International Journal of Latest Engineering Research and Applications (IJLERA) ISSN: 2455-7137

Volume – 08, Issue – 01, January 2023, PP – 46-53

A High-Order Fuzzy Time Series Forecasting Method Based on an Intersection Operation for Forecasting USD Foreign Exchange Rate

Herawati, N.[1], Chandra, D.[2], Rizki, D.N.Y.[1], Nisa, K.[1]

[1](Department of Mathematics, University of Lampung, Bandar Lampung, Indonesia)

[2](Department of Management, Prasetiya Mandiri Maritime Institute, Bandar Lampung, Indonesia)

Abstract: Fuzzy Time Series is a forecasting method using the concept of fuzzy sets as the basis for forecasting modeling. In an approach based on fuzzy set theory, uncertainty modeling is done by utilizing membership values. Fuzzy C-Means is a fuzzy cluster method to obtain membership values from the input model by using a membership function. Artificial Neural Network is a fuzzy logic-based method that can model uncertainty using membership values. To overcome the many operations of the matrix R with a large number in the first-order model method, it can be overcome by high-order fuzzy time series. This study aims to forecast the USD exchange rate using the FCM-based high-order fuzzy time series forecasting method on the fuzzification step and ANN with multiple inputs and multiple outputs to identify fuzzy relationships. The results showed that MAPE (Mean Absolute Percentage Error) and MdAPE (Median Absolute Percentage Error) were small. This indicates that the high-order fuzzy time series is able to predict the USD data foreign exchange rate very well.

Keywords: Artificial Neural Network, Fuzzy C-Means, High Order Fuzzy Time Series, Intersection, MAPE, MdAPE

I. INTRODUCTION

Forecasting is a technique for estimating a value in the future based on past data. The forecasting process is very important on time series data in the decision-making process and making predictions for the future, which can be achieved based on accurate analysis. The classic time series approach with the stochastic model requires several assumptions such as the number of observations, normal distribution, and linearity. Thus, this approach can lead to misleading forecasting results when this assumption is not met. Therefore, a non-stochastic approach has been proposed as an alternative to the stochastic time series forecasting model. In recent years, the fuzzy time series approach has prominent among non-stochastic time series forecasting methods. Fuzzy-based methods such as fuzzy time series forecasting models do not require the assumptions required by the stochastic model.

Fuzzy Time Series (FTS) was first introduced by [1]. This method uses the concept of fuzzy sets as the basis for forecasting modeling [1, 2, 3]. The fuzzy time series forecasting model consists of three stages, that is fuzzification, determination of fuzzy relationships, and defuzzification. In an approach based on fuzzy set theory, uncertainty modeling is done by utilizing membership values. The membership value is obtained from the input model by using a membership function or empirical methods such as Fuzzy C-Means (FCM). Fuzzy logic-based methods can model uncertainty using membership values, so it has important advantages compared to other techniques such as Artificial Neural Networks (ANN) [4, 5]. Thus, fuzzy logic-based approaches can yield better results than those obtained with other soft computational methods such as artificial neural networks in terms of handling uncertainty due to the nature of this approach. Research using high-level fuzzy methods based on artificial neural networks has been carried out by several researchers [6-9]. In this study, the exchange rate of Indonesian foreign currencies against the USD will be predicted using high-order fuzzy time series forecasting. High-level fuzzy time series forecasting is proposed using FCM in the fuzzification step and ANN with multiple inputs and multiple outputs to identify fuzzy relationships. The intersection operation on membership values is applied to each observation that has a time-delay fuzzy time series to eliminate problems related to the number of ANN inputs, which may occur in high-order models [10]. The accuracy of the forecasting method can be seen from the MAPE (Mean Absolute Percentage Error) and MdAPE (Median Absolute Percentage Error) values Percentage Error) values.

II. HIGH- ORDER FUZZY TIME SERIES

An algorithm based on a first-order model for forecasting time invariant fuzzy time series $F(t)$was proposed by [1]. In this algorithm, the fuzzy relationship matrix $R\ (t, t-1) = R$is obtained through many matrix operations. Fuzzy forecasts are also obtained using the max-min composition, given by:

$$F(t) = F(t-1)\,{}^{\circ}R \tag{1}$$

The dimension of the matrix R is equal to the number of fuzzy sets. The number of fuzzy sets is also equal to the number of sub-intervals that make up the universe of discourse. If more fuzzy sets are used, more matrix operations are needed to obtain an R matrix. This will increase the computational cost significantly when the Song and Chissom method is performed using a large number of fuzzy sets.

Definition. Let a time-invariant fuzzy time series be $F(t)$. If $F(t)$is defined as a lagged fuzzy time series $F(t-1), F(t-2)$, …, and $F(t-n)$, then the fuzzy logical relationship is

$$F(t-n), \ldots, F(t-2), F(t-1) \rightarrow F(t) \tag{2}$$

For a high-order model, one can use the intersection operation for fuzzy forecasting, as follows:

$$F(t) = (F(t-n) \cap \ldots \cap F(t-2)\, F(t-1))^{\circ}R \tag{3}$$

which is called the *n* th-order fuzzy time series forecasting model.

The fuzzy time series forecasting model consists of three stages, that is fuzzification, defining fuzzy relationships using Feed Forward Artificial Neural Network (FFANN), and defuzzification. In high order fuzzy time series method, we avoid subjective decisions by using FCM technique in fuzzification stage and Feed Forward Artificial Neural Network (FFANN) to identify the fuzzy relations and takes all membership values into account in defuzzification stage. According to [6-8], the steps for forecasting with the high-order fuzzy time series method are as follows:

1. Fuzzify observations of time series by using FCM clustering method. This technique was firstly introduced by [2] and the most widely used clustering algorithm. In this technique, fuzzy clustering is conducted by minimizing the least squared errors within groups. Let c be the number of fuzzy sets, such that $2 \le c \le n$ where n is the number of observations. FCM clustering algorithm in which the number of fuzzy sets is c is applied to the time series consists of crisp values. Then, after the center of each fuzzy set is determined, the degrees for each observation, which denote a degree of belonging to a fuzzy set for that observation, are calculated with respect to the obtained center values of fuzzy sets. Finally, ordered fuzzy sets L_r $(r = 1,2, \ldots, c)$ are obtained according to the ascending ordered centers, which are denoted by v_r $(r = 1,2, \ldots, c)$.

   Let μ_{ij} be the membership values, v_i be the center of cluster, n be the number of variables, and c be the number of clusters. Then the objective function, which is tried to be minimized in fuzzy clustering, is

   $$J_\beta(X, V, U) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^{\beta}\, d^2(x_j, v_i) \tag{4}$$

   with,X = the data matrix; V = the matrix of the cluster centers; U = the matrix of membership values
   β = fuzziness index, which is taken as $\beta > 1$; $d(x_j, v_i)$ = similarity measure between an observation and corresponding fuzzy cluster center.The objective function J_β is minimized subject to constraints given below,

   $$0 \le \mu_{ij} \le 1, \forall i, j$$
   $$0 \le \sum_{i=1}^{c} \mu_{ij} = 1,\ \forall i$$
   $$\sum_{i=1}^{c} \mu_{ij} = 1, \forall j \tag{5}$$

   In FCM clustering method, to solve the minimization problem given above, an iterative algorithm is used. In each iteration, the values of v_i and μ_{ij} are updated by using the formulas given in (6) and (7), respectively.

   $$v_i = \frac{\sum_{j=1}^{n} \mu_{ij}^{\beta} x_j}{\sum_{j=1}^{n} \mu_{ij}^{\beta}} \tag{6}$$

   $$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d(x_j, v_i)}{d(x_j, v_k)}\right)^{\frac{2}{(\beta-1)}}} \tag{7}$$

2. **Defining the fuzzy relationship with FFANN**

The Feed Forward Artificial Neural Network (FFANN) is the simplest and most widely used Artificial Neural Network (ANN). The membership values are used as inputs and targets values for the FFANN, so the number of FFANN inputs and outputs used for determining fuzzy relationships is equal to the number of fuzzy sets, c. In this process, the point to take into consideration is to avoid a possible loss in ability of generalization of the feed forward neural network. The neural network inputs are the membership value vectors obtained by applying the intersection operation to the membership value vectors of each lagged time series observation that belongs to each fuzzy set.

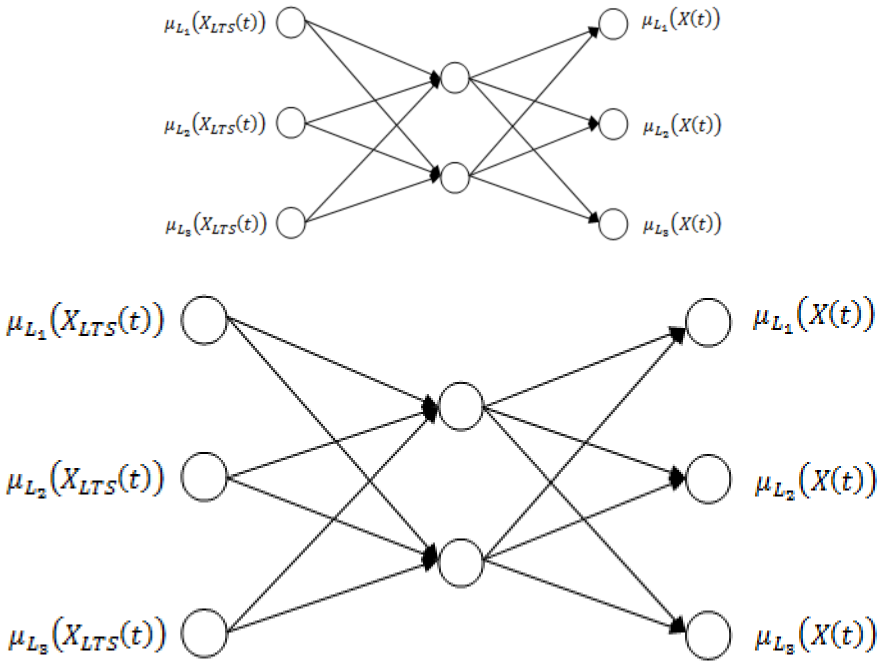

Figure 1. FFANN architecture with 2 neurons in hidden layers.

The numbers of neurons in the hidden layer are determined by trial and error to improve the generalizability of the FFANN. In all of the neurons in the neural network, the sigmoid function is used as an activation function, given by:

$$f(x) = (1 + \exp(-x))^{-1} \quad (8)$$

In the FFANN with this architectural structure optimal weights are obtained by training with learning algorithm Levenberg-Marquardt. Thus, the trained FFANN have learned the relationship among the membership degrees of the subsequent observations of time series.

3. Defuzzification

The defuzzification processes in this step are explained as follows. The outputs are generated using the membership degrees of belonging to c fuzzy sets for the observations in the lagged time series at $t(\mu_{L_i}(X_{LTS}(t)))$ as FFANN inputs. The FFANN outputs represent the fuzzy forecasts, which comprise the membership degrees for the observation at t. It should be noted that in contrast to FCM, the sum of these membership degrees is not equal to 1, but the defuzzification operation requires that these fuzzy forecasts are standardized by transforming them into weights using Eq. (9).

$$w_{it} = \frac{\hat{\mu}_{L_i}(X(t))}{\hat{\mu}_{L_1}(X(t)) + \hat{\mu}_{L_2}(X(t)) + \cdots + \hat{\mu}_{L_c}(X(t))} \quad (9)$$

Then, the forecasting results are obtained as follows:

$$\hat{X}_t = \sum_{i=1}^{c} w_{it} v_i \quad (10)$$

where $\hat{\mu}_{L_i}(X(t))$, i=1,2,...,c represents the fuzzy forecast for the observation at t, which comprises the membership values as the FFANN outputs, where te weights used to defuzzify the fuzzy forecasts are represented by w_{it}, t=1,2,...,c.

To evaluate the performance of the high order fuzzy time series method in this study, used Mean Absolute Percentage Error (MAPE) and Median Absolute Percentage Error (MdAPE). If the resulting error rate is getting smaller, then the forecasting results will be closer to the actual value.

$$MAPE = \frac{1}{N}\sum_{t=1}^{N} \left| \frac{Y_t - F_t}{Y_t} \right| \times 100\% \quad (11)$$

$$MdAPE = Median \left| \frac{Y_t - F_t}{Y_t} \right| \quad (12)$$

If MAPE and MdAPE are below 10%, we can say that the method can predict the data very well.

III. RESULTS AND DISCUSSION

3.1 Fuzzify Observations of Time Series by Using FCM

The center of the cluster and membership values in fuzzy sets was determined by choosing the number of clusters, weighting power, maximum iteration, smallest error, initial objective function, and initial iteration. In this study, six clusters with 2 weights were used and the maximum iteration used to reach a convergent value was 100 and the smallest error is 10^{-9}. The initial objective function is 0 and the initial iteration is 1. So, we present the results of the centers of cluster for each cluster using FCM in Table 1.

Table 1. Centers of clusters by FCM

Centers of Clusters					
v_1	v_2	v_3	v_4	v_5	v_6
14080,20	14607,11	16366,78	15054,35	13690,06	14257,68

Then we generated random number μ_{ij} as membership value for each observation. In generating random numbers μ_{ij}, the sum of these membership degrees is equal to 1. For example, the sum of the membership values of period June 2019 as follow,

$$\sum_{i=1}^{6} \mu_{ij} = 0{,}7622 + 0{,}0130 + 0{,}0006 + 0{,}0034 + 0{,}0139 + 0{,}2070 = 1$$

The results of the membership values of observation using FCM is presented in Table 2.

Table 2. Membership of observation

t	Period	$X(t)$	Membership Values					
			L_1	L_2	L_3	L_4	L_5	L_6
1	Jun-2019	14141	0,7622	0,0130	0,0006	0,0034	0,0139	0,2070
2	Jul-2019	14026	0,9151	0,0080	0,0005	0,0025	0,0238	0,0501
3	Aug-2019	14237	0,0170	0,0031	0,0001	0,0006	0,0014	0,9778
4	Sep-2019	14174	0,4247	0,0199	0,0008	0,0048	0,0160	0,5338
5	Oct-2019	14008	0,8655	0,0126	0,0008	0,0041	0,0446	0,0724
6	Nov-2019	14102	0,9757	0,0018	0,0001	0,0005	0,0027	0,0191
7	Dec-2019	13901	0,4836	0,0311	0,0026	0,0117	0,3490	0,1221
8	Jan-2020	13662	0,0045	0,0009	0,0001	0,0004	0,9920	0,0022
9	Feb-2020	14234	0,0230	0,0039	0,0001	0,0008	0,0018	0,9703
10	Mar-2020	16367	0,0000	0,0000	1,0000	0,0000	0,0000	0,0000
11	Apr-2020	15157	0,0085	0,0326	0,0067	0,9354	0,0046	0,0122
12	May-2020	14733	0,0290	0,7804	0,0046	0,1198	0,0114	0,0547
13	Jun-2020	14302	0,0373	0,0197	0,0004	0,0032	0,0049	0,9344
14	Jul-2020	14653	0,0062	0,9653	0,0007	0,0126	0,0022	0,0130
15	Aug-2020	14554	0,0118	0,9429	0,0008	0,0106	0,0036	0,0303
16	Sep-2020	14918	0,0206	0,1500	0,0069	0,7796	0,0096	0,0332
17	Oct-2020	14690	0,0166	0,8958	0,0022	0,0464	0,0062	0,0329
18	Nov-2020	14128	0,8614	0,0086	0,0004	0,0023	0,0103	0,1171
19	Dec-2020	14105	0,9679	0,0024	0,0001	0,0007	0,0035	0,0255
20	Jan-2021	14084	0,9994	0,0001	0,0000	0,0000	0,0001	0,0005
21	Feb-2021	14229	0,0355	0,0055	0,0002	0,0012	0,0027	0,9550
22	Mar-2021	14572	0,0050	0,9758	0,0004	0,0052	0,0015	0,0122
23	Apr-2021	14468	0,0775	0,6025	0,0032	0,0339	0,0193	0,2636
24	May-2021	14310	0,0473	0,0283	0,0006	0,0045	0,0065	0,9128
25	Jun-2021	14496	0,0529	0,7403	0,0026	0,0293	0,0141	0,1609

To define the fuzzy relationship with FFANN, we inputted the vectors values which were obtained by applying the intersection operation to the vector of membership values of observations. The number of fuzzy sets are 6. For example, the sample membership values for a third-order fuzzy time series forecasting model, to obtain a forecast for $X(t = 4)$:

$$\mu\big(X(t-1=3)\big) = (0{,}7622 \quad 0{,}0130 \quad 0{,}0006 \quad 0{,}0034 \quad 0{,}0139 \quad 0{,}2070)$$

$$\mu(X(t-2=2)) = (0{,}9151 \quad 0{,}0080 \quad 0{,}0005 \quad 0{,}0025 \quad 0{,}0238 \quad 0{,}0501)$$
$$\mu(X(t-3=1)) = (0{,}0170 \quad 0{,}0031 \quad 0{,}0001 \quad 0{,}0006 \quad 0{,}0014 \quad 0{,}9778)$$

Then,

$$\mu(X_{LTS}(4)) = \left(\mu(X(t-1=3)) \cap \mu(X(t-2=2)) \cap \mu(X(t-3=1))\right)$$
$$= (0{,}0170 \quad 0{,}0031 \quad 0{,}0001 \quad 0{,}0006 \quad 0{,}0014 \quad 0{,}0501)$$

Table 3. Fuzzy Relationship (Input)

t	Input					
	1	2	3	4	5	6
4	0,0170	0,0031	0,0001	0,0006	0,0014	0,0501
5	0,0170	0,0031	0,0001	0,0006	0,0014	0,0501
6	0,0170	0,0031	0,0001	0,0006	0,0014	0,0724
7	0,4247	0,0018	0,0001	0,0005	0,0027	0,0191
8	0,4836	0,0018	0,0001	0,0005	0,0027	0,0191
9	0,0045	0,0009	0,0001	0,0004	0,0027	0,0022
10	0,0045	0,0009	0,0001	0,0004	0,0018	0,0022
11	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
12	0,0000	0,0000	0,0001	0,0000	0,0000	0,0000
13	0,0000	0,0000	0,0046	0,0000	0,0000	0,0000
14	0,0085	0,0197	0,0004	0,0032	0,0046	0,0122
15	0,0062	0,0197	0,0004	0,0032	0,0022	0,0130
16	0,0062	0,0197	0,0004	0,0032	0,0022	0,0130
17	0,0062	0,1500	0,0007	0,0106	0,0022	0,0130
18	0,0118	0,1500	0,0008	0,0106	0,0036	0,0303
19	0,0166	0,0086	0,0004	0,0023	0,0062	0,0329
20	0,0166	0,0024	0,0001	0,0007	0,0035	0,0255
21	0,8614	0,0001	0,0000	0,0000	0,0001	0,0005
22	0,0355	0,0001	0,0000	0,0000	0,0001	0,0005
23	0,0050	0,0001	0,0000	0,0000	0,0001	0,0005
24	0,0050	0,0055	0,0002	0,0012	0,0015	0,0122
25	0,0050	0,0283	0,0004	0,0045	0,0015	0,0122

3.2 Defuzzifying

To determine defuzzification, the data in Table 3 were analyzed by FFANN. As a result, the number of membership values does not have to be equal to 1. The output can be seen in Table 4.

Table 4. Defuzzifyingfuzzy forecast

t	Output					
	1	2	3	4	5	6
4	0,4771	0,2377	0,0021	0,1926	0,4457	0,4526
5	0,3054	0,4771	0,0000	0,1752	0,6325	0,0076
6	0,4741	0,1721	0,0021	0,1943	0,5421	0,4520
7	0,2295	0,0003	0,0000	0,0000	0,3635	0,4696
8	0,2016	0,0030	0,0000	0,0000	0,3496	0,0009
9	0,4947	0,0029	0,1190	0,0177	0,5329	0,6203
10	0,0000	0,0000	1,0000	0,4040	0,0000	0,0000
11	0,4999	0,0003	0,4762	0,4998	0,0000	0,5001
12	0,4999	0,1826	0,4998	0,4998	0,0009	0,5001
13	0,4944	0,4944	0,4913	0,4928	0,4911	0,5039
14	0,4656	0,4656	0,1449	0,1911	0,1892	0,2363
15	0,4676	0,4676	0,1679	0,2208	0,1896	0,2364
16	0,4676	0,2208	0,2006	0,4676	0,0009	0,2364
17	0,3091	0,6493	0,0097	0,3784	0,0003	0,2561
18	0,3035	0,3035	0,0006	0,0011	0,1595	0,0000
19	0,4712	0,1865	0,0000	0,0000	0,3466	0,3740
20	0,4810	0,0000	0,0000	0,0000	0,1709	0,3209
21	0,0822	0,0822	0,0000	0,0000	0,4002	0,5167
22	0,4750	0,4750	0,1185	0,1815	0,1105	0,1566
23	0,4963	0,4963	0,0938	0,3200	0,3892	0,5407
24	0,4872	0,4872	0,0000	0,0000	0,0000	0,9993
25	0,4577	0,4577	0,1185	0,1815	0,1105	0,1566

The next step was to forecast the data using high-order forecasting method. The results can be seen in Table 5.

Table 5. Forecast obtained by the high order fuzzy time series

Period	$X(t)$	Weight						Forecast
		1	2	3	4	5	6	
Sep-2019	14174	0,2639	0,1315	0,0012	0,1065	0,2465	0,2504	14204,17
Oct-2019	14008	0,1911	0,2986	0,0000	0,1097	0,3959	0,0048	14190,76
Nov-2019	14102	0,2581	0,0937	0,0011	0,1058	0,2951	0,2461	14163,77
Dec-2019	13901	0,2159	0,0003	0,0000	0,0000	0,3420	0,4418	14025,34
Jan-2020	13662	0,3632	0,0054	0,0000	0,0000	0,6298	0,0016	13837,63
Feb-2020	14234	0,2768	0,0016	0,0666	0,0099	0,2981	0,3470	14188,20
Mar-2020	16367	0,0000	0,0000	0,7123	0,2877	0,0000	0,0000	15989,13
Apr-2020	15157	0,2529	0,0002	0,2410	0,2529	0,0000	0,2530	14922,51
May-2020	14733	0,2290	0,0836	0,2289	0,2289	0,0004	0,2291	14911,28
Jun-2020	14302	0,1666	0,1666	0,1655	0,1660	0,1655	0,1698	14673,82
Jul-2020	14653	0,2751	0,2751	0,0856	0,1129	0,1118	0,1396	14512,02
Aug-2020	14554	0,2672	0,2672	0,0959	0,1262	0,1083	0,1351	14545,01
Sep-2020	14918	0,2934	0,1385	0,1259	0,2934	0,0006	0,1483	14752,86
Oct-2020	14690	0,1928	0,4051	0,0061	0,2361	0,0002	0,1598	14565,73
Nov-2020	14128	0,3951	0,3951	0,0008	0,0014	0,2076	0,0000	14210,55
Dec-2020	14105	0,3419	0,1353	0,0000	0,0000	0,2515	0,2713	14101,55
Jan-2021	14084	0,4944	0,0000	0,0000	0,0000	0,1757	0,3299	14070,21
Feb-2021	14229	0,0760	0,0760	0,0000	0,0000	0,3701	0,4779	14060,67
Mar-2021	14572	0,3131	0,3131	0,0781	0,1196	0,0728	0,1032	14530,23
Apr-2021	14468	0,2124	0,2124	0,0401	0,1370	0,1666	0,2314	14393,45
May-2021	14310	0,2468	0,2468	0,0000	0,0000	0,0000	0,5063	14300,13
Jun-2021	14496	0,3087	0,3087	0,0799	0,1224	0,0745	0,1056	14534,58
MAPE	0,824%							
MdAPE	0,715%							

From Table 5 we can see that forecasting foreign exchange rate of USD data using high order fuzzy time series method has an accuracy of forecasting below 10%, that is MAPE= 0,824% and MdAPE= 0,715%. This values indicate that the high order fuzzy time series method in this study has a very good level of accuracy.

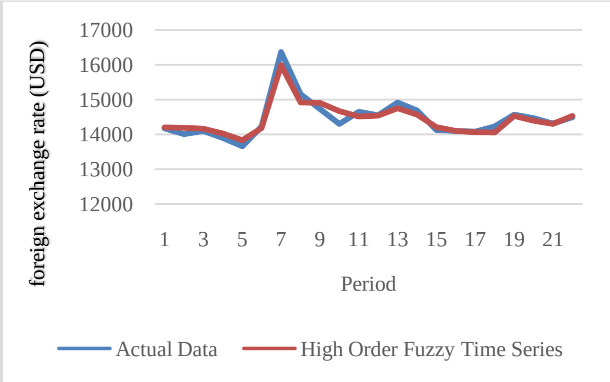

Figure 2. Comparison of forecasting results between the actual data and high-order fuzzy time series.

Based on Fig. 2, it can be seen that the forecasting results using high order fuzzy time series. has a value that is not much different from the actual data, which indicates that the plot of forecasting results using the high order fuzzy time series method predict the data very well.

For example, to determine the foreign exchange rate of USD for July 2021, we inputed the vectors values obtained by applying the intersection operation (April 2021-June 2021) to the membership value vector. The FFANN forecast for July 2021 $X(t=26)$ is as follows:

$$\mu(X_{Jun\,21}) = \mu(X(t-1=25)) = (0{,}0529 \quad 0{,}7403 \quad 0{,}0026 \quad 0{,}0293 \quad 0{,}0141 \quad 0{,}1609)$$
$$\mu(X_{May\,21}) = \mu(X(t-2=24)) = (0{,}0473 \quad 0{,}0283 \quad 0{,}0006 \quad 0{,}0045 \quad 0{,}0065 \quad 0{,}9128)$$
$$\mu(X_{Apr\,21}) = \mu(X(t-3=23)) = (0{,}0775 \quad 0{,}6025 \quad 0{,}0032 \quad 0{,}0339 \quad 0{,}0193 \quad 0{,}2636)$$

Then,

$$\mu(X_{LTS}(26)) = \Big(\mu(X(t-1=25)) \cap \mu(X(t-2=24)) \cap \mu(X(t-3=23))\Big)$$
$$= (0{,}0473 \quad 0{,}0283 \quad 0{,}0006 \quad 0{,}0045 \quad 0{,}0065 \quad 0{,}1609)$$

After determining the input, the output FFANN be obtained is 0,4028; 0,4570; 0,0520; 0,1815; 0,0172; 0,5273. Then the weights can be obtained as follows:

$$w_1 = \frac{0{,}4028}{0{,}4028 + 0{,}4570 + 0{,}0520 + 0{,}1815 + 0{,}0172 + 0{,}5273} = 0{,}2459$$
$$w_2 = \frac{0{,}4570}{0{,}4028 + 0{,}4570 + 0{,}0520 + 0{,}1815 + 0{,}0172 + 0{,}5273} = 0{,}2790$$
$$w_3 = \frac{0{,}0520}{0{,}4028 + 0{,}4570 + 0{,}0520 + 0{,}1815 + 0{,}0172 + 0{,}5273} = 0{,}0317$$
$$w_4 = \frac{0{,}1815}{0{,}4028 + 0{,}4570 + 0{,}0520 + 0{,}1815 + 0{,}0172 + 0{,}5273} = 0{,}1108$$
$$w_5 = \frac{0{,}0172}{0{,}4028 + 0{,}4570 + 0{,}0520 + 0{,}1815 + 0{,}0172 + 0{,}5273} = 0{,}0105$$
$$w_6 = \frac{0{,}5273}{0{,}4028 + 0{,}4570 + 0{,}0520 + 0{,}1815 + 0{,}0172 + 0{,}5273} = 0{,}3220$$

Then, the forecasting results using high order Fuzzy time series for July 2021 is:

$$X(26) = 0{,}2459 \times 14080{,}20 + 0{,}2790 \times 14607{,}11 + 0{,}0317 \times 16366{,}78 + 0{,}1108 \times 15054{,}35 + 0{,}0105 \times 13690{,}06 + 0{,}3220 \times 14257{,}68 = 14460{,}82$$

IV. CONCLUSION

Although fuzzy time series approaches provide some advantages, it also have some disadvantages. Some of them require subjective decisions in fuzzification and ignore the value of membership. This weakness directly affects forecasting performance. In this study, a new high-order fuzzy time series forecasting model is proposed to overcome this problem. The method can determine membership values systematically using the FCM technique at the fuzzification stage. This membership value is used in the stage of determining the fuzzy relationship so that better forecasting results can be obtained. In this study using the high order time series method to predict the USD exchange rate gives good results in forecasting the exchange rate in the next period. This can be seen from the deviation value of the forecast data with the original data which is quite small, namely MAPE = 0.824% and MdAPE = 0.715%. From these results it can be concluded that the high-order fuzzy time series method can predict USD exchange rate data very well. For the July 2021 period, the USD exchange rate is predicted to be 14460.82 USD.

REFERENCES

[1]. Q. Song and B. S. Chissom, 1993. Forecasting Enrollment with Fuzzy Time Series Part I. *Fuzzy sets and systems*. *54(1), 1993, 1-9.*

[2]. J.C. Bezdek, *Pattern Recognition with Fuzzy Objective Function Algorthms*, N Y: Plenum Press, 1981.

[3]. L.A. Zadeh, Fuzzy Sets. *Information and Control*. *8, 1965, 338-353.*

[4]. C.H. Aladag, M. A. Basaran, E. Egrioglu, U. Yolcu, and V. R. Uslu, Forecasting in High Order Fuzzy Time Series by Using Neural Networks to Define Fuzzy Relations,*Expert System Application*. 36, 2009, 4228-4231.

[5]. K. Huarang, and T.H.K. Yu, The Application of Neural Networks to Forecast Fuzzy Time Series, *Physica A*. 363, 2006, 481-491

[6]. S.M. Chen, Forecasting Enrollments Based on High-Order Fuzzy Time Series, *Cybernetics and Systems*, 33(1), 2002, 1-16.

[7]. S.M. Chen andN.Y. Chung, Forecasting enrollments using high-order fuzzy time series and genetic algorithms, *Int. J. Intell. Syst*., 21 (5), 2006, 485-501

[8]. N. Herawati, S. Saidi, E. Setiawan, K. Nisa and Ropiudin, Performance of High-Order Chen Fuzzy Time Series Forecasting Method and FeedforwardBackpropagation Neural Network Method in Forecasting Composite Stock Price Index,*American Journal of Computational and Applied Mathematics*, 12(1), 2022, 1-7

[9]. O.C. Yolcu, U. Yolcu, E. Egrioglu, and C. H. Aladag, High-Order Fuzzy Time Series Forecasting Method Based on An Intersection Operation,*Applied Mathematical Modelling*, 40, 2016, 8750-8765.

[10]. `E. Bas, U. Yolcu, E. Egrioglu, and H. Aladag, A Fuzzy 4, 5, 6, 7, Time Series Forecasting Method Based On Operation of Union and Feed Forward Artificial Neural Network,*American Journal of Intelligent Systems*. 5(3), 2015, 81-91.

