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On the Comparison of the Methods of Parameter Estimation for Pareto Distribution

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On the Comparison of the Methods of Parameter Estimation for Pareto Distribution

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Abstract. The main purposes of this study is to asses on comparison of parameter estimation methods of the Pareto distribution. The estimation methods include moment, maximum likelihood estimation, probabilityweightedmoment, and generalizedmoment methods. Based on unbiasness, variance, and consistency properties, the results demonstrate that in estimating parameters of the Pareto distribution, the maximum likelihood method is the one of the best estimation methods.

1. Introduction

The Pareto distribution is one of a continue probability distribution with scale parameter $\beta$and shift parameter k where$\beta > 0$and$k > 0$ [1].Pareto distribution introduced by an Italian economist Vilfredo Pareto, he was found that 80% land in Italia owned by no more than 20% of population. Based on those fact, then the Pareto's law come up which stated that 20% effort will earn as much as 80%, this law also known as 20/80 or*law of the few* [2]. Probability density function of Pareto distribution is as follows [3]:

$$f(x;\beta,k) = \frac{k\beta^k}{x^{k+1}}; x \geq \beta, \beta > 0, k > \quad (1)$$

And the cumulative distribution of Pareto [3], is as follows:

$$F(x;\beta,k) = 1 - \left(\frac{\beta}{x}\right)^k; x \geq \beta, \beta > 0, k > \quad (2)$$

The estimation of parameters is a process by using a sample to estimate the unknown parameters of a population [4]. There are some methods that can be used to estimates the parameters, namely: methods of moment, method of maximum likelihood, method of probability weighted moment and method of generalized moment. An estimation of a parameter has attained the properties of unbiased, minimum variance, consistency, sufficient statistics and completeness.

In this study the comparison of those methods will be discussed to find the best method that can be used to estimate the parameters based on the criteria: of unbiased, minimum variance, consistency, sufficient statistics and completeness.

2. Materials and Methods

2.1 Method

The steps of the method conducted in this study:

1. Creating the curve of probability density function of Pareto distribution with parameter (β, κ) using software R
2. Estimating Pareto parameter (β, κ) using Method of Moments, Maximum Likelihood Estimation Method, Probability Weight Moment Method, and Generalized Method of Moments.
3. Examining the characteristics of unbiased from estimatorof each parameter β and κ of the Method of Moments, Maximum Likelihood Estimation Method, Probability Weight Moment Method, and Generalized Method of Moments.
4. Examining the characteristics of consistencyfrom estimator of each parameter β and κ of the Method of Moments, Maximum Likelihood Estimation Method, Probability Weighted Moment Method, and Generalized Method of Moments.
5. Examining the characteristics of minimum varianceof Pareto distribution.
6. Examining the characteristics of sufficient statistic of Pareto distribution.
7. Examining the characteristics of and completeness of Pareto distribution.
8. Simulating using Software R for the Method of Moments, Maximum Likelihood Estimation Method, Probability Weight Moment Method, and Generalized Method of Moments.

3. Result and Discussion

3.1 The Curve of Probability of Pareto Distribution (β, κ)

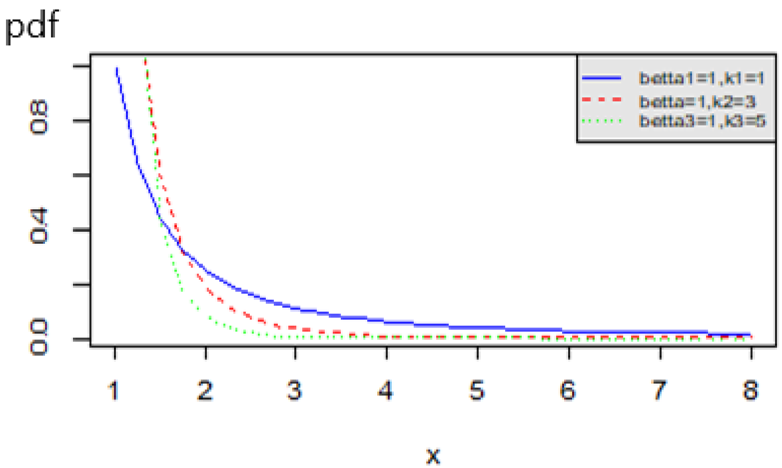

Figure 1. Pareto Distribution Probability Density Function

Pareto distribution probability density function is conducted using different parameter k which are $k = 1$, $k = 3$, $k = 5$ and the value of parameter $\beta = 1$ which is shown by the Figure 1. Parameter k is a shape parameter which numerical parameter to point the shape of curve. Meanwhile, parameter β is a scale parameter. It can be seen that the three graphs in Figure 1 has similar data variation which is caused by the same value of scale parameters.

3.2 Estimation of Parameter of Pareto Distribution

3.2.1 Method of Moment
The method of moments is a method of estimation of parameters. First, it starts with deriving equations which relate the population moments (namely, the expected values of powers of the random variable under consideration) to the parameters of interest. Second, a sample is drawn and the population moments are estimated from the sample. The equations are then solved for the parameters of interest, using the sample moments in place of the (unknown) population moments. This results in estimates of those parameters [5]. The results are as follows:

$$\hat{k} = \frac{{m_1}^2}{{m_1}^2 - m_2} \text{and} \hat{\beta} = \left(\frac{m_2}{m_1}\right) \quad (3)$$

3.2.2 Maximum Likelihood Estimation Method
The likelihood function of Pareto distribution is as follows:

$$L(\beta, k) = k\beta^k \prod_{i=1}^{n} \frac{1}{{x_i}^{k+1}} \quad (4)$$

Next, we take logarithm on both sides, we have:

$$\ln L(a,b) = n \ \ln k \quad + n - (k+1) \sum_{i=1}^{n} \ln x_i \quad (5)$$

The estimation of the parameters can be attain by the derivative with respect to the parameters β and kand then set equal to zero [6]. So that we found the estimation as follows:

$$\hat{k} = \frac{n}{\sum_{i=1}^{n} \ln x_i - n \ln \beta} \text{and} \hat{\beta} = \min x_i \quad (6)$$

3.2.3 Probability Weighted Moment
To estimate the parameters of Paretodistribution by probability weighted moment, first we looking for the inverse of its cumulative distribution function [7], and we have:

$$]x = \frac{\beta}{(1-F(x))^{1/k}} \quad (7)$$

Next, we looking for the-t moment by the following formula:

$$M_t = M_{1,0,t} = \int_0^1 \left(X(F)\right) (1 - F(x))^t \, dF = \int_0^1 \left(\frac{\beta}{(1-F(x))^{1/k}}\right) (1 - F(x))^t \, dF \quad (8)$$

$$M_t = M_{1,0,t} = -\frac{\beta k}{tk + k - 1} \quad (9)$$

By substitution $t = 0$and$t = 1$ we have:

$$M_0 = -\frac{\beta k}{k-1} \text{and } M_1 = -\frac{\beta k}{2k-1} \quad (10)$$

Next, the estimation of parameter k is: $\hat{k} = \frac{M_0 - M_1}{M_0 - 2M_1}$ and the estimation of parameter $\beta$is:$\hat{\beta} = \frac{M_1 \, M_0}{M_0 - M_1}$

3.2.4. Generalized Method of Moment
The estimation of parameter Pareto distribution by using generalized method of moment [8] is found by the following formula:

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$$M_{l,r} = \int_0^1 x^l\,[F(x)]^r = \int_0^1 \left[\frac{\beta}{\left(1-F(x)\right)^{1/k}}\right]^l [F(x)]^r \tag{11}$$

If the value of $r = 0$ and l is arbitrary, we have:

$$M_l = \frac{k\,\beta^l}{(l-k\,)} \tag{12}$$

If the values of $l = l_1, l_2 (l_1 \neq l_2)$ we have:

$$M_{l_1} = \frac{k\beta^{l_1}}{(l_1-k\,)} \qquad and \quad M_{l_2} = \frac{k\beta^{l_2}}{(l_2-k\,)} \tag{13}$$

So that we have:

$$\hat{\beta} = \left(\frac{M_{l_1}l_1 - M_{l_1}k}{k}\right)^{\frac{1}{l_1}} \qquad and \quad \hat{k} = \frac{M_{l_2}l_2}{(\beta^{l_2}+M_{l_2})} \tag{14}$$

3.3 Unbiasedness

The estimation of parameters Pareto distribution by using those methods, then we check the unbiasedness as follows:

3.3.1 Method of Moment

The estimation parameter k ($\hat{k}$)

$$E(\hat{k}) = E\left[\frac{{m_1}^2}{{m_1}^2-m_2}\right] = \frac{\frac{1}{n}\sum_{i=1}^n X_i^1}{\frac{1}{n}\sum_{i=1}^n X_i^1-\beta} = k \tag{15}$$

The estimation parameter β ($\hat{\beta}$)

$$E(\hat{\beta}) = E\left[\frac{m_2}{m_1}\right] = \frac{\frac{1}{n}\sum_{i=1}^n X_i^2}{\frac{1}{n}\sum_{i=1}^n X_1} = \beta \tag{16}$$

3.3.2 Probability weighted moment

The estimation parameter k ($\hat{k}$)

$$E(\hat{k}) = E\left[\frac{M_0 - M_1}{M_0 - 2M_1}\right] = \frac{-\frac{\beta k}{k-1} - \left(-\frac{\beta k}{2k-1}\right)}{-\frac{\beta k}{k-1} - \left(2\left(-\frac{\beta k}{2k-1}\right)\right)} = k \tag{17}$$

The estimation parameter β ($\hat{\beta}$)

$$E(\hat{\beta}) = E\left[\frac{M_1\,M_0}{M_0 - M_1}\right] = \frac{\left(-\frac{\beta k}{2k-1}\right)\left(-\frac{\beta k}{k-1}\right)}{\left(-\frac{\beta k}{k-1}\right) - \left(-\frac{\beta k}{2k-1}\right)} = \beta \tag{18}$$

3.3.3. Generalized Method of Moment

The estimation parameter k ($\hat{k}$)

$$E(\hat{k}) = E\left[\frac{M_{l_2}l_2}{(\beta^{l_2}+M_{l_2})}\right] = \frac{k\beta^{l_2}l_2}{\beta^{l_2}l_2} = k \tag{19}$$

The estimation parameter β ($\hat{\beta}$)

$$E(\hat{\beta}) = E\left[\left(\frac{M_{l_1}l_1 - M_{l_1}k}{k}\right)^{\frac{1}{l_1}}\right] = \left(\frac{\frac{k\beta^{l_1}}{(l_1-k)}(l_1-k)}{k}\right)^{\frac{1}{l_1}} = \beta \tag{20}$$

So that the estimation $\hat{\beta}$ and $\hat{k}$ are unbiased for β and k.

3.4 Consistency

To check the consistency, the estimation of parameter Pareto distribution by using Chebyshev theorem,

$$P(|\hat{k} - k| \geq \varepsilon) \leq \frac{Var\,(\hat{k})}{\varepsilon^2} \text{for} \forall\ \varepsilon > 0 \text{ [6]}, \tag{21}$$

as follows:

3.4.1 Method of Moment

The estimation parameter k ($\hat{k}$)

$$P(|\hat{k} - k| \geq \varepsilon) \leq \frac{Var\left(\frac{{m_1}^2}{{m_1}^2 - m_2}\right)}{\varepsilon^2} \tag{22}$$

$$\lim_{n\to\infty}\left\{\left[\left(\frac{{m_1}^2}{{m_1}^2 - m_2}\right) - k\right]\right\} \leq 0 \tag{23}$$

The estimation parameter β ($\hat{\beta}$)

$$P(|\hat{\beta} - \beta| \geq \varepsilon) \leq \frac{Var\left(\frac{m_2}{m_1}\right)}{\varepsilon^2} \tag{24}$$

$$\lim_{n\to\infty}\left\{\left[\left(\frac{m_2}{m_1}\right) - \beta\right]\right\} \leq 0 \tag{25}$$

3.4.2 Probability Weighted Moment

The estimation parameter k ($\hat{k}$)

$$P(|\hat{k} - k| \geq \varepsilon) \leq \frac{Var\left(\frac{M_0 - M_1}{M_0 - 2M_1}\right)}{\varepsilon^2} \tag{26}$$

$$\lim_{n\to\infty}\left\{\left[\left(\frac{M_0 - M_1}{M_0 - 2M_1}\right) - k\right]\right\} \leq 0 \tag{27}$$

The estimation parameter β ($\hat{\beta}$)

$$P(|\hat{\beta} - \beta| \geq \varepsilon) \leq \frac{Var\left(\frac{M_1\,M_0}{M_0 - M_1}\right)}{\varepsilon^2} \tag{28}$$

$$\lim_{n\to\infty}\left\{\left[\left(\frac{M_1\,M_0}{M_0 - M_1}\right) - \beta\right]\right\} \leq 0 \tag{29}$$

3.4.3 Method of Generalized Moment

The estimation parameter k ($\hat{k}$)

$$P(|\hat{k} - k| \geq \varepsilon) \leq \frac{Var\left(\frac{M_{l_2}l_2}{(\beta^{l_2} + M_{l_2})}\right)}{\varepsilon^2} \tag{30}$$

$$\lim_{n\to\infty}\left\{\left[\left(\frac{M_{l_2}l_2}{(\beta^{l_2}+M_{l_2})}\right)-k\right]\right\}\leq 0 \quad (31)$$

The estimation parameterβ ($\hat{\beta}$)

$$P\left(\left|\hat{\beta}-\beta\right|\geq\varepsilon\right)\leq\frac{Var\left(\left(\frac{M_{l_1}l_1-M_{l_1}k}{k}\right)^{\frac{1}{l_1}}\right)}{\varepsilon^2} \quad (32)$$

$$\lim_{n\to\infty}\left\{\left[\left(\left(\frac{M_{l_1}l_1-M_{l_1}k}{k}\right)^{\frac{1}{l_1}}\right)-\beta\right]\right\}\leq 0 \quad (33)$$

So that the estimation $\hat{\beta}$ and$\hat{k}$are consistent estimation for $\beta$and$k$.

3.5 Check for Minimum Variance

To check the minimum variance of Pareto distribution, first we find the Fisher information matrix [6], as follows:

$$l_n(\beta,k)=-\begin{bmatrix}E\left[\frac{\partial}{\partial\beta}\left(\frac{\partial\ln L}{\partial\beta}\right)\right] & E\left[\frac{\partial}{\partial\beta}\left(\frac{\partial\ln L}{\partial k}\right)\right]\\ E\left[\frac{\partial}{\partial k}\left(\frac{\partial\ln L}{\partial\beta}\right)\right] & E\left[\frac{\partial}{\partial k}\left(\frac{\partial\ln L}{\partial k}\right)\right]\end{bmatrix} \quad (34)$$

where

$$\ln L(\beta,k)=\text{n}\ln k+nk\,ln\,\beta-(k+1)\sum_{i=1}^{n}\ln x \quad (35)$$

So that we have the Fisher information as follows:

$$l_n^{-1}(\beta,k)=\frac{\beta^2k^2}{(n^2k-n^2k^2)}\begin{bmatrix}\frac{n}{k^2} & -\frac{n}{\beta}\\ -\frac{n}{\beta} & \frac{nk}{\beta^2}\end{bmatrix} \quad (36)$$

Then we calculate the Cramer-Rao inequality [9] as follows:

$$Var\left(\hat{\theta}\right)\geq\frac{1}{E\left[\left(\frac{\partial lnf(x;\theta)}{\partial\theta}\right)^2\right]}=\frac{1}{I(\theta)}=I^{-1}(\theta) \quad (37)$$

$$Var\left(\hat{\beta},\hat{k}\right)\geq l_n^{-1}(\beta,k)=\frac{\beta^2k^2}{(n^2k-n^2k^2)}\begin{bmatrix}\frac{n}{k^2} & -\frac{n}{\beta}\\ -\frac{n}{\beta} & \frac{nk}{\beta^2}\end{bmatrix} \quad (38)$$

The estimation parameter $\left(\hat{\beta},\hat{k}\right)$is an efficient estimation since the variance is attains the minimum Cramer-Rao inequality.

3.6 Check for sufficiency

To find the sufficient statistic of Pareto distribution, we used Fisher-Neymann factorization [10, 11],as follows:

$$f\left(\underline{X};\beta,k\right)=\prod_{i=1}^{n}\frac{k\beta^k}{x_i^{\,k+1}} \quad (39)$$

$$f\left(\underline{X};\beta,k\right)=\underbrace{k\beta^k x_i^{-kn}}_{k_1\left(U_i\left(\underline{X}\right);\beta,k\right)}\underbrace{x_i^{-n}}_{k_2\left(\underline{X}\right)}$$

Since$k_2\left(\underline{X}\right)$is independent of$\beta,k$, so$X_i$is sufficient statistic.

3.7 *Check for completeness*

It will be shown that X_i is complete statistic if $E\big(g(x)\big) = 0$ and $P(g(x) = 0) = 1$ [6].

$$E\big(g(x)\big) = \sum g(x)\,.\,f\big(\underline{X};\beta,k\big) = \sum g(x)\,.\,k\beta^k {x_i}^{-kn} {x_i}^{-n} \tag{40}$$

If $E\big(g(x)\big) = 0$ then $\sum g(x)\,.\,k\beta^k {x_i}^{-kn} {x_i}^{-n} = 0$. This means that $k\beta^k {x_i}^{-kn} {x_i}^{-n}$ is not equal to 0. If $x = 0$ *then* $g(0) = 0$, and if $x = 1$ *then* $g(1) = 0$ and so on, if we take $x = x$ then $g(x) = 0$. So that $P(g(x) = 0) = 1$ and so it is a complete statistic.

3.8 *Simulation for the estimation of parameters β and k*

Simulation of the estimation of parameters of Pareto distribution β and k by using software R version 3.3.2 based on the form of the curve of probability density function, for the value of parameters β and k, where $\beta = 1$ and $k = 1, 3, 5$. The sample size are n = 10,20, 40, 80 and 100. In this simulation the values of mean and *Mean Square Error* (MSE) are as follows:

Table 1. Estimation values of Parameter β = 1 and k = 1

	β=1				**k=1**				
n		**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**
10	Mean	100.8153	0.03265	0.379776	66.66712	0.001478	0.180645	0.424009	0.052365
	MSE	353755.8	1.060587	1.27014	0.43768	1.01088	1.086232	1.39387	0.82726
20	Mean	4.84897	0.006766	0.23841	0.978156	0.000216	0.067132	0.225295	0.035723
	MSE	356.4836	1.024123	1.20852	0.42578	1.00521	1.07413	1.30743	0.85548
40	Mean	3.08625	0.001123	0.069839	0.561284	5.23E-05	0.039886	0.125745	0.024695
	MSE	109.1997	1.013938	1.18725	0.38859	1.00166	1.024169	1.24523	0.901242
80	Mean	1.62732	0.000401	0.04998	0.618115	5E-06	0.015129	0.079663	0.023902
	MSE	9.763389	1.009246	1.17962	0.43796	1.00165	1.239304	1.24462	0.87375
100	Mean	1.37139	0.000171	0.043115	0.477711	6.99E-06	0.00817	0.079434	0.018452
	MSE	16.035	0.03265	0.379776	66.66712	0.001478	0.180645	0.424009	0.052365

Table 2. Estimation values of parameters β=1 and k=3

	β=1				**k=3**				
n		**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**
10	Mean	1.959268	1.03774	1.019456	1.205927	1.012386	3.601367	3.777597	3.593276
	MSE	1.796609	0.00317	0.005265	0.067104	3.950619	1.547171	2.854884	0.384108
20	Mean	1.74417	1.017385	0.99741	1.207944	1.00293	3.352905	3.38725	3.598216

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n	β=1				k=3				
		MM ($\hat{\beta}$)	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**
	MSE	0.676059	0.000572	0.003433	0.064176	3.98827	0.699084	1.312927	0.383282
40	Mean	1.82381	1.008734	1.0012	1.223368	1.00076	3.059885	3.31467	3.618071
	MSE	0.870265	0.000152	0.000555	0.068908	3.996954	0.227937	0.478858	0.40283
80	Mean	1.88181	1.004525	1.0043	1.200214	1.000197	3.043481	3.13774	3.597016
	MSE	1.006656	0.0000423	0.000258	0.045736	3.999213	0.109883	0.217478	0.364045
100	Mean	1.95285	1.002995	0.99914	1.198911	1.00013	3.081881	3.07644	3.595051
	MSE	1.252917	0.0000155	0.00021	0.04639	3.999482	0.109554	0.205944	0.362135

Table 3. The estimation values of parameters β = 1 and k = 5

n	β=1				k=				
		MM ($\hat{\beta}$)	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**	**MM ($\hat{\beta}$)**	**MLE ($\hat{\beta}$)**
10	Mean	1.338033	1.019809	1.001434	0.995711	1.010654	6.302275	6.16258	5.53851
	MSE	0.186855	0.000687	0.002055	0.004872	15.91489	6.709471	8.06903	0.31854
20	Mean	1.32645	1.009209	0.99742	0.99932	1.00265	5.589863	5.31536	5.54942
	MSE	0.126053	0.000193	0.000785	0.003325	15.97882	2.074026	2.84416	0.32142
40	Mean	1.318704	1.005312	1.0012	0.99303	1.000661	5.26881	5.35951	5.53617
	MSE	0.108494	0.0000619	0.000555	0.001596	15.99471	0.699832	1.42526	0.29794
80	Mean	1.336775	1.0022	1.0043	1.00361	1.000166	5.099307	5.21972	5.56326
	MSE	0.118444	0.0000097	0.000258	0.000908	15.99867	0.427755	0.47236	0.32578
100	Mean	1.334248	1.002085	0.99914	1.00379	1. 000107	5.118131	5.09333	5.56402
	MSE	0.116986	0.00000956	0.00021	0.000659	15.99915	0.291194	0.40821	0.32205

Based on the tables above, it was found that the values of estimation for mean of the parameters β and k by using method of moment, methods of maximum likelihood, probability weighted moment, and method of generalized moment, the best method is the maximum likelihood method, since it is close to the real values of parameters (β, k) and with the minimum *Mean Square Error* (MSE).

4. Conclusion

Based on the result above, we conclude that:

1. The estimation of parameters$(\beta, k)$by using method of moment, method of maximum likelihood, method of probability weighted moment, and generalized moment are the estimation methods that can be use since all the method attains the properties of unbiasedness, minimum variance, consistency, sufficient statistic and completeness.
2. Based on the simulation result by using software R the values of *mean square error*for the method of maximum likelihood has the smallest values compared with the others methods. Therefore, we can conclude that the method of maximum likelihood is the best method to estimate the parameters of Pareto distribution if the sample size is large.

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