-319.

On the locating chromatic number of certain barbell graphs

Asmiati[1], I Ketut Sadha Gunce Yana[1], Lyra Yulianti[2]

[1]Mathematics Departement, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No.1 Bandar Lampung, Indonesia
asmiati.1976@fmipa.unila.ac.id; sikesaguya412@gmail.com

[2]Mathematics Departement, Faculty of Mathematics and Natural Sciences, Andalas University, Kampus UNAND Limau Manis, Padang 25163, Indonesia
lyra@sci.unand.com

Abstract

The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion.

In this paper we investigate the locating chromatic number for two families of barbell graphs.

Keywords: locating chromatic number, barbell graph, complete graph, generalized Petersen graph

1 Introduction

The partition dimension was introduced by Chartrand et al. [8] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [11], the optimization of threat detecting sensors [10] and chemical data classification [9]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [6]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \to \{1, 2, \ldots, k\}$ where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$ where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G

is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$ where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [6]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Theorem 1.1. [6] *Let c be a locating coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u, t) = d(v, t)$ for all $t \in V(G) - \{u, v\}$, then $c(u) \neq c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph G.

Corollary 1.1. [6] *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \geq k + 1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand, et al. in [6] have determined all graphs of order n with locating chromatic number n, namely a complete multipartite graph of n vertices. Moreover, Chartrand et al. [7] have succeeded in constructing tree on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n - 1)$. Then Behtoei and Omoomi [5] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [3] obtained the locating chromatic number of the generalized Petersen graph $P(n, 1)$ for $n \geq 3$. Baskoro and Asmiati [4] have characterized all trees with locating-chromatic number 3. In [12] were characterized all trees of order n with locating chromatic number $n - t$, for any integers n and t, where $n > t + 3$ and $2 \leq t < \frac{n}{2}$. Asmiati et al. in [1] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [2] for firecracker graphs. Next, Wellyyanti et al. [14] determined the locating chromatic number for complete n-ary trees.

The generalized Petersen graph $P(n, m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n - 1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [13]. Let us note that the generalized Petersen graph $P(n, 1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n, 1)$.

Theorem 1.2. [7] *For $n \geq 2$, the locating chromatic number of complete graph K_n is n.*

Theorem 1.3. [3] *The locating chromatic number of generalized Petersen Graph $P(n, 1)$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.*

The *barbell graph* is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number for barbell graph $B_{m,n}$ for $m, n \geq 3$, where G and H are complete graphs on m and n vertices, respectively. Secondly, we determine the locating chromatic number of barbell graph $B_{P(n,1)}$ for $n \geq 3$, where G and H are two isomorphic copies of the generalized Petersen graph $P(n, 1)$.

2 Results and Discussion

Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.

Theorem 2.1. *Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n + 1$.*

Proof Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1} \{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains two isomorphic copies of a complete graph K_n, then with respect to Theorem 1.2 we have that $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n + 1$.

To show that $n+1$ is an upper bound for the locating chromatic number of barbell graph $B_{n,n}$ it suffices to prove the existence of an optimal locating coloring $c : V(B_{n,n}) \rightarrow \{1, 2, \ldots, n+1\}$. For $n \geq 3$ we construct the function c in the following way:

$$c(u_i) = i, \qquad 1 \leq i \leq n$$

$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases}$$

By using the coloring c, we obtain the color codes of $V(B_{n,n})$ as follows:

$$c_{\Pi}(u_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component},\ 1 \leq i \leq n \\ 2, & \text{for } (n+1)^{th} \text{ component},\ 1 \leq i \leq n-1 \\ 1, & \text{otherwise,} \end{cases}$$

$$c_{\Pi}(v_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component},\ 2 \leq i \leq n-1 \\ & \text{for } n^{th} \text{ component},\ i = 1, \text{and} \\ & \text{for } (n+1)^{th} \text{ component},\ i = n, \\ 3, & \text{for } 1^{st} \text{ component},\ 1 \leq i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ component},\ i = n \\ 1, & \text{otherwise.} \end{cases}$$

Since all vertices in $V(B_{n,n})$ have distinct color codes, then the coloring c is desired locating coloring. Thus, $\chi_L(B_{n,n}) = n + 1$. □

Corollary 2.1. *For $n, m \geq 3$ and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is*

$$\chi_L(B_{m,n}) = \max\{m, n\}.$$

Next theorem provides the exact value of the locating chromatic number for barbell graph $B_{P(n,1)}$.

Theorem 2.2. *Let $B_{P(n,1)}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{P(n,1)}$ is*

$$\chi_L(B_{P(n,1)}) = \begin{cases} 4, & \text{for odd } n \\ 5, & \text{for even } n. \end{cases}$$

Proof Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_iu_{i+1}, u_{n+i}u_{n+i+1}, w_iw_{i+1}, w_{n+i}w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_nu_1, u_{2n}u_{n+1}, w_nw_1, w_{2n}w_{n+1}\} \cup \{u_iu_{n+i}, w_iw_{n+i} : 1 \leq i \leq n\} \cup \{u_nw_n\}$.

Let us distinguish two cases.

Case 1, n odd. According to Theorem 1.3 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

$$c(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i,\ i \geq 2 \\ 4, & \text{for odd } i,\ i \geq 3. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i,\ i \geq 3 \\ 4, & \text{for even } i,\ i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for odd } i,\ i \leq n-2 \\ 2, & \text{for even } i,\ i \leq n-1 \\ 3, & \text{for } i = n. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i,\ i \leq n-1 \\ 2, & \text{for odd } i,\ i \leq n-2 \\ 4, & \text{for } i = n. \end{cases}$$

For n odd the color codes of $V(B_{P(n,1)})$ are:

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ component},\ i \leq \frac{n+1}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n+1}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \frac{n+1}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \frac{n+1}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ i \geq 2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ i \geq 3 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component}, \ i \leq \frac{n+1}{2} \\ i-1, & \text{for } 2^{nd} \text{ component}, \ i \leq \frac{n+1}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ component}, \ i > \frac{n+1}{2} \\ n-i+2, & \text{for } 1^{st} \text{ component}, \ i > \frac{n+1}{2} \\ 0, & \text{for } 4^{th} \text{ component}, \ i \text{ even}, \ i \geq 2 \\ & \text{for } 3^{th} \text{ component}, \ i \text{ odd}, \ i \geq 3 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_i) = \begin{cases} i, & \text{for } 3^{th} \text{ component}, \ i \leq \frac{n-1}{2} \\ i+1, & \text{for } 4^{th} \text{ component}, \ i \leq \frac{n-1}{2} \\ n-i, & \text{for } 3^{th} \text{ component}, \ i \geq \frac{n+1}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component}, \ i \geq \frac{n+1}{2} \\ 0, & \text{for } 2^{nd} \text{ component}, \ i \text{ even}, \ i \leq n-1 \\ & \text{for } 1^{st} \text{ component}, \ i \text{ odd}, \ i \leq n-2 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_{n+i}) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, \ i \leq \frac{n-1}{2} \\ i+1, & \text{for } 3^{th} \text{ component}, \ i \leq \frac{n-1}{2} \\ n-i, & \text{for } 4^{th} \text{ component}, \ i \geq \frac{n+1}{2} \\ n-i+1, & \text{for } 3^{th} \text{ component}, \ i \geq \frac{n+1}{2} \\ 0, & \text{for } 1^{st} \text{ component}, \ i \text{ even}, \ i \leq n-1 \\ & \text{for } 2^{nd} \text{ component}, \ i \text{ odd}, \ i \leq n-2 \\ 1, & \text{otherwise.} \end{cases}$$

Since all vertices in $B_{P(n,1)}$ have distinct color codes, then the coloring c with 4 colors is an optimal locating coloring and it proves that $\chi_L(B_{P(n,1)}) \leq 4$.

Case 2, n even. In view of the lower bound from Theorem 2.2 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \to \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n \geq 4$, we describe the locating coloring in the following way:

$$c(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i, \ 2 \leq i \leq n-2 \\ 4, & \text{for odd } i, \ 3 \leq i \leq n-1 \\ 5, & \text{for } i = n. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i, \ i \geq 3 \\ 4, & \text{for even } i, \ i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for odd } i,\ i \le n-3 \\ 2, & \text{for even } i,\ i \le n-2 \\ 3, & \text{for } i = n-1 \\ 4, & \text{for } i = n. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i,\ i \le n-2 \\ 2, & \text{for odd } i,\ i \le n-1 \\ 5, & \text{for } i = n. \end{cases}$$

In fact, our locating coloring of $B_{P(n,1)}$, n even, has been chosen in such a way that the color codes are:

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i \le \frac{n}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \le \frac{n}{2} \\ n-i, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ 2 \le i \le n-2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ 3 \le i \le n-1 \\ 2, & \text{for } 4^{th} \text{ component},\ i = 1 \\ & \text{for } 3^{th} \text{ component},\ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component},\ i \le \frac{n}{2} \\ i-1, & \text{for } 2^{nd} \text{ component},\ i \le \frac{n}{2} \\ n+i, & \text{for } 5^{th} \text{ component},\ i \le \frac{n}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 1^{th} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ 3 \le i \le n-1 \\ & \text{for } 4^{th} \text{ component},\ i \text{ even},\ 2 \le i \le n \\ 2, & \text{for } 3^{th} \text{ component},\ i = 1 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, \ i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component}, \ i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component}, \ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 4^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component}, \ \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component}, \ i \text{ odd}, \ i \leq n-3 \\ & \text{for } 2^{nd} \text{ component}, \ i \text{ even}, \ i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n-1 \\ & \text{for } 2^{nd} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_{n+i}) = \begin{cases} i, & \text{for } 5^{th} \text{ component}, \ i \leq \frac{n}{2} \\ i+1, & \text{for } 4^{th} \text{ component}, \ i \leq \frac{n}{2} \\ i+2 & \text{for } 3^{th} \text{ component}, \ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 3^{th} \text{ component}, \ \frac{n}{2} \leq i \leq n-1 \\ & \text{for } 5^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component}, \ i > \frac{n}{2} \\ 0, & \text{for } 1^{st} \text{ component}, \ i \text{ even}, \ i \leq n-2 \\ & \text{for } 2^{nd} \text{ component}, \ i \text{ odd}, \ i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ and } 3^{th} \text{ components}, \ i = n \\ 1, & \text{otherwise.} \end{cases}$$

Since for n even all vertices of $B_{P(n,1)}$ have distinct color codes then our locating coloring has the required properties and $\chi_L(B_{P(n,1)}) \leq 5$. This concludes the proof. $\square$

Acknowledgement

We are thankful to DRPM Dikti for the Fundamental Grant 2018.

References

[1] Asmiati, Assiyatun, H., and Baskoro, E.T., Locating-chromatic number of amalgamation of stars, ITB J. Sci. 43(1) (2011), pp. 1-8.

[2] Asmiati, Assiyatun, H., Baskoro, E.T., Suprijanto, D., Simanjuntak, R., Uttunggadewa, S., Locating-chromatic number of firecracker graphs, Far East Journal of Mathematical Sciences, 63(1) (2012), pp. 11-23.

[3] Asmiati, Wamiliana, Devriyadi, and Yulianti, L., On Some Petersen graphs having locating-chromatic number four or five, Far East Journal of Mathematical Sciences 102(4) (2017), pp. 769-778.

[4] Baskoro, E.T., Asmiati, Characterizing all trees with locating-chromatic number 3, Electronic Journal of Graph Theory and Applications 1(2) (2013), pp. 109-117.

[5] Behtoei, A., and Omoomi, B., On the locating chromatic number of Kneser graphs, Discrete Applied Mathematics 159 (2011), pp. 2214-2221.

[6] Chartrand, G., Erwin, D., Henning, M.A., Slater, P.J., and Zhang, P., The locating-chromatic number of a graph, Bull. Inst. Combin. Appl. 36 (2002), pp. 89-101.

[7] Chartrand, G., Erwin, D., Henning, M.A., Slater, P.J., Zhang, P. Graph of order n with locating-chromatic number $n-1$, Discrete Math. 269 (2003), pp. 65-79.

[8] Chartrand, G., Salehi, E., and Zhang, P., On the partition dimension of graph, Congr. Numer. 130 (1998), pp. 157-168.

[9] Chartrand, G., Zhang, P., The theory and applications of resolvability in graphs: a survey, Congr. Numer. 160 (2003), pp. 47-68.

[10] Johnson, M.A., Structure-activity maps for visualizing the graph variables arising in drug design, J. Biopharm. Statist. 3 (1993), pp. 203-236.

[11] Saenpholphat, V., Zhang, P., Conditional resolvability: a survey, Internat. J. Math. Sci. 38 (2004), pp. 1997-2017.

[12] Syofyan, D.K., Baskoro, E.T., Assiyatun, H., Trees with Certain Locating-Chromatic Number, J. Math. Fund. Sci. 48(1) (2016), pp. 39-47.

[13] Watkins, M.E., A theorem on Tait colorings with an application to the generalized Petersen graphs, J. Combin. Theory 6 (1969), pp. 152-164.

[14] Wellyanti, D., Baskoro, E.T., Simanjuntak, R., Uttunggadewa, S., On locating chromatic number of complete n-ary tree, ACKE Int. J. Graphs Comb. 10(3) (2013), pp. 309-319.

VII. MINOR REVISION REQUIRED

1 Juli 2018

5327504: Minor Revision Required

Yahoo/Email M...

Dalibor Froncek <ijmms@hindawi.com>

Min, 1 Jul 2018 jam 17.19

Kepada: asmiati308@yahoo.com

Cc: dfroncek@d.umn.edu, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

Following the review of your Research Article titled "On The Locating Chromatic Number Of Some Barbell Graphs," by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti, I recommend that it should be revised taking into account the changes requested by the reviewer(s). Please login to the Manuscript Tracking System to read the submitted review report(s) and submit the revised version of your manuscript not later than Sunday, July 15, 2018.

To submit your revised manuscript, please access "Current Manuscripts" in your account and upload the PDF file of your revised manuscript. You are also asked to submit your replies to the reviewer(s) comments as an additional PDF file.

Best regards,

Dalibor Froncek
dfroncek@d.umn.edu

https://mail.yahoo.com/d/search/keyword=hindawi/messages/AIDNfQIZ9irDWziqqwHBOBdxqGA?.intl=id&.lang=id-ID&.partner=none&.src=fp

PENILAIAN REVIEWER (REVISI MINOR)

Hindawi
International Journal of Mathematics and Mathematical Sciences
Article ID 5327504

Research Article

On the Locating Chromatic Number of Certain Barbell Graphs

Asmiati,[1] I. Ketut Sadha Gunce Yana,[1] and Lyra Yulianti[2]

[1] *Mathematics Department, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No.1 Bandar Lampung, Indonesia*
[2] *Mathematics Department, Faculty of Mathematics and Natural Sciences, Andalas University, Kampus UNAND Limau Manis, Padang 25163, Indonesia*

Correspondence should be addressed to Asmiati; asmiati308@yahoo.com

Received 27 March 2018; Revised 26 June 2018; Accepted 22 July 2018

Academic Editor: Dalibor Froncek

The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In this paper we investigate the locating chromatic number for two families of barbell graphs.

1

1. Introduction

The partition dimension was introduced by Chartrand et al. [1] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [2], the optimization of threat detecting sensors [3], and chemical data classification [4]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [5]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \longrightarrow \{1, 2, \ldots, k\}$, where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$, where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$, where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [5]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Theorem 1 (see [5]). *Let c be a locating coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u, t) = d(v, t)$ for all $t \in V(G) - \{u, v\}$, then $c(u) \neq c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph G.

Corollary 2 (see [5]). *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \geq k + 1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand et al. in [5] have determined all graphs of order n with locating chromatic number n, namely, a complete multipartite graph of n vertices. Moreover, Chartrand et al.

[6] have succeeded in constructing tree on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [7] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [8] obtained the locating chromatic number of the generalized Petersen graph $P(n,1)$ for $n \geq 3$. Baskoro and Asmiati [9] have characterized all trees with locating chromatic number 3. In [10] were characterized all trees of
2 order n with locating chromatic number $n-t$, for any integers n and t, where $n > t+3$ and $2 \leq t < n/2$. Asmiati et al. in [11] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [12] for firecracker graphs. Next, Wellyyanti et al. [13] determined the locating chromatic number for complete n-ary trees.

The generalized Petersen graph $P(n,m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [14]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Theorem 3 (see [6]). *For $n \geq 2$, the locating chromatic number of complete graph K_n is n.*

Theorem 4 (see [8]). *The locating chromatic number of generalized Petersen graph $P(n,1)$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.*

The *barbell graph* is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number for barbell graph $B_{m,n}$ for $m,n \geq 3$, where G and H are complete graphs on m and n vertices, respectively. Secondly, we determine the locating chromatic number of barbell graph $B_{P(n,1)}$ for $n \geq 3$, where G and H are two isomorphic copies of the generalized Petersen graph $P(n,1)$.

2. Results and Discussion

Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.

Theorem 5. *Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n+1$.*

Proof. Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1} \{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains two isomorphic copies of a complete graph K_n, then with respect to Theorem 3 we have $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n+1$.

To show that $n+1$ is an upper bound for the locating chromatic number of barbell graph $B_{n,n}$ it suffices to prove the existence of an optimal locating coloring $c : V(B_{n,n}) \longrightarrow \{1, 2, \ldots, n+1\}$. For $n \geq 3$ we construct the function c in the following way:

$$c(u_i) = i, \quad 1 \leq i \leq n$$
$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases} \tag{1}$$

By using the coloring c, we obtain the color codes of $V(B_{n,n})$ as follows:

$$c_\Pi(u_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 1 \leq i \leq n \\ 2, & \text{for } (n+1)^{th} \text{ component}, \ 1 \leq i \leq n-1 \\ 1, & \text{otherwise}, \end{cases}$$
$$c_\Pi(v_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 2 \leq i \leq n-1 \\ & \text{for } n^{th} \text{ component}, \ i = 1, \text{ and} \\ & \text{for } (n+1)^{th} \text{ component}, \ i = n, \\ 3, & \text{for } 1^{st} \text{ component}, \ 1 \leq i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases} \tag{2}$$

Since all vertices in $V(B_{n,n})$ have distinct color codes, then the coloring c is desired locating coloring. Thus, $\chi_L(B_{n,n}) = n+1$. □

Corollary 6. *For $n, m \geq 3$, and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is*

$$\chi_L(B_{m,n}) = \max\{m, n\}. \tag{3}$$

Next theorem provides the exact value of the locating chromatic number for barbell graph $B_{P(n,1)}$.

Theorem 7. *Let $B_{P(n,1)}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{P(n,1)}$ is*

$$\chi_L(B_{P(n,1)}) = \begin{cases} 4, & \text{for odd } n \\ 5, & \text{for even } n. \end{cases} \tag{4}$$

Proof. Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_i u_{i+1}, u_{n+i} u_{n+i+1}, w_i w_{i+1}, w_{n+i} w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1, u_{2n} u_{n+1}, w_n w_1, w_{2n} w_{n+1}\} \cup \{u_i u_{n+i}, w_i w_{n+i} : 1 \leq i \leq n\} \cup \{u_n w_n\}$.

Let us distinguish two cases.

Case 1 (n odd). According to Theorem 4 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

$$c(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i,\ i \geq 2 \\ 4, & \text{for odd } i,\ i \geq 3. \end{cases}$$
$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i,\ i \geq 3 \\ 4, & \text{for even } i,\ i \geq 2. \end{cases}$$
$$c(w_i) = \begin{cases} 1, & \text{for odd } i,\ i \leq n-2 \\ 2, & \text{for even } i,\ i \leq n-1 \\ 3, & \text{for } i = n. \end{cases}$$
$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i,\ i \leq n-1 \\ 2, & \text{for odd } i,\ i \leq n-2 \\ 4, & \text{for } i = n. \end{cases} \tag{5}$$

For n odd the color codes of $V(B_{P(n,1)})$ are

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ component},\ i \leq \dfrac{n+1}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \leq \dfrac{n+1}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \dfrac{n+1}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \dfrac{n+1}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ i \geq 2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ i \geq 3 \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component},\ i \leq \dfrac{n+1}{2} \\ i-1, & \text{for } 2^{nd} \text{ component},\ i \leq \dfrac{n+1}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ component},\ i > \dfrac{n+1}{2} \\ n-i+2, & \text{for } 1^{st} \text{ component},\ i > \dfrac{n+1}{2} \\ 0, & \text{for } 4^{th} \text{ component},\ i \text{ even},\ i \geq 2 \\ & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ i \geq 3 \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 3^{th} \text{ component},\ i \leq \dfrac{n-1}{2} \\ i+1, & \text{for } 4^{th} \text{ component},\ i \leq \dfrac{n-1}{2} \\ n-i, & \text{for } 3^{th} \text{ component},\ i \geq \dfrac{n+1}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component},\ i \geq \dfrac{n+1}{2} \\ 0, & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i \leq n-1 \\ & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i \leq n-2 \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i, & \text{for } 4^{th} \text{ component},\ i \leq \dfrac{n-1}{2} \\ i+1, & \text{for } 3^{th} \text{ component},\ i \leq \dfrac{n-1}{2} \\ n-i, & \text{for } 4^{th} \text{ component},\ i \geq \dfrac{n+1}{2} \\ n-i+1, & \text{for } 3^{th} \text{ component},\ i \geq \dfrac{n+1}{2} \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i \leq n-1 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i \leq n-2 \\ 1, & \text{otherwise}. \end{cases} \tag{6}$$

Since all vertices in $B_{P(n,1)}$ have distinct color codes, then the coloring c with 4 colors is an optimal locating coloring and it proves that $\chi_L(B_{P(n,1)}) \leq 4$.

Case 2 (n even). In view of the lower bound from Theorem 7 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \longrightarrow \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$

have distinct color codes. For n even, $n \geq 4$, we describe the locating coloring in the following way:

$$c(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i,\ 2 \leq i \leq n-2 \\ 4, & \text{for odd } i,\ 3 \leq i \leq n-1 \\ 5, & \text{for } i = n. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i,\ i \geq 3 \\ 4, & \text{for even } i,\ i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for odd } i,\ i \leq n-3 \\ 2, & \text{for even } i,\ i \leq n-2 \\ 3, & \text{for } i = n-1 \\ 4, & \text{for } i = n. \end{cases} \quad (7)$$

$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i,\ i \leq n-2 \\ 2, & \text{for odd } i,\ i \leq n-1 \\ 5, & \text{for } i = n. \end{cases}$$

In fact, our locating coloring of $B_{P(n,1)}$, n even, has been chosen in such a way that the color codes are

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i \leq \frac{n}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n}{2} \\ n-i, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ 2 \leq i \leq n-2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ 3 \leq i \leq n-1 \\ 2, & \text{for } 4^{th} \text{ component},\ i = 1 \\ & \text{for } 3^{th} \text{ component},\ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n}{2} \\ i-1, & \text{for } 2^{nd} \text{ component},\ i \leq \frac{n}{2} \\ n+i, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 1^{th} \text{ component}, i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ 3 \leq i \leq n-1 \\ & \text{for } 4^{th} \text{ component},\ i \text{ even},\ 2 \leq i \leq n \\ 2, & \text{for } 3^{th} \text{ component},\ i = 1 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component},\ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 4^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component},\ \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i \leq n-3 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component},\ i = n-1 \\ & \text{for } 2^{nd} \text{ component},\ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+1, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+2 & \text{for } 3^{th} \text{ component},\ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 3^{th} \text{ component},\ \frac{n}{2} \leq i \leq n-1 \\ & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i \leq n-2 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ and } 3^{th} \text{ components},\ i = n \\ 1, & \text{otherwise.} \end{cases} \quad (8)$$

Since for n even all vertices of $B_{P(n,1)}$ have distinct color codes then our locating coloring has the required properties and $\chi_L(B_{P(n,1)}) \leq 5$. This concludes the proof. □

Data Availability

3 The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors are thankful to DRPM Dikti for the Fundamental Grant 2018.

References

[1] G. Chartrand, P. Zhang, and E. Salehi, "On the partition dimension of a graph," vol. 130, pp. 157–168.

[2] V. Saenpholphat and P. Zhang, "Conditional resolvability: a survey," *International Journal of Mathematics and Mathematical Sciences*, vol. 38, pp. 1997–2017, 2004.

[3] M. Johnson, "Structure-activity maps for visualizing the graph variables arising in drug design," *Journal of Biopharmaceutical Statistics*, vol. 3, no. 2, pp. 203–236, 1993.

[4] G. Chartrand and P. Zhang, "THE theory and applications of resolvability in graphs. A survey," vol. 160, pp. 47–68.

[5] G. Chartrand, D. Erwin, M. A. Henning, P. J. Slater, and P. Zhang, "The locating-chromatic number of a graph," *Bulletin of the Institute of Combinatorics and Its Applications*, vol. 36, pp. 89–101, 2002.

[6] G. Chartrand, D. Erwin, M. A. Henning, P. J. Slater, and P. Zhang, "Graphs of order n-1," *Discrete Mathematics*, vol. 269, no. 1-3, pp. 65–79, 2003.

[7] A. Behtoei and B. Omoomi, "On the locating chromatic number of Kneser graphs," *Discrete Applied Mathematics: The Journal of Combinatorial Algorithms, Informatics and Computational Sciences*, vol. 159, no. 18, pp. 2214–2221, 2011.

[8] Asmiati, Wamiliana, Devriyadi, and L. Yulianti, "On some petersen graphs having locating chromatic number four or five," *Far East Journal of Mathematical Sciences*, vol. 102, no. 4, pp. 769–778, 2017.

[9] E. T. Baskoro and Asmiati, "Characterizing all trees with locating-chromatic number 3," *Electronic Journal of Graph Theory and Applications. EJGTA*, vol. 1, no. 2, pp. 109–117, 2013.

[10] D. K. Syofyan, E. T. Baskoro, and H. Assiyatun, "Trees with certain locating-chromatic number," *Journal of Mathematical and Fundamental Sciences*, vol. 48, no. 1, pp. 39–47, 2016.

[11] Asmiati, H. Assiyatun, and E. T. Baskoro, "Locating-chromatic number of amalgamation of stars," *ITB Journal of Science*, vol. 43A, no. 1, pp. 1–8, 2011.

[12] Asmiati, H. Assiyatun, E. T. Baskoro, D. Suprijanto, R. Simanjuntak, and S. Uttunggadewa, "The locating-chromatic number of firecracker graphs," *Far East Journal of Mathematical Sciences (FJMS)*, vol. 63, no. 1, pp. 11–23, 2012.

[13] D. Welyyanti, E. T. Baskoro, R. Simanjuntak, and S. Uttunggadewa, "On locating-chromatic number of complete n-ary tree," *AKCE International Journal of Graphs and Combinatorics*, vol. 10, no. 3, pp. 309–315, 2013.

[14] M. E. Watkins, "A theorem on tait colorings with an application to the generalized Petersen graphs," *Journal of Combinatorial Theory*, vol. 6, no. 2, pp. 152–164, 1969.

Composition Comments

1. Please check and confirm the author(s) first and last names and their order which exist in the last page.

2. The highlighted part is grammatically unclear/incorrect. Please rephrase this part for the sake of clarity and correctness.

3. Please carefully check your Data Statement, as it may have been added/rephrased by our editorial staff. If you'd like assistance in making further revisions, please refer to our Research Data policy at https://www.hindawi.com/research.data/#statement.

Author(s) Name(s)

It is very important to confirm the author(s) last and first names in order to be displayed correctly on our website as well as in the indexing databases:

Author 1
Given Names:
Last Name: Asmiati

Author 2
Given Names: I. Ketut
Last Name: Sadha Gunce Yana

Author 3
Given Names: Lyra
Last Name: Yulianti

It is very important for each author to have a linked ORCID (Open Researcher and Contributor ID) account on MTS. ORCID aims to solve the name ambiguity problem in scholarly communications by creating a registry of persistent unique identifiers for individual researchers.

To register a linked ORCID account, please go to the Account Update page (http://mts.hindawi.com/update/) in our Manuscript Tracking System and after you have logged in click on the ORCID link at the top of the page. This link will take you to the ORCID website where you will be able to create an account for yourself. Once you have done so, your new ORCID will be saved in our Manuscript Tracking System automatically.

VIII. REVISED VERSION RECEIVED FOR MINOR REVISION

8 Juli 2018

5327504: Revised Version Received

Yahoo/Email M...

International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hinda

Min, 8 Jul 2018 jam 15.15

Kepada: asmiati308@yahoo.com

Cc: ahmed.khaled@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

The revised version of Research Article 5327504 titled "On The Locating Chromatic Number Of Some Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti has been received. The editor assigned to handle the review process of your manuscript will inform you as soon as a decision is reached.

Thank you for submitting your work to International Journal of Mathematics and Mathematical Sciences.

Best regards,

--
****
Ahmed Khaled
Editorial Office
Hindawi
http://www.hindawi.com
****

https://mail.yahoo.com/d/search/keyword=hindawi/messages/AJsNq_oR_Ba0W0HILgF-0Lbc2nA?.intl=id&.lang=id-ID&.partner=none&.src=fp

Re: 5327504: Revised Version Received

Yahoo/Terkirim

Asmiati Asmiati <asmiati308@yahoo.com>

Jum, 13 Jul 2018 jam 11.56

Kepada: International Journal of Mathematics and Mathematical Sciences

Dear Prof. Ahmed Khaled,

We have sent some files about our response in revised the manuscript.
Thank you very much for your cooperation.

Best regards,
Asmiati

Hindawi
International Journal of Mathematics and Mathematical Sciences
Volume 2018, Article ID 5327504, 5 pages
https://doi.org/10.1155/2018/5327504

Research Article

On the Locating Chromatic Number of Certain Barbell Graphs

Asmiati,[1] **I. Ketut Sadha Gunce Yana,**[1] **and Lyra Yulianti**[2]

[1] *Mathematics Department, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No.1 Bandar Lampung, Indonesia*
[2] *Mathematics Department, Faculty of Mathematics and Natural Sciences, Andalas University, Kampus UNAND Limau Manis, Padang 25163, Indonesia*

Correspondence should be addressed to Asmiati; asmiati308@yahoo.com

Received 27 March 2018; Revised 26 June 2018; Accepted 22 July 2018; Published 5 August 2018

Academic Editor: Dalibor Froncek

The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In this paper we investigate the locating chromatic number for two families of barbell graphs.

1. Introduction

The partition dimension was introduced by Chartrand et al. [1] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [2], the optimization of threat detecting sensors [3], and chemical data classification [4]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [5]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \longrightarrow \{1, 2, \ldots, k\}$, where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$, where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$, where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [5]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Theorem 1 (see [5]). *Let c be a locating coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u, t) = d(v, t)$ for all $t \in V(G) - \{u, v\}$, then $c(u) \neq c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph G.

Corollary 2 (see [5]). *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \geq k + 1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand et al. in [5] have determined all graphs of order n with locating chromatic number n, namely, a complete multipartite graph of n vertices. Moreover, Chartrand et al. [6] have succeeded in constructing tree on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [7] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [8] obtained the locating chromatic number of the generalized Petersen graph $P(n,1)$ for $n \geq 3$. Baskoro and Asmiati [9] have characterized all trees with locating chromatic number 3. In [10] all trees of order n with locating chromatic number $n-1$ were characterized, for any integers n and t, where $n > t+3$ and $2 \leq t < n/2$. Asmiati et al. in [11] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [12] for firecracker graphs. Next, Wellyyanti et al. [13] determined the locating chromatic number for complete n-ary trees.

The generalized Petersen graph $P(n,m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [14]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Theorem 3 (see [6]). *For $n \geq 2$, the locating chromatic number of complete graph K_n is n.*

Theorem 4 (see [8]). *The locating chromatic number of generalized Petersen graph $P(n,1)$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.*

The *barbell graph* is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number for barbell graph $B_{m,n}$ for $m,n \geq 3$, where G and H are complete graphs on m and n vertices, respectively. Secondly, we determine the locating chromatic number of barbell graph $B_{P(n,1)}$ for $n \geq 3$, where G and H are two isomorphic copies of the generalized Petersen graph $P(n,1)$.

2. Results and Discussion

Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.

Theorem 5. *Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n+1$.*

Proof. Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1} \{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains two isomorphic copies of a complete graph K_n, then with respect to Theorem 3 we have $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n+1$.

To show that $n+1$ is an upper bound for the locating chromatic number of barbell graph $B_{n,n}$ it suffices to prove the existence of an optimal locating coloring $c : V(B_{n,n}) \longrightarrow \{1, 2, \ldots, n+1\}$. For $n \geq 3$ we construct the function c in the following way:

$$c(u_i) = i, \quad 1 \leq i \leq n$$
$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases} \tag{1}$$

By using the coloring c, we obtain the color codes of $V(B_{n,n})$ as follows:

$$c_\Pi(u_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 1 \leq i \leq n \\ 2, & \text{for } (n+1)^{th} \text{ component}, \ 1 \leq i \leq n-1 \\ 1, & \text{otherwise,} \end{cases}$$
$$c_\Pi(v_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 2 \leq i \leq n-1 \\ & \text{for } n^{th} \text{ component}, \ i = 1, \text{ and} \\ & \text{for } (n+1)^{th} \text{ component}, \ i = n, \\ 3, & \text{for } 1^{st} \text{ component}, \ 1 \leq i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases} \tag{2}$$

Since all vertices in $V(B_{n,n})$ have distinct color codes, then the coloring c is desired locating coloring. Thus, $\chi_L(B_{n,n}) = n+1$. □

Corollary 6. *For $n, m \geq 3$, and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is*

$$\chi_L(B_{m,n}) = \max\{m, n\}. \tag{3}$$

Next theorem provides the exact value of the locating chromatic number for barbell graph $B_{P(n,1)}$.

Theorem 7. *Let $B_{P(n,1)}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{P(n,1)}$ is*

$$\chi_L\left(B_{P(n,1)}\right)=\begin{cases}4, & \text{for odd } n\\ 5, & \text{for even } n.\end{cases} \tag{4}$$

Proof. Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_i u_{i+1}, u_{n+i}u_{n+i+1}, w_i w_{i+1}, w_{n+i}w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1, u_{2n}u_{n+1}, w_n w_1, w_{2n}w_{n+1}\} \cup \{u_i u_{n+i}, w_i w_{n+i} : 1 \leq i \leq n\} \cup \{u_n w_n\}$.

Let us distinguish two cases.

Case 1 (n odd). According to Theorem 4 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

$$\begin{gathered}
c\left(u_i\right)=\begin{cases}1, & \text{for } i=1\\ 3, & \text{for even } i,\ i\geq 2\\ 4, & \text{for odd } i,\ i\geq 3.\end{cases}\\
c\left(u_{n+i}\right)=\begin{cases}2, & \text{for } i=1\\ 3, & \text{for odd } i,\ i\geq 3\\ 4, & \text{for even } i,\ i\geq 2.\end{cases}\\
c\left(w_i\right)=\begin{cases}1, & \text{for odd } i,\ i\leq n-2\\ 2, & \text{for even } i,\ i\leq n-1\\ 3, & \text{for } i=n.\end{cases}\\
c\left(w_{n+i}\right)=\begin{cases}1, & \text{for even } i,\ i\leq n-1\\ 2, & \text{for odd } i,\ i\leq n-2\\ 4, & \text{for } i=n.\end{cases}
\end{gathered} \tag{5}$$

For n odd the color codes of $V(B_{P(n,1)})$ are

$$\begin{gathered}
c_\Pi\left(u_i\right)\\
=\begin{cases}i, & \text{for } 2^{nd} \text{ component},\ i\leq \dfrac{n+1}{2}\\
i-1, & \text{for } 1^{st} \text{ component},\ i\leq \dfrac{n+1}{2}\\
n-i+1, & \text{for } 1^{st} \text{ component},\ i> \dfrac{n+1}{2}\\
n-i+2, & \text{for } 2^{nd} \text{ component},\ i> \dfrac{n+1}{2}\\
0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ i\geq 2\\
 & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ i\geq 3\\
1, & \text{otherwise.}\end{cases}\\
c_\Pi\left(u_{n+i}\right)\\
=\begin{cases}i, & \text{for } 1^{st} \text{ component},\ i\leq \dfrac{n+1}{2}\\
i-1, & \text{for } 2^{nd} \text{ component},\ i\leq \dfrac{n+1}{2}\\
n-i+1, & \text{for } 2^{nd} \text{ component},\ i> \dfrac{n+1}{2}\\
n-i+2, & \text{for } 1^{st} \text{ component},\ i> \dfrac{n+1}{2}\\
0, & \text{for } 4^{th} \text{ component},\ i \text{ even},\ i\geq 2\\
 & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ i\geq 3\\
1, & \text{otherwise.}\end{cases}\\
c_\Pi\left(w_i\right)\\
=\begin{cases}i, & \text{for } 3^{th} \text{ component},\ i\leq \dfrac{n-1}{2}\\
i+1, & \text{for } 4^{th} \text{ component},\ i\leq \dfrac{n-1}{2}\\
n-i, & \text{for } 3^{th} \text{ component},\ i\geq \dfrac{n+1}{2}\\
n-i+1, & \text{for } 4^{th} \text{ component},\ i\geq \dfrac{n+1}{2}\\
0, & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i\leq n-1\\
 & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i\leq n-2\\
1, & \text{otherwise.}\end{cases}\\
c_\Pi\left(w_{n+i}\right)\\
=\begin{cases}i, & \text{for } 4^{th} \text{ component},\ i\leq \dfrac{n-1}{2}\\
i+1, & \text{for } 3^{th} \text{ component},\ i\leq \dfrac{n-1}{2}\\
n-i, & \text{for } 4^{th} \text{ component},\ i\geq \dfrac{n+1}{2}\\
n-i+1, & \text{for } 3^{th} \text{ component},\ i\geq \dfrac{n+1}{2}\\
0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i\leq n-1\\
 & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i\leq n-2\\
1, & \text{otherwise.}\end{cases}
\end{gathered} \tag{6}$$

Since all vertices in $B_{P(n,1)}$ have distinct color codes, then the coloring c with 4 colors is an optimal locating coloring and it proves that $\chi_L(B_{P(n,1)}) \leq 4$.

Case 2 (n even). In view of the lower bound from Theorem 7 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \longrightarrow \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n \geq 4$, we describe the locating coloring in the following way:

$$c\left(u_i\right)=\begin{cases}1, & \text{for } i=1\\ 3, & \text{for even } i,\ 2\leq i\leq n-2\\ 4, & \text{for odd } i,\ 3\leq i\leq n-1\\ 5, & \text{for } i=n.\end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i,\ i \geq 3 \\ 4, & \text{for even } i,\ i \geq 2. \end{cases}$$
$$c(w_i) = \begin{cases} 1, & \text{for odd } i,\ i \leq n-3 \\ 2, & \text{for even } i,\ i \leq n-2 \\ 3, & \text{for } i = n-1 \\ 4, & \text{for } i = n. \end{cases}$$
$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i,\ i \leq n-2 \\ 2, & \text{for odd } i,\ i \leq n-1 \\ 5, & \text{for } i = n. \end{cases} \tag{7}$$

In fact, our locating coloring of $B_{P(n,1)}$, n even, has been chosen in such a way that the color codes are

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i \leq \frac{n}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n}{2} \\ n-i, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ 2 \leq i \leq n-2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ 3 \leq i \leq n-1 \\ 2, & \text{for } 4^{th} \text{ component},\ i = 1 \\ & \text{for } 3^{th} \text{ component},\ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n}{2} \\ i-1, & \text{for } 2^{nd} \text{ component},\ i \leq \frac{n}{2} \\ n+i, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 1^{th} \text{ component}, i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ 3 \leq i \leq n-1 \\ & \text{for } 4^{th} \text{ component},\ i \text{ even},\ 2 \leq i \leq n \\ 2, & \text{for } 3^{th} \text{ component},\ i = 1 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component},\ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 4^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component},\ \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i \leq n-3 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component},\ i = n-1 \\ & \text{for } 2^{nd} \text{ component},\ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+1, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+2 & \text{for } 3^{th} \text{ component},\ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 3^{th} \text{ component},\ \frac{n}{2} \leq i \leq n-1 \\ & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i \leq n-2 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ and } 3^{th} \text{ components},\ i = n \\ 1, & \text{otherwise.} \end{cases} \tag{8}$$

Since for n even all vertices of $B_{P(n,1)}$ have distinct color codes then our locating coloring has the required properties and $\chi_L(B_{P(n,1)}) \leq 5$. This concludes the proof. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors are thankful to DRPM Dikti for the Fundamental Grant 2018.

References

[1] G. Chartrand, P. Zhang, and E. Salehi, "On the partition dimension of a graph," *Congressus Numerantium*, vol. 130, pp. 157–168, 1998.

[2] V. Saenpholphat and P. Zhang, "Conditional resolvability: a survey," *International Journal of Mathematics and Mathematical Sciences*, vol. 38, pp. 1997–2017, 2004.

[3] M. Johnson, "Structure-activity maps for visualizing the graph variables arising in drug design," *Journal of Biopharmaceutical Statistics*, vol. 3, no. 2, pp. 203–236, 1993.

[4] G. Chartrand and P. Zhang, "THE theory and applications of resolvability in graphs. A survey," vol. 160, pp. 47–68.

[5] G. Chartrand, D. Erwin, M. A. Henning, P. J. Slater, and P. Zhang, "The locating-chromatic number of a graph," *Bulletin of the Institute of Combinatorics and Its Applications*, vol. 36, pp. 89–101, 2002.

[6] G. Chartrand, D. Erwin, M. A. Henning, P. J. Slater, and P. Zhang, "Graphs of order *n*-1," *Discrete Mathematics*, vol. 269, no. 1-3, pp. 65–79, 2003.

[7] A. Behtoei and B. Omoomi, "On the locating chromatic number of Kneser graphs," *Discrete Applied Mathematics: The Journal of Combinatorial Algorithms, Informatics and Computational Sciences*, vol. 159, no. 18, pp. 2214–2221, 2011.

[8] Asmiati, Wamiliana, Devriyadi, and L. Yulianti, "On some petersen graphs having locating chromatic number four or five," *Far East Journal of Mathematical Sciences*, vol. 102, no. 4, pp. 769–778, 2017.

[9] E. T. Baskoro and Asmiati, "Characterizing all trees with locating-chromatic number 3," *Electronic Journal of Graph Theory and Applications. EJGTA*, vol. 1, no. 2, pp. 109–117, 2013.

[10] D. K. Syofyan, E. T. Baskoro, and H. Assiyatun, "Trees with certain locating-chromatic number," *Journal of Mathematical and Fundamental Sciences*, vol. 48, no. 1, pp. 39–47, 2016.

[11] Asmiati, H. Assiyatun, and E. T. Baskoro, "Locating-chromatic number of amalgamation of stars," *ITB Journal of Science*, vol. 43A, no. 1, pp. 1–8, 2011.

[12] Asmiati, H. Assiyatun, E. T. Baskoro, D. Suprijanto, R. Simanjuntak, and S. Uttunggadewa, "The locating-chromatic number of firecracker graphs," *Far East Journal of Mathematical Sciences (FJMS)*, vol. 63, no. 1, pp. 11–23, 2012.

[13] D. Welyyanti, E. T. Baskoro, R. Simanjuntak, and S. Uttunggadewa, "On locating-chromatic number of complete *n*-ary tree," *AKCE International Journal of Graphs and Combinatorics*, vol. 10, no. 3, pp. 309–315, 2013.

[14] M. E. Watkins, "A theorem on tait colorings with an application to the generalized Petersen graphs," *Journal of Combinatorial Theory*, vol. 6, no. 2, pp. 152–164, 1969.

IX MANUSCRIPT HAS BEEN ACCEPTED
22 Juli 2018

Dalibor Froncek <ijmms@hindawi.com> Min, 22 Jul 2018 jam 21.09
Kepada: asmiati308@yahoo.com
Cc: dfroncek@d.umn.edu, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

The review process of Research Article 5327504 titled "On The Locating Chromatic Number Of Certain Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti submitted to International Journal of Mathematics and Mathematical Sciences has been completed. I am pleased to inform you that your manuscript has now been accepted for publication in the journal.

The publication process of your manuscript will be initiated upon the receipt of electronic files. Please log in to the Manuscript Tracking System at the link below using your username and password, and upload the electronic files of your final accepted version within the next 2-3 days.

http://mts.hindawi.com/author/5327504/upload.files/

The electronic files should include the following:

1- Source file of the final accepted manuscript (Word or TeX/LaTeX).
2- PDF file of the final accepted manuscript.
3- Editable figure files (each figure in a separate EPS/PostScript/Word file) if any, taking into consideration that TIFF, JPG, JPEG, BMP formats are not editable.

Thank you again for submitting your manuscript to International Journal of Mathematics and Mathematical Sciences.

Best regards,

Dalibor Froncek
dfroncek@d.umn.edu

https://mail.yahoo.com/d/search/keyword=ijmms/messages/ALtFcsZ25OJrW2gvbAe0qIrKNww?.intl=id&.lang=id-ID&.partner=none&.src=fp

International Journal of Mathematics and Mathematical Sciences <nada.nemr@hindawi.com>   Min, 22 Jul 2018 jam 21.09

Kepada: asmiati308@yahoo.com

Cc: nada.nemr@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

As an open access journal, International Journal of Mathematics and Mathematical Sciences levies Article Processing Charges that amount to USD 750. The total charges for your manuscript (5327504), before any taxes that may apply, are USD 750.

You can access the invoice for your article and make payments through the following URL:

https://invoice.hindawi.com/4AC9FB69-D63A-4993-9411-0EB14796ED5C/

You will need to login to your account on the Manuscript Tracking System to access the link. After entering your billing address information, you will be able to pay by credit card or bank transfer.

If paying by bank transfer, please refer to invoice number 5327504 and return a scanned copy of the payment authorisation by email to facilitate our tracking of your payment.

The invoice is payable upon receipt and your prompt action would be appreciated.

If the payment will be made by an alternative source (e.g. your institution), you can provide contact details for the person who will be arranging payment via the invoice page.

If I can be of any assistance with the payment process, please let me know.

Kind regards,

****
Nada Nemr
Accounts Receivable Specialist
Hindawi
http://www.hindawi.com
****

X. ACKNOWLEDGING RECEIPT OF ELECTRONIC FILES
24 Juli 2018

5327504: Acknowledging Receipt of Electronic Files

Yahoo/Email M...

International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hinda

Sel, 24 Jul 2018 jam 17.44

Kepada: asmiati308@yahoo.com

Cc: ahmed.khaled@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

This is to confirm the receipt of the electronic files of Research Article 5327504 titled "On The Locating Chromatic Number Of Certain Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti. We will check all the uploaded files and contact you if anything else is needed.

Thank you for your cooperation.

Best regards,

--
****
Ahmed Khaled
Editorial Office
Hindawi
http://www.hindawi.com
****

https://mail.yahoo.com/d/search/keyword=hindawi/messages/AJVpHettxBwTW1cC_gXYGIo9fV8?.intl=id&.lang=id-ID&.partner=none&.src=fp

XI. GALLEY PROOFS

26 Juli 2018

5327504: Galley Proofs

Yahoo/Email M...

International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@h

Kam, 26 Jul 2018 jam 13.15

Kepada: asmiati308@yahoo.com

Cc: lyrayulianti@gmail.com, sikesaguya412@gmail.com

Dear Dr. Asmiati,

I am pleased to let you know that the first set of galley proofs of your Research Article 5327504 titled "On The Locating Chromatic Number Of Certain Barbell Graphs," is ready. You can apply your corrections directly to the manuscript with the Online Proofing System (OPS).

Using the OPS, you can quickly and easily make corrections directly to your galley proofs and submit these corrections with a single click.

https://ops.hindawi.com/author/5327504/

To expedite the publication of your manuscript, please send us your corrected galley proofs within three days.

Best regards,

--
****
Ahmed Khaled
Editorial Office
Hindawi
https://www.hindawi.com
****

https://mail.yahoo.com/d/search/keyword=hindawi/messages/ALKKkNc9uu2rW1lm_gy16M2TJrg?.intl=id&.lang=id-ID&.partner=none&.src=fp

XII. ARTICLE PROCESSING CHARGES

28 Juli 2018

5327504: Article Processing Charges

Yahoo/Email M...

Nada Nemr <nada.nemr@hindawi.com>

Sab, 28 Jul 2018 jam 00.50

Kepada: asmiati308@yahoo.com

Cc: nada.nemr@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

This is a reminder concerning the unpaid invoice for manuscript 5327504 in the amount of USD 750.

Please use the following URL to access payment details for this invoice.

https://invoice.hindawi.com/4ac9fb69-d63a-4993-9411-0eb14796ed5c

If I can be of any assistance with the payment process, please let me know.

Kind regards,

****

Nada Nemr
Accounts Receivable Specialist
Hindawi
http://www.hindawi.com
****

https://mail.yahoo.com/d/search/keyword=hindawi/messages/ABUkT8xcq1FmW1tbWQbxWGnxQsQ?.intl=id&.lang=id-ID&.partner=none&.src=fp

Kembali Arsipkan Pindahkan Hapus Spam

Re: 5327504: Article Processing Charges

Yahoo/Terkirim

Asmiati Asmiati <asmiati308@yahoo.com>

Kepada: Nada Nemr

Sel, 31 Jul 2018 jam 15.13

Dear Nada Nemr,

Thank you very much for your email.

Best regards,
Asmiati

BUKTI PEMBAYARAN PAPER

H 4G 0K/s 19:56 4G2 75%

billing_08201...

Lembar Penagihan Kartu Kredit BNI

Yth. Bapak/Ibu
ASMIATI
PURNAWIRAWAN III GG PERINTIS
NO 53 RT 005

GUNUNG TERANG LANGKAPURA
BANDAR LAMPUNG
35152

Tanggal Cetak
19-08-2018

VISA mastercard

Halaman 01 dari 01

Tanggal Transaksi	Tanggal Pembukuan	Rincian Transaksi Anda	Jumlah (Rp.)
		TAGIHAN BULAN LALU	0
426-4000-1274-3818		ASMIATI	
1-08-2018	02-08-2018	HINDAWI LIMITED LONDON GB 750.00 (USD1=IDR14873.94)	11.155.457
		TOTAL TAGIHAN BULAN INI	11.155.457

Informasi Kredit	
BATAS KREDIT	15.000.000
BATAS PENARIKAN TUNAI	7.500.000
SISA KREDIT	3.844.500
SISA PENARIKAN TUNAI	3.844.500

Ringkasan Belanja dan Pembayaran	
TAGIHAN BULAN LALU	0
PEMBAYARAN	0
PEMBELANJAAN	11.155.457
PENARIKAN TUNAI	0
BIAYA ADM & BUNGA	0
TAGIHAN BULAN INI	11.155.457
PEMBAYARAN MINIMUM	1.115.600
TANGGAL JATUH TEMPO	08-09-2018
KOLEKTIBILITAS KREDIT	1

Informasi BNI Reward Points		
JUMLAH POIN BULAN LALU	:	0
JUMLAH POIN BULAN INI	:	4462
JUMLAH POIN DITUKARKAN	:	0 (-)
JUMLAH POIN TERSEDIA	:	4462

UNTUK KENYAMANAN DAN KEAMANAN TRANSAKSI ANDA, BNI AKAN MENERAPKAN SMS NOTIFIKASI KARTU KREDIT DENGAN BIAYA BERLANGGANAN PADA BULAN AGUSTUS 2018

Info dan Promo Bulan Ini

Untuk kenyamanan dan keamanan saat bertransaksi online, pastikan no handphone Anda dan kartu tambahan Anda telah terdaftar di BNI.

Gunakan PIN di setiap transaksi kartu kredit Anda. Dapatkan PIN Anda, kirim SMS ke 3346 dg format: RPIN (spasi) 16 Digit No kartu (spasi) tgl lahir (ddmmyyyy)

Daftarkan Perlindungan PerisaiPlus di KK BNI Anda untuk mendapatkan manfaat perlindungan tagihan hanya dgn membayar Premi 0.69% dari saldo tagihan setiap bulan

Kartu Kredit BNI

5327504: Galley Proof Corrections

Yahoo/Email M...

International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hinda

Sel, 31 Jul 2018 jam 14.16

Kepada: asmiati308@yahoo.com

Cc: lyrayulianti@gmail.com, sikesaguya412@gmail.com

Dear Dr. Asmiati,

This is to confirm the receipt of the first galley proof corrections of Research Article 5327504 titled "On The Locating Chromatic Number Of Certain Barbell Graphs,".

We will address your comments and send you another set of galley proofs.

Thank you for your cooperation.

Best regards,

--
****
Ahmed Khaled
Editorial Office
Hindawi
https://www.hindawi.com
****

← Kembali | Arsipkan | Pindahkan | Hapus | Spam | •••

Bls: 5327504: Galley Proofs

Yahoo/Terkirim

Asmiati Asmiati <asmiati308@yahoo.com>

Kepada: ahmed.khaled@hindawi.com

Kam, 2 Agu 2018 jam 16.56

Dear Ahmed Khaled,

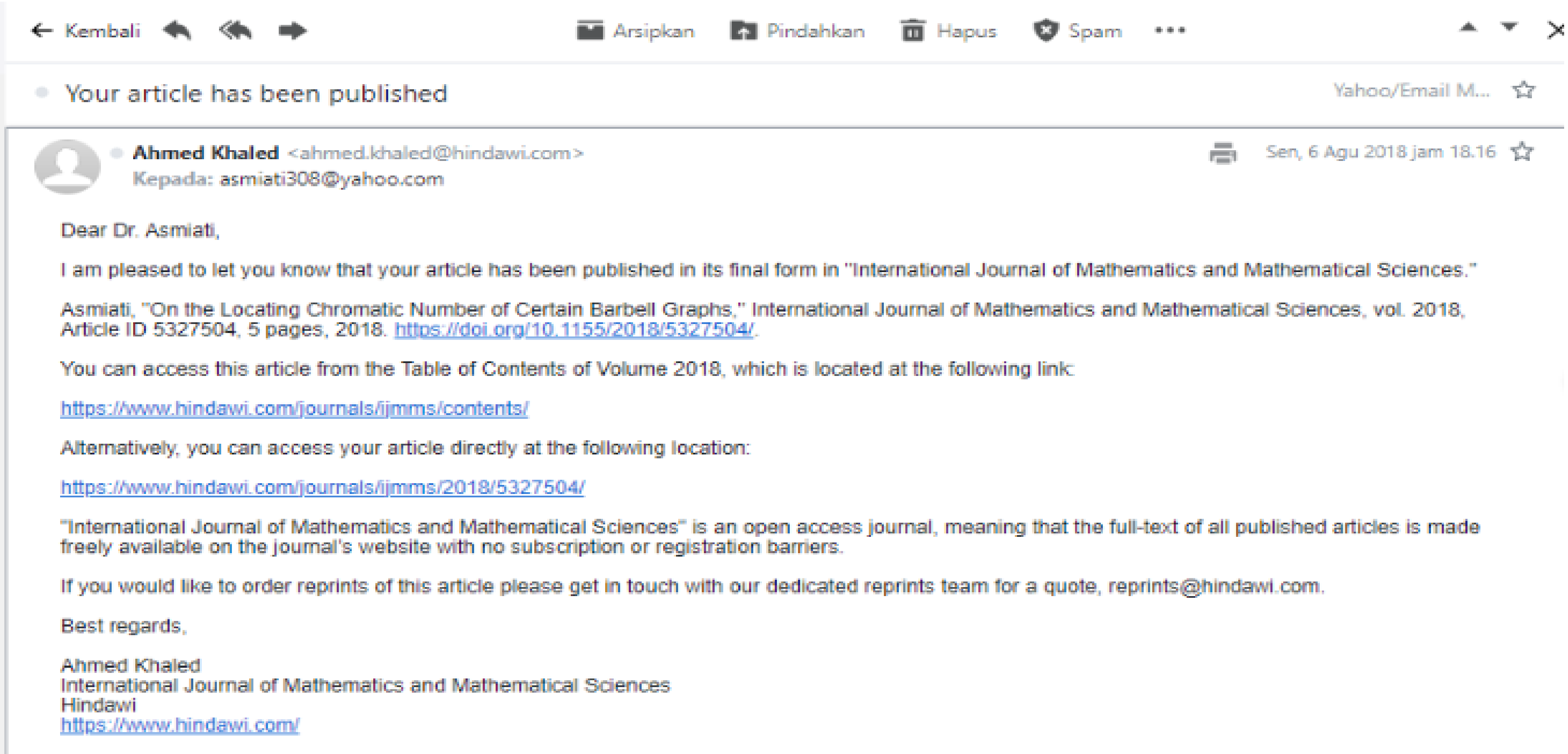
We have checked our second round manuscript, and we believe that we have corrected all the mistakes in the paper.

Best regards,
Asmiati

Dikirim dari Yahoo Mail di Android

XIII. PUBLISHED

6 Agustus 2018



← Kembali Arsipkan Pindahkan Hapus Spam •••

Your article has been published Yahoo/Email M...

Ahmed Khaled <ahmed.khaled@hindawi.com> Sen, 6 Agu 2018 jam 18.16
Kepada: asmiati308@yahoo.com

Dear Dr. Asmiati,

I am pleased to let you know that your article has been published in its final form in "International Journal of Mathematics and Mathematical Sciences."

Asmiati, "On the Locating Chromatic Number of Certain Barbell Graphs," International Journal of Mathematics and Mathematical Sciences, vol. 2018, Article ID 5327504, 5 pages, 2018. https://doi.org/10.1155/2018/5327504/.

You can access this article from the Table of Contents of Volume 2018, which is located at the following link:

https://www.hindawi.com/journals/ijmms/contents/

Alternatively, you can access your article directly at the following location:

https://www.hindawi.com/journals/ijmms/2018/5327504/

"International Journal of Mathematics and Mathematical Sciences" is an open access journal, meaning that the full-text of all published articles is made freely available on the journal's website with no subscription or registration barriers.

If you would like to order reprints of this article please get in touch with our dedicated reprints team for a quote, reprints@hindawi.com.

Best regards,

Ahmed Khaled
International Journal of Mathematics and Mathematical Sciences
Hindawi
https://www.hindawi.com/

https://mail.yahoo.com/d/search/keyword=hindawi/messages/APjCo3J8eMcsW2guHgo5MKxJW9c?.intl=id&.lang=id-ID&.partner=none&.src=fp

Kembali Arsipkan Pindahkan Hapus Spam

Bls: Your article has been published

Yahoo/Terkirim

Asmiati Asmiati <asmiati308@yahoo.com>

Kepada: ahmed.khaled@hindawi.com

Sen, 6 Agu 2018 jam 18.22

Dear Prof. Ahmed Khaled,

Thank you very much for your information.

Best regards,
Asmiati

Dikirim dari Yahoo Mail di Android

Pada Sen, 6 Agt 2018 pada 15:16, Ahmed Khaled
<ahmed.khaled@hindawi.com> menulis:

Hindawi
International Journal of Mathematics and Mathematical Sciences
Volume 2018, Article ID 5327504, 5 pages
https://doi.org/10.1155/2018/5327504

Research Article

On the Locating Chromatic Number of Certain Barbell Graphs

Asmiati,[1] **I. Ketut Sadha Gunce Yana,**[1] **and Lyra Yulianti**[2]

[1] *Mathematics Department, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No.1 Bandar Lampung, Indonesia*
[2] *Mathematics Department, Faculty of Mathematics and Natural Sciences, Andalas University, Kampus UNAND Limau Manis, Padang 25163, Indonesia*

Correspondence should be addressed to Asmiati; asmiati308@yahoo.com

Received 27 March 2018; Revised 26 June 2018; Accepted 22 July 2018; Published 5 August 2018

Academic Editor: Dalibor Froncek

The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In this paper we investigate the locating chromatic number for two families of barbell graphs.

1. Introduction

The partition dimension was introduced by Chartrand et al. [1] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [2], the optimization of threat detecting sensors [3], and chemical data classification [4]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [5]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \longrightarrow \{1, 2, \ldots, k\}$, where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$, where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$, where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [5]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Theorem 1 (see [5]). *Let c be a locating coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u, t) = d(v, t)$ for all $t \in V(G) - \{u, v\}$, then $c(u) \neq c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph G.

Corollary 2 (see [5]). *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \geq k + 1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand et al. in [5] have determined all graphs of order n with locating chromatic number n, namely, a complete multipartite graph of n vertices. Moreover, Chartrand et al. [6] have succeeded in constructing tree on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [7] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [8] obtained the locating chromatic number of the generalized Petersen graph $P(n,1)$ for $n \geq 3$. Baskoro and Asmiati [9] have characterized all trees with locating chromatic number 3. In [10] all trees of order n with locating chromatic number $n-1$ were characterized, for any integers n and t, where $n > t+3$ and $2 \leq t < n/2$. Asmiati et al. in [11] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [12] for firecracker graphs. Next, Wellyyanti et al. [13] determined the locating chromatic number for complete n-ary trees.

The generalized Petersen graph $P(n,m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [14]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Theorem 3 (see [6]). *For $n \geq 2$, the locating chromatic number of complete graph K_n is n.*

Theorem 4 (see [8]). *The locating chromatic number of generalized Petersen graph $P(n,1)$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.*

The *barbell graph* is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number for barbell graph $B_{m,n}$ for $m,n \geq 3$, where G and H are complete graphs on m and n vertices, respectively. Secondly, we determine the locating chromatic number of barbell graph $B_{P(n,1)}$ for $n \geq 3$, where G and H are two isomorphic copies of the generalized Petersen graph $P(n,1)$.

2. Results and Discussion

Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.

Theorem 5. *Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n+1$.*

Proof. Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1}\{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1}\{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains two isomorphic copies of a complete graph K_n, then with respect to Theorem 3 we have $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n+1$.

To show that $n+1$ is an upper bound for the locating chromatic number of barbell graph $B_{n,n}$ it suffices to prove the existence of an optimal locating coloring $c : V(B_{n,n}) \longrightarrow \{1,2,\ldots,n+1\}$. For $n \geq 3$ we construct the function c in the following way:

$$c(u_i) = i, \quad 1 \leq i \leq n$$
$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases} \tag{1}$$

By using the coloring c, we obtain the color codes of $V(B_{n,n})$ as follows:

$$c_\Pi(u_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \; 1 \leq i \leq n \\ 2, & \text{for } (n+1)^{th} \text{ component}, \; 1 \leq i \leq n-1 \\ 1, & \text{otherwise,} \end{cases}$$
$$c_\Pi(v_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \; 2 \leq i \leq n-1 \\ & \text{for } n^{th} \text{ component}, \; i = 1, \text{ and} \\ & \text{for } (n+1)^{th} \text{ component}, \; i = n, \\ 3, & \text{for } 1^{st} \text{ component}, \; 1 \leq i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ component}, \; i = n \\ 1, & \text{otherwise.} \end{cases} \tag{2}$$

Since all vertices in $V(B_{n,n})$ have distinct color codes, then the coloring c is desired locating coloring. Thus, $\chi_L(B_{n,n}) = n+1$. □

Corollary 6. *For $n,m \geq 3$, and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is*

$$\chi_L(B_{m,n}) = \max\{m,n\}. \tag{3}$$

Next theorem provides the exact value of the locating chromatic number for barbell graph $B_{P(n,1)}$.

Theorem 7. *Let $B_{P(n,1)}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{P(n,1)}$ is*

$$\chi_L\left(B_{P(n,1)}\right) = \begin{cases} 4, & \textit{for odd } n \\ 5, & \textit{for even } n. \end{cases} \tag{4}$$

Proof. Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_i u_{i+1}, u_{n+i}u_{n+i+1}, w_i w_{i+1}, w_{n+i}w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1, u_{2n}u_{n+1}, w_n w_1, w_{2n}w_{n+1}\} \cup \{u_i u_{n+i}, w_i w_{n+i} : 1 \leq i \leq n\} \cup \{u_n w_n\}$.

Let us distinguish two cases.

Case 1 (n odd). According to Theorem 4 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

$$c\left(u_i\right) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i,\ i \geq 2 \\ 4, & \text{for odd } i,\ i \geq 3. \end{cases}$$
$$c\left(u_{n+i}\right) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i,\ i \geq 3 \\ 4, & \text{for even } i,\ i \geq 2. \end{cases}$$
$$c\left(w_i\right) = \begin{cases} 1, & \text{for odd } i,\ i \leq n-2 \\ 2, & \text{for even } i,\ i \leq n-1 \\ 3, & \text{for } i = n. \end{cases} \tag{5}$$
$$c\left(w_{n+i}\right) = \begin{cases} 1, & \text{for even } i,\ i \leq n-1 \\ 2, & \text{for odd } i,\ i \leq n-2 \\ 4, & \text{for } i = n. \end{cases}$$

For n odd the color codes of $V(B_{P(n,1)})$ are

$$c_\Pi\left(u_i\right) = \begin{cases} i, & \text{for } 2^{nd} \text{ component},\ i \leq \dfrac{n+1}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \leq \dfrac{n+1}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \dfrac{n+1}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \dfrac{n+1}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ i \geq 2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ i \geq 3 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi\left(u_{n+i}\right) = \begin{cases} i, & \text{for } 1^{st} \text{ component},\ i \leq \dfrac{n+1}{2} \\ i-1, & \text{for } 2^{nd} \text{ component},\ i \leq \dfrac{n+1}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ component},\ i > \dfrac{n+1}{2} \\ n-i+2, & \text{for } 1^{st} \text{ component},\ i > \dfrac{n+1}{2} \\ 0, & \text{for } 4^{th} \text{ component},\ i \text{ even},\ i \geq 2 \\ & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ i \geq 3 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi\left(w_i\right) = \begin{cases} i, & \text{for } 3^{th} \text{ component},\ i \leq \dfrac{n-1}{2} \\ i+1, & \text{for } 4^{th} \text{ component},\ i \leq \dfrac{n-1}{2} \\ n-i, & \text{for } 3^{th} \text{ component},\ i \geq \dfrac{n+1}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component},\ i \geq \dfrac{n+1}{2} \\ 0, & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i \leq n-1 \\ & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i \leq n-2 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi\left(w_{n+i}\right) = \begin{cases} i, & \text{for } 4^{th} \text{ component},\ i \leq \dfrac{n-1}{2} \\ i+1, & \text{for } 3^{th} \text{ component},\ i \leq \dfrac{n-1}{2} \\ n-i, & \text{for } 4^{th} \text{ component},\ i \geq \dfrac{n+1}{2} \\ n-i+1, & \text{for } 3^{th} \text{ component},\ i \geq \dfrac{n+1}{2} \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i \leq n-1 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i \leq n-2 \\ 1, & \text{otherwise.} \end{cases} \tag{6}$$

Since all vertices in $B_{P(n,1)}$ have distinct color codes, then the coloring c with 4 colors is an optimal locating coloring and it proves that $\chi_L(B_{P(n,1)}) \leq 4$.

Case 2 (n even). In view of the lower bound from Theorem 7 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \longrightarrow \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n \geq 4$, we describe the locating coloring in the following way:

$$c\left(u_i\right) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i,\ 2 \leq i \leq n-2 \\ 4, & \text{for odd } i,\ 3 \leq i \leq n-1 \\ 5, & \text{for } i = n. \end{cases}$$

$$c\left(u_{n+i}\right)=\begin{cases}2, & \text{for } i=1\\ 3, & \text{for odd } i,\ i\geq 3\\ 4, & \text{for even } i,\ i\geq 2.\end{cases}$$
$$c\left(w_{i}\right)=\begin{cases}1, & \text{for odd } i,\ i\leq n-3\\ 2, & \text{for even } i,\ i\leq n-2\\ 3, & \text{for } i=n-1\\ 4, & \text{for } i=n.\end{cases}$$
$$c\left(w_{n+i}\right)=\begin{cases}1, & \text{for even } i,\ i\leq n-2\\ 2, & \text{for odd } i,\ i\leq n-1\\ 5, & \text{for } i=n.\end{cases} \tag{7}$$

In fact, our locating coloring of $B_{P(n,1)}$, n even, has been chosen in such a way that the color codes are

$$c_{\Pi}\left(u_{i}\right)=\begin{cases}i, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i\leq \frac{n}{2}\\ i-1, & \text{for } 1^{st} \text{ component},\ i\leq \frac{n}{2}\\ n-i, & \text{for } 5^{th} \text{ component},\ i>\frac{n}{2}\\ n-i+1, & \text{for } 1^{st} \text{ component},\ i>\frac{n}{2}\\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i>\frac{n}{2}\\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ 2\leq i\leq n-2\\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ 3\leq i\leq n-1\\ 2, & \text{for } 4^{th} \text{ component},\ i=1\\ & \text{for } 3^{th} \text{ component},\ i=n\\ 1, & \text{otherwise.}\end{cases}$$

$$c_{\Pi}\left(u_{n+i}\right)=\begin{cases}i, & \text{for } 1^{st} \text{ component},\ i\leq \frac{n}{2}\\ i-1, & \text{for } 2^{nd} \text{ component},\ i\leq \frac{n}{2}\\ n+i, & \text{for } 5^{th} \text{ component},\ i\leq \frac{n}{2}\\ n-i+1, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i>\frac{n}{2}\\ n-i+2, & \text{for } 1^{th} \text{ component}, i>\frac{n}{2}\\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ 3\leq i\leq n-1\\ & \text{for } 4^{th} \text{ component},\ i \text{ even},\ 2\leq i\leq n\\ 2, & \text{for } 3^{th} \text{ component},\ i=1\\ 1, & \text{otherwise.}\end{cases}$$

$$c_{\Pi}\left(w_{i}\right)=\begin{cases}i, & \text{for } 4^{th} \text{ component},\ i\leq \frac{n}{2}\\ i+1, & \text{for } 5^{th} \text{ component},\ i\leq \frac{n}{2}\\ & \text{for } 3^{th} \text{ component},\ i\leq \frac{n}{2}-1\\ n-i, & \text{for } 4^{th} \text{ component},\ i>\frac{n}{2}\\ n-i+1, & \text{for } 5^{th} \text{ component},\ i>\frac{n}{2}\\ n-i-1, & \text{for } 3^{th} \text{ component},\ \frac{n}{2}\leq i\leq n-1\\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i\leq n-3\\ & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i\leq n-2\\ 2, & \text{for } 1^{st} \text{ component},\ i=n-1\\ & \text{for } 2^{nd} \text{ component},\ i=n\\ 1, & \text{otherwise.}\end{cases}$$

$$c_{\Pi}\left(w_{n+i}\right)=\begin{cases}i, & \text{for } 5^{th} \text{ component},\ i\leq \frac{n}{2}\\ i+1, & \text{for } 4^{th} \text{ component},\ i\leq \frac{n}{2}\\ i+2 & \text{for } 3^{th} \text{ component},\ i\leq \frac{n}{2}-1\\ n-i, & \text{for } 3^{th} \text{ component},\ \frac{n}{2}\leq i\leq n-1\\ & \text{for } 5^{th} \text{ component},\ i>\frac{n}{2}\\ n-i+1, & \text{for } 4^{th} \text{ component},\ i>\frac{n}{2}\\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i\leq n-2\\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i\leq n-1\\ 2, & \text{for } 1^{st} \text{ and } 3^{th} \text{ components},\ i=n\\ 1, & \text{otherwise.}\end{cases} \tag{8}$$

Since for n even all vertices of $B_{P(n,1)}$ have distinct color codes then our locating coloring has the required properties and $\chi_L(B_{P(n,1)})\leq 5$. This concludes the proof. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors are thankful to DRPM Dikti for the Fundamental Grant 2018.

References

[1] G. Chartrand, P. Zhang, and E. Salehi, "On the partition dimension of a graph," *Congressus Numerantium*, vol. 130, pp. 157–168, 1998.

[2] V. Saenpholphat and P. Zhang, "Conditional resolvability: a survey," *International Journal of Mathematics and Mathematical Sciences*, vol. 38, pp. 1997–2017, 2004.

[3] M. Johnson, "Structure-activity maps for visualizing the graph variables arising in drug design," *Journal of Biopharmaceutical Statistics*, vol. 3, no. 2, pp. 203–236, 1993.

[4] G. Chartrand and P. Zhang, "THE theory and applications of resolvability in graphs. A survey," vol. 160, pp. 47–68.

[5] G. Chartrand, D. Erwin, M. A. Henning, P. J. Slater, and P. Zhang, "The locating-chromatic number of a graph," *Bulletin of the Institute of Combinatorics and Its Applications*, vol. 36, pp. 89–101, 2002.

[6] G. Chartrand, D. Erwin, M. A. Henning, P. J. Slater, and P. Zhang, "Graphs of order n-1," *Discrete Mathematics*, vol. 269, no. 1-3, pp. 65–79, 2003.

[7] A. Behtoei and B. Omoomi, "On the locating chromatic number of Kneser graphs," *Discrete Applied Mathematics: The Journal of Combinatorial Algorithms, Informatics and Computational Sciences*, vol. 159, no. 18, pp. 2214–2221, 2011.

[8] Asmiati, Wamiliana, Devriyadi, and L. Yulianti, "On some petersen graphs having locating chromatic number four or five," *Far East Journal of Mathematical Sciences*, vol. 102, no. 4, pp. 769–778, 2017.

[9] E. T. Baskoro and Asmiati, "Characterizing all trees with locating-chromatic number 3," *Electronic Journal of Graph Theory and Applications. EJGTA*, vol. 1, no. 2, pp. 109–117, 2013.

[10] D. K. Syofyan, E. T. Baskoro, and H. Assiyatun, "Trees with certain locating-chromatic number," *Journal of Mathematical and Fundamental Sciences*, vol. 48, no. 1, pp. 39–47, 2016.

[11] Asmiati, H. Assiyatun, and E. T. Baskoro, "Locating-chromatic number of amalgamation of stars," *ITB Journal of Science*, vol. 43A, no. 1, pp. 1–8, 2011.

[12] Asmiati, H. Assiyatun, E. T. Baskoro, D. Suprijanto, R. Simanjuntak, and S. Uttunggadewa, "The locating-chromatic number of firecracker graphs," *Far East Journal of Mathematical Sciences (FJMS)*, vol. 63, no. 1, pp. 11–23, 2012.

[13] D. Welyyanti, E. T. Baskoro, R. Simanjuntak, and S. Uttunggadewa, "On locating-chromatic number of complete n-ary tree," *AKCE International Journal of Graphs and Combinatorics*, vol. 10, no. 3, pp. 309–315, 2013.

[14] M. E. Watkins, "A theorem on tait colorings with an application to the generalized Petersen graphs," *Journal of Combinatorial Theory*, vol. 6, no. 2, pp. 152–164, 1969.