5327504: Acknowledging Receipt

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International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hind Sel, 27 Mar 2018 jam 09.11

Kepada: asmiati308@yahoo.com

Cc: ahmed.khaled@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

The Research Article titled "ON THE LOCATING CHROMATIC NUMBER OF SOME BARBELL GRAPHS," by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti has been received and assigned the number 5327504.

All authors will receive a copy of all the correspondences regarding this manuscript.

Thank you for submitting your work to International Journal of Mathematics and Mathematical Sciences.

Best regards,

--
****
Ahmed Khaled
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5327504: Authors' Feedback Needed

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ahmed.khaled@hindawi.com
Kepada: asmiati308@yahoo.com
Cc: sikesaguya412@gmail.com, lyrayulianti@gmail.com

Min, 1 Apr 2018 jam 20.18

Dear Dr. Asmiati,

This is regarding manuscript 5327504 titled "ON THE LOCATING CHROMATIC NUMBER OF SOME BARBELL GRAPHS" submitted to International Journal of Mathematics and Mathematical Sciences. While checking your manuscript, we had comments regarding the following points:

Please note that the reference list should include a diversity of sources that support the scholarly content of the manuscript. Concentrating on the author's own work could be misinterpreted as an attempt to increase citations to that author's work. To avoid such a misinterpretation, please update the reference list to include more diverse citations and reduce citations to the work of [cited author].

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We look forward to your quick response.

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Best regards,

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5327504: Authors' Data Needed

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ahmed.khaled@hindawi.com
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Min, 1 Apr 2018 jam 20.18

Dear Dr. Asmiati,

This is regarding manuscript 5327504 titled "ON THE LOCATING CHROMATIC NUMBER OF SOME BARBELL GRAPHS" submitted to International Journal of Mathematics and Mathematical Sciences. While checking the data of the authors for verification, we had the following comment(s):

We were unable to find any academic history for I Ketut Sanda Gunce Yana, and Lyra Yulianti.

In order to proceed with the review process of manuscript 5327504, please provide us with institutional email addresses for I Ketut Sanda Gunce Yana, and Lyra Yulianti, along with their institutional web pages, and a list of their previous publications.

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Ahmed Khaled <ahmed.khaled@hindawi.com> Rab, 4 Apr 2018 jam 14.36

Kepada: asmiati308@yahoo.com

Cc: sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

Please confirm the receipt of my previous email, and provide your response as soon as possible.

Your prompt response is needed in order to avoid any further delay in the review process.

Best regards,

Ahmed

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Response For Hindawi

Yahoo/Terkirim

Asmiati Asmiati <asmiati308@yahoo.com>
Kepada: ahmed.khaled@hindawi.com

Kam, 5 Apr 2018 jam 08.33

Dear Prof. Ahmed Khaled
Editorial Office Hindawi,

Hereby I attach some files related to our manuscript. Thank you very much for your attention.

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Asmiati

Unduh semua lampiran sebagai file zip

Respon For ... .pdf
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Paper Barb... .docx
207.3kB

Paper Barbe... .pdf
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Ahmed Khaled <ahmed.khaled@hindawi.com>

Kam, 5 Apr 2018 jam 13.08

Kepada: asmiati308@yahoo.com

Cc: sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

Thank you for your response. Please submit your updated version within the next 24 hours. If you need more time, you can withdraw the current version and resubmit the manuscript to undergo the review process.

I look forward to your response.

Best regards,

Ahmed

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5327504: Major Revision Required

Yahoo/Email M...

Dalibor Froncek <ijmms@hindawi.com>
Kepada: asmiati308@yahoo.com
Cc: dfroncek@d.umn.edu, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Jum, 8 Jun 2018 jam 15.15

Dear Dr. Asmiati,

Following the review of Research Article titled "On The Locating Chromatic Number Of Some Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti, I recommend that it should be revised taking into account the changes requested by the reviewer(s). Since the requested changes are major, the revised manuscript will undergo a second round of review by the same reviewer(s). Please login to the Manuscript Tracking System to read the submitted review report(s) and submit the revised version of your manuscript no later than Friday, July 06, 2018.

To submit the revised version of your manuscript, please access "Author Activities" in your account and upload the PDF file of your revised manuscript. Also, please submit your replies to the comments of the reviewer(s) as an additional PDF file.

Best regards,

Dalibor Froncek
dfroncek@d.umn.edu

5327504: Minor Revision Required

Yahoo/Email M...

Dalibor Froncek <ijmms@hindawi.com>

Min, 1 Jul 2018 jam 17.19

Kepada: asmiati308@yahoo.com

Cc: dfroncek@d.umn.edu, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

Following the review of your Research Article titled "On The Locating Chromatic Number Of Some Barbell Graphs," by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti, I recommend that it should be revised taking into account the changes requested by the reviewer(s). Please login to the Manuscript Tracking System to read the submitted review report(s) and submit the revised version of your manuscript not later than Sunday, July 15, 2018.

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Dalibor Froncek
dfroncek@d.umn.edu

5327504: Revised Version Received

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International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hinda

Min, 8 Jul 2018 jam 15.15

Kepada: asmiati308@yahoo.com

Cc: ahmed.khaled@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

The revised version of Research Article 5327504 titled "On The Locating Chromatic Number Of Some Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti has been received. The editor assigned to handle the review process of your manuscript will inform you as soon as a decision is reached.

Thank you for submitting your work to International Journal of Mathematics and Mathematical Sciences.

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Re: 5327504: Revised Version Received

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Asmiati Asmiati <asmiati308@yahoo.com>

Jum, 13 Jul 2018 jam 11.56

Kepada: International Journal of Mathematics and Mathematical Sciences

Dear Prof. Ahmed Khaled,

We have sent some files about our response in revised the manuscript.
Thank you very much for your cooperation.

Best regards,
Asmiati

Dalibor Froncek <ijmms@hindawi.com>
Kepada: asmiati308@yahoo.com
Cc: dfroncek@d.umn.edu, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Min, 22 Jul 2018 jam 21.09

Dear Dr. Asmiati,

The review process of Research Article 5327504 titled "On The Locating Chromatic Number Of Certain Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti submitted to International Journal of Mathematics and Mathematical Sciences has been completed. I am pleased to inform you that your manuscript has now been accepted for publication in the journal.

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Thank you again for submitting your manuscript to International Journal of Mathematics and Mathematical Sciences.

Best regards,

Dalibor Froncek
dfroncek@d.umn.edu

International Journal of Mathematics and Mathematical Sciences <nada.nemr@hindawi.com> Min, 22 Jul 2018 jam 21.09

Kepada: asmiati308@yahoo.com

Cc: nada.nemr@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

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International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hinda

Sel, 24 Jul 2018 jam 17.44

Kepada: asmiati308@yahoo.com

Cc: ahmed.khaled@hindawi.com, sikesaguya412@gmail.com, lyrayulianti@gmail.com

Dear Dr. Asmiati,

This is to confirm the receipt of the electronic files of Research Article 5327504 titled "On The Locating Chromatic Number Of Certain Barbell Graphs" by Asmiati Asmiati, I Ketut Sanda Gunce Yana and Lyra Yulianti. We will check all the uploaded files and contact you if anything else is needed.

Thank you for your cooperation.

Best regards,

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5327504: Galley Proofs

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International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@h

Kam, 26 Jul 2018 jam 13.15

Kepada: asmiati308@yahoo.com

Cc: lyrayulianti@gmail.com, sikesaguya412@gmail.com

Dear Dr. Asmiati,

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Sab, 28 Jul 2018 jam 00.50

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5327504: Galley Proof Corrections

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International Journal of Mathematics and Mathematical Sciences <ahmed.khaled@hinda

Sel, 31 Jul 2018 jam 14.16

Kepada: asmiati308@yahoo.com

Cc: lyrayulianti@gmail.com, sikesaguya412@gmail.com

Dear Dr. Asmiati,

This is to confirm the receipt of the first galley proof corrections of Research Article 5327504 titled "On The Locating Chromatic Number Of Certain Barbell Graphs,".

We will address your comments and send you another set of galley proofs.

Thank you for your cooperation.

Best regards,

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> Pada Sen, 6 Agt 2018 pada 15:16, Ahmed Khaled
> <ahmed.khaled@hindawi.com> menulis:
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> Dear Dr. Asmiati,
>
> I am pleased to let you know that your article has been published in its final form in "International Journal of Mathematics and Mathematical Sciences."
>
> Asmiati, "On the Locating Chromatic Number of Certain Barbell Graphs," International Journal of Mathematics and Mathematical Sciences, vol. 2018, Article ID 5327504, 5 pages, 2018. https://doi.org/10.1155/2018/5327504/.
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> Best regards,
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> Ahmed Khaled

Dear Prof. Ahmed Khaled

Editorial Office Hindawi,

We have fixed manuscript according to your comments:

1. The institutional email address for Lyra is lyra@sci.unand.ac.id, and the institutional web is : http://matematika.fmipa.unand.ac.id

   List of Publications:
   a) Asmiati, Wamiliana, Devriyadi, **Lyra Yulianti,** On Some Petersen Graphs Having Locating Chromatic Number Four Or Five, *Far East Journal of Mathematical Sciences* 102 (4) : 769 – 778 (2017)
   b) **Lyra Yulianti**, Nirmala Santi, Admi Nazra, Ramsey Minimal Graphs for $2K_2$ versus $2C_n$, *Applied Mathematical Sciences* 9 (85): 4211 – 4217 (2015)
   c) Kristiana Wijaya, **Lyra Yulianti**, Edy Tri Baskoro, Hilda Assiyatun, Djoko Suprijanto, All Ramsey $(2K_2,C_4)$-Minimal Graphs, *Journal of Algorithms and Computation* 46 : 9 – 25 (2015).
   d) Syafrizal Sy, Gema Histamedika, **Lyra Yulianti** , The Rainbow Connection of Fan and Sun, *Applied Mathematical Sciences* 7 (64): 3155 – 3159 (2013).
   e) **Lyra Yulianti**, The asymptotic distribution of the number of 3-star factors in random d-regular graphs, *Discrete Mathematics, Algorithms and Applications* 3(2) : 203 – 222 (2011)
   f) Edy Tri Baskoro, **Lyra Yulianti**, Ramsey minimal graphs for $2K_2$ versus P_n, *Advances and Applications of Discrete Mathematics* 8(2) : 83 – 90 (2011)
   g) **Lyra Yulianti**, Hilda Assiyatun, Saladin Uttunggadewa, Edy Tri Baskoro, On Ramsey $(K_{1,2},P_4)$-minimal graphs, *Far East Journal of Mathematical Sciences* 40(1) : 23 – 36 (2010)
   h) Tomas Vetrik, **Lyra Yulianti**, Edy Tri Baskoro, On Ramsey $(K_{1,2},C_4)$-minimal graphs, *Discussiones Mathematicae Graph Theory* 30(4) : 637 – 649 (2010)
   i) Tomas Vetrik, Edy Tri Baskoro, **Lyra Yulianti**, A Note on Ramsey $(K_{1,2},C_4)$-minimal Graphs of diameter 2, *Proceeding of the International Conference 70 years of FCE STU Bratislava*, pp 1 – 4 (2008)
   j) Edy Tri Baskoro, **Lyra Yulianti**, Hilda Assiyatun, Ramsey $(K_{1,2},C_4)$-minimal graphs, *Journal of Combinatorial Mathematics and Combinatorial Computing* 65: 79 – 90 (2008)

2. I Ketut Sadha Gunce Yana is my student in Mathematics Department, Lampung University, and he does not have the institutional email address and does not have the previous publications yet.
3. I have reduced my paper in the reference list, but there are three important references that I keep because those papers conducted as the references of my previous research.

Thank you very much for your kindest attention,

Regards,

Asmiati

ON THE LOCATING CHROMATIC NUMBER OF SOME BARBELL GRAPHS

Asmiati[1], I Ketut Sadha Gunce Yana[2], Lyra Yulianti[3]
[1]Mathematics Departement, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No.1 Bandar Lampung, Indonesia.
[2] Student of Mathematics Departement, Faculty of Mathematics and Natural Sciences, Lampung University, Jl. Brodjonegoro No.1 Bandar Lampung, Indonesia.
[3] Mathematics Departement, Faculty of Mathematics and Natural Sciences, Andalas University, Kampus UNAND Limau Manis, Padang 25163, Indonesia.
Email: asmiati308@yahoo.com, asmiati.1976@fmipa.unila.ac.id, sikesaguya412@gmail.com, lyra@sci.unand.com

Abstract. The locating chromatic number of a graph is the minimal color required so that it qualifies for a locating coloring. In this paper we will discuss about the locating chromatic number of barbell graph; where both of them contain a complete graph K_n or Petersen graph $P_{n,1}$ for $n \geq 3$.

Keyword: locating chromatic number, barbell graph, complete graph, Petersen graph.

1. Introduction

The partition dimension was introduced by Chartrand et al. [5] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [11], the optimization of threat detecting sensors [10], chemical data classification [8]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [6]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Consider $G=(V,E)$ as the given connected graph and c as the proper coloring of G using k colors $1,2,\ldots,k$ for some positive integer k. We denote $\Pi=\{C_1,C_2,...,C_k\}$ as the partition of $V(G)$, where C_i is the color class, i.e the set of vertices that given the i-th color, for $i\in[1,k]$. For an arbitrary vertex $v \in V(G)$, the color code $c_\Pi(v)$ is defined as the ordered k-tuple

$$c_\pi(v) = (d(v,C_1), d(v,C_2), \ldots, d(v,C_k)),$$

where $d(v,C_i)=\min\{d(v,x) \mid x\in C_i\}$ for $i\in[1,k]$. If for every two vertices $u,v \in V(G)$, their color codes are different, $c_\pi(u) \neq c_\pi(v)$, then c is defined as the locating coloring of G using k colors. The locating chromatic number of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem about the locating chromatic number of a graph, proven by Chartrand et al. [6]. The neighborhood of vertex s in a connected graph G, denoted by $N(s)$, is the set of vertices adjacent to s.

Theorem 1.1 [6] *Let c be a locating coloring in a connected graph G. If s and t are distinct vertices of G such that $d(s,u) = d(t,u)$ for all $u \in V(G) - \{s,t\}$, then $c(s) \neq c(t)$. In particular, if s and t are non-adjacent vertices of G such that $N(s) = N(t)$, then $c(s) \neq c(t)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph *G*.
Corollary 1.1 [6] *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \geq k+1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on some certain graph classes. Chartrand et al. [7] has successed in constructing tree on *n* vertices, $n \geq 5$ with locating chromatic numbers varying from *3* to *n*, except for $(n-1)$. Then Behtoei and Omoomi [4] have obtained the locating chromatic number of the Kneser graph. Recently, Asmiati et al.[1] obtained the locating chromatic number of Petersen Graph, $P_{n,1}$, for n ≥ 3.

There are some recent results for some special cases of trees as follows. Asmiati et al. [3] has successed in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and Asmiati et al. [2] for firecracker graphs. Next, Des Wellyyanti et al.[9] determined the locating chromatic number for complete n-ary tree.

The following definition of Petersen graph is taken from [1]. Let $\{u_1, u_2, \ldots, u_n\}$ be the set of vertices in the outer cycle and $\{v_1, v_2, \ldots, v_n\}$ be the set of vertices in the inner cycle, for n ≥ 3. From the definition, we have that the Petersen graph, denoted by $P_{n,k}$, for n ≥ 3 and $1 \leq k \leq \left\lfloor \frac{n-1}{2} \right\rfloor$, has $2n$ vertices and *3n* edges.

Theorem 1.2 and Theorem 1.3 gave the locating chromatic numbers for complete graph and Petersen graph.

Theorem 1.2 [7]
For n ≥ 2, the locating chromatic number of complete graph K_n is n.

Theorem 1.3 [1]
The locating chromatic number of Petersen Graph $P_{n,1}$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.

The barbell graph is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number of barbell graph $B_{m,n}$ for $m,n \geq 3$, where *G* and *H* are two copies of complete graph on *m* and *n* vertices, K_m and K_n, respectively. If $m = n$, we denote the barbell graph by $B_{n,n}$. Secondly, we obtain the locating chromatic number of barbell graph $B_{P_{n,1}}$ for $n \geq 3$, where *G* and *H* are two copies of Petersen graphs $P_{n,1}$.

2. Results and Discussion

Theorem 2.1

The locating chromatic number of Barbell Graph $B_{n,n}$ is $n+1$, for $n \geq 3$.

Proof:

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains the complete graph K_n, then by Theorem 1.2, we have $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is the locating coloring using n colors. It is clear that there are two vertices have the same color codes, a contrary. Thus, we have that $\chi_L(B_{n,n}) \geq n+1$.

Next, we construct the upper bound of the locating chromatic number for barbell graph $B_{n,n}$. The set of vertices of the first complete graph is denoted by $V(K_n^1) = \{u_i;\ i \in [1,n]\}$, whereas the set of vertices of the second complete graph is denoted by $V(K_n^2) = \{v_i;\ i \in [1,n]\}$.

Let c be a coloring on $B_{n,n}$ using $n+1$ colors. We assign the following colors of $V(B_{n,n})$:

$$c(u_i) = i \qquad ; 1 \leq i \leq n$$

$$c(v_i) = \begin{cases} i & , 2 \leq i \leq n-1; \\ n & , i = 1; \\ n+1 & , \text{otherwise.} \end{cases}$$

By using this coloring, we obtain the color codes of $V(B_{n,n})$ as follows.

$$c_\Pi(u_i) = \begin{cases} 0 & , \text{(i)th} - \text{component for } 1 \leq i \leq n; \\ 2 & , \text{(n + 1)th} - \text{component for } 1 \leq i \leq n-1; \\ 1 & , \text{otherwise.} \end{cases}$$

$$c_\Pi(v_i) = \begin{cases} 0 & , \text{(i)th} - \text{component for } 2 \leq i \leq n-1, \text{ or} \\ & \text{(n)th} - \text{component for } i = 1, \text{ or} \\ & (n+1) - \text{component for } i = n; \\ 3 & , \text{(1)st} - \text{component for } 1 \leq i \leq n-1; \\ 2 & , \text{(1)st} - \text{component for } i = n; \\ 1 & , \text{otherwise.} \end{cases}$$

Since all vertices on $V(B_{n,n})$ have distinct color codes, then c is a locating coloring. Thus, $\chi_L(B_{n,n}) \leq n+1$. □

The following figure is a minimum locating coloring of barbell graph $B_{6,6}$.

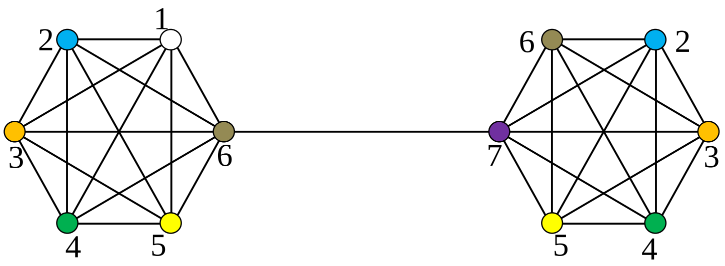

Figure 1. A minimum locating coloring of barbell graph $B_{6,6}$

The following Corollary 2.2 is the direct consequence of Theorem 2.1.

Corollary 2.2
For $n, m \geq 3$ and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is

$$\chi_L(B_{m,n}) = max\{n, m\}.$$

Theorem 2.3
For $n \geq 3$, the locating chromatic number of barbell graph $B_{P_{n,1}}$ is

$$\chi_L(B_{P_{n,1}}) = \begin{cases} 4, for\ odd\ n \\ 5, for\ even\ n \end{cases}.$$

Proof. To prove this theorem, we consider two cases as follows.

Case 1. $\chi_L(B_{P_{n,1}}) = 4$, for odd n.

Since the barbell graph $B_{P_{n,1}}$ contains Petersen Graph $P_{n,1}$ for odd n, then by Theorem 1.3, we have $\chi_L(B_{P_{n,1}}) \geq 4$.

Next, we determine the upper bound of the locating chromatic number of $B_{P_{n,1}}$. For odd n, let $\{u_i, u_{n+i}\ ;\ i \in [1, n]\}$ be the set of vertices of the first Petersen Graph and $\{w_i, w_{n+i}\ ;\ i \in [1, n]\}$ be the set of vertices of the second Petersen Graph.

Let c be a coloring of $V(B_{P_{n,1}})$ using 4 colors, defined as follows:

$$c(u_i) = \begin{cases} 1 & , i = 1; \\ 3 & , \text{for even } i, i \geq 2; \\ 4 & , \text{for odd } i, i \geq 3. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2 & , i = 1; \\ 3 & , for \text{ odd } i \geq 3; \\ 4 & , \text{for even } i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1 & , \text{odd } i < n-1; \\ 2 & , \text{even } i \leq n-1; \\ 3 & , i = \text{n}. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1 & , \text{even } i \leq n-1; \\ 2 & , \text{odd } i < n-1; \\ 4 & , i = n. \end{cases}$$

The color codes of $V(B_{P_{n,1}})$ for odd n are:

$$c_\Pi(u_i) = \begin{cases} i & , (2)\text{nd} - \text{component for } i \leq \frac{n+1}{2}; \\ i-1 & , (1)\text{st} - \text{component for } i \leq \frac{n+1}{2}; \\ n-i+1 & , (1)\text{st} - \text{component for } i > \frac{n+1}{2}. \\ \\ n-i+2 & , (2)\text{nd} - \text{component } i > \frac{n+1}{2}; \\ 0 & , (3)\text{th} - \text{component for even } i \geq 2; \\ & (4)\text{th} - \text{component for odd } i > 2; \\ 1 & , \text{ otherwise.} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i & , (1)\text{st} - \text{component for } i \leq \frac{n+1}{2}; \\ i-1 & , (2)\text{nd} - \text{component for } i \leq \frac{n+1}{2}; \\ n-i+1 & , (2)\text{nd} - \text{component for } i > \frac{n+1}{2}. \\ \\ n-i+2 & , (1)\text{st} - \text{component for } i > \frac{n+1}{2}; \\ 0 & , (4)\text{th} - \text{component for even } \geq 2; \\ & (3)\text{th} - \text{component for odd } i \geq 2; \\ 1 & , \text{ otherwise.} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i & , (3)\text{th} - \text{component for } i \leq \frac{n-1}{2}; \\ i+1 & , (4)\text{th} - \text{component for } i \leq \frac{n-1}{2}; \\ n-i & , (3)\text{th} - \text{component for } i \geq \frac{n+1}{2}. \\ \\ n-i+1 & , (4)\text{th} - \text{component for } i \geq \frac{n+1}{2}; \\ 0 & , (2)\text{nd} - \text{component for even } i \leq n-1; \\ & (1)\text{st} - \text{component for odd } i \leq n-1; \\ 1 & , \text{otherwise.} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i & , (4)\text{th} - \text{component for } i \leq \frac{n-1}{2}; \\ i+1 & , (3)\text{th} - \text{component for } i \leq \frac{n-1}{2}; \\ n-i & , (4)\text{th} - \text{component for } i \geq \frac{n+1}{2}; \\ \\ n-i+1 & , (3)\text{th} - \text{component for } i \geq \frac{n+1}{2}; \\ 0 & , (1)\text{th} - \text{component for even } i \leq n-1; \\ & \ \ (2)\text{th} - \text{component for odd } i \leq n-1; \\ 1 & ; \text{otherwise.} \end{cases}$$

Since all vertices on $V(B_{P_{n,1}})$ have distinct color codes, then c is a locating coloring. As the result, we have that $\chi_L(B_{P_{n,1}}) \leq 4$.

Case 2. $\chi_L(B_{P_{n,1}}) = 5$, for even n.

Since the barbell graph $B_{P_{n,1}}$ contains Petersen Graph $P_{n,1}$ for even n, then by Theorem 1.3, we have $\chi_L(B_{P_{n,1}}) \geq 5$.

Next, we determine the upper bound of the locating chromatic number of $B_{P_{n,1}}$ for even n. Let c be a coloring of $B_{P_{n,1}}$ using 5 colors as follows:

$$c(u_i) = \begin{cases} 1 & , i = 1; \\ 3 & , \text{even } 2 \leq i \leq n-1; \\ \\ 4 & , \text{odd } 2 < i \leq n-1; \\ 5 & , i = n. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2 & , i = 1; \\ 3 & , \text{odd } i > 2; \\ 4 & , \text{even } i \geq 2; \end{cases}$$

$$c(w_i) = \begin{cases} 1 & , \text{odd } i \leq n-2; \\ 2 & , \text{even } i \leq n-2. \\ \\ 3 & , i = n-1; \\ 4 & , i = n. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1 & , \text{even } i \leq n-1; \\ 2 & , \text{odd } i \leq n-1; \\ 5 & , i = n. \end{cases}$$

The color codes of $V(B_{P_{n,1}})$ for even n are:

$$c_{\Pi}(u_i)=\begin{cases} i & ,\ (2)\text{nd}, (5)\text{th} - \text{component for } i \le \frac{n}{2};\\ i-1 & ,\ (1)\text{st} - \text{component for } i \le \frac{n}{2};\\ n-i & ,\ (5)\text{th} - \text{component for } i > \frac{n}{2};\\ n-i+1 & ,\ (1)\text{st} - \text{component for } i > \frac{n}{2};\\ n-i+2 & ,\ (2)\text{nd} - \text{component for } i > \frac{n}{2};\\ 0 & ,\ (3)\text{th} - \text{component for even } 2 \le i \le n-1;\\ & \quad (4)\text{th} - \text{component for odd } 2 < i \le n-1;\\ 2 & ,\ (4)\text{th} - \text{component for } i = 1;\\ & \quad (3)\text{th} - \text{component for } i = n;\\ 1 & ,\ \text{otherwise.} \end{cases}$$

$$c_{\Pi}(u_{n+i})=\begin{cases} i & ,\ (1)\text{st} - \text{component for } i \le \frac{n}{2};\\ i-1 & ,\ (2)\text{nd} - \text{component for } i \le \frac{n}{2};\\ i+1 & ,\ (5)\text{th} - \text{component for } i \le \frac{n}{2};\\ n-i+1 & ,\ (2)\text{nd and } (5) - \text{components for } i > \frac{n}{2};\\ n-i+2 & ,\ (1)\text{th} - \text{component for } i > \frac{n}{2};\\ 0 & ,\ (3)\text{th} - \text{component for odd } 2 \le i \le n;\\ & \quad (4)\text{th} - \text{component for even } 2 \le i \le n;\\ 2 & ,\ (3)\text{th} - \text{component for } i = 1;\\ 1 & ,\ \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_i)=\begin{cases} i & ,\ (4)\text{th} - \text{component for } i \le \frac{n}{2};\\ i+1 & ,\ (5)\text{th} - \text{component for } i \le \frac{n}{2};\\ & \quad (3)\text{th} - \text{component for } i \le \left(\frac{n}{2}\right) - 1;\\ n-i & ,\ (4)\text{th} - \text{component for } i > \frac{n}{2};\\ n-i+1 & ,\ (5)\text{th} - \text{component for } i > \frac{n}{2}.\\ n-i-1 & ,\ (3)\text{th} - \text{component for } \frac{n}{2} \le i \le n-1;\\ 0 & ,\ (1)\text{st} - \text{component for odd } i \le n-2;\\ & \quad (2)\text{nd} - \text{component for odd } i \le n-2;\\ 2 & ,\ (1)\text{st} - \text{component for } i = n-1;\\ & \quad (2)\text{nd} - \text{component for } i = n;\\ 1 & ,\ \text{otherwise.} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i & \text{, (5)th} - \text{component for } i \le \frac{n}{2}; \\ i+1 & \text{, (4)th} - \text{component for } i \le \frac{n}{2}; \\ i+2 & \text{, (3)th} - \text{component for } i \le \left(\frac{n}{2}\right) - 1; \\ n-i & \text{, (3)th} - \text{component for } \frac{n}{2} \le i \le n-1; \\ & \quad \text{(5)th} - \text{component for } i > \frac{n}{2}; \\ n-i+1 & \text{, (4)th} - \text{component for } i > \frac{n}{2}; \\ 0 & \text{, (1)th} - \text{component for even } i \le n-1; \\ & \quad \text{(2)th} - \text{component for odd } i \le n-1; \\ 2 & \text{, (1)st and (3)th} - \text{component for } i = n; \\ 1 & \text{, otherwise.} \end{cases}$$

Since all vertices have distinct color codes on $V(B_{P_{n,1}})$ for even n, then c is a locating coloring. Thus, we have that $\chi_L(B_{P_{n,1}}) \le 5$. □

The following figure is a minimum locating coloring of barbell graph $B_{P_{5,1}}$.

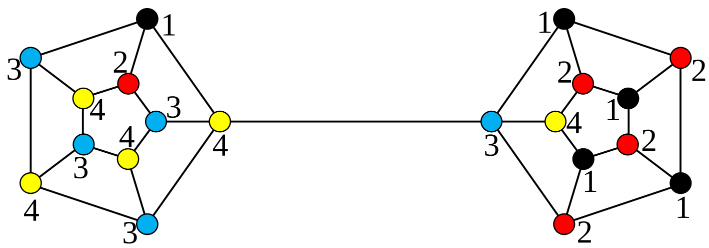

Figure 2. A minimum locating coloring of $B_{P_{5,1}}$

3. Acknowledgement

We are thankful to DRPM Dikti for the Fundamental Grant 2018.

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REFEREE'S REPORT

on the paper 5327504

Title : On the locating chromatic number of some barbell graphs

Authors: Asmiati, I Ketut Sadha Gunce Yana and Lyra Yulianti

The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In the present paper the authors investigate the locating chromatic number for two families of barbell graphs.

The topic is actual and the results are interesting. Due to the fact that no general theorem for determining the locating chromatic number of graphs is known, it make sense to investigate the locating chromatic number for families of graphs.

The present version of the paper is not prepared carefully and contains several incorrectness and formal mistakes.
Therefore I do not recommend the publication of the paper as it is. A revised version of the paper prepared by the comments below can be accepted for publication.

Comments:

Page 1, title: write "certain" instead "some"

Page 1: Rewrite Abstract with using the definition on locating coloring.

Page 2, after Corollary 1.1: Complete information of the paper [Baskoro, E.T., Asmiati, Characterizing all trees with locating-chromatic number 3, Electronic Journal of Graph Theory and Applications 1(2) (2013), pp. 109-117.], where are characterized all trees with locating-chromatic number 3.

Page 2, Petersen graph: The Petersen graph contains only 10 vertices and 15 edges. You want to consider the generalized Petersen graph $P(n, m)$ with $2n$ vertices and $3n$ edges which was introduced in [Watkins, M.E., A theorem on Tait colorings with an application to the generalized Petersen graphs, J. Combin. Theory 6 (1969), pp. 152-164.]

Page 2, Theorem 1.3: complete "generalized" before "Petersen"

Page 2, line -4: after $m, n \geq 3$ write "where G and H are complete graphs on m and n vertices, respectively."

Page 3, Proof of Theorem 2.1 start as follows: Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1} \{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

Page 3, in the proof of Theorem 2.1 and also in the proof of Theorem 2.3: use "i^{th}" instead "$(i)th$"

Page 4, Corollary 2.2: "$\max\{n, m\}$" should be "$\max\{n, m\} + 1$"

Page 5, line 1 and line 5: "$i < n - 1$" change for "$i \leq n - 2$"

Page 5, line 13: "$i > 2$" change for "$i \geq 3$"

Page 5, line -2 and on page 6, lines 6 and 16: "$i \leq n - 1$" change for "$i \leq n - 2$"

Page 7, line 16: write "$3 \leq i \leq n - 1$" instead "$2 \leq i \leq n$"

Page 7, line -5: "$i \leq n - 2$" change for "$i \leq n - 3$"

Page 7, line -4: write "for even $i \leq n - 2$" instead "for odd $i \leq n - 2$"

Page 8, line 7: "$i \leq n - 1$" change for "$i \leq n - 2$"

REFEREE'S REPORT

on the revised version of the paper 5327504.v2

Title : On the locating chromatic number of certain barbell graphs

Authors: Asmiati, I Ketut Sadha Gunce Yana and Lyra Yulianti

Again the revised version of the paper is not prepared carefully and the authors did not accept all suggestions and recommendations given in the referee's report. Therefore I do not recommend the publication of the paper as it is. A revised version of the paper prepared by the comments below can be accepted for publication.

Comments:

Page 1, Abstract rewrite by the following way: The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion.
In this paper we investigate the locating chromatic number for two families of barbell graphs.

Page 1, lines from -1 to -6 and on page 2 lines from 1 up to 7 - rewrite by the following way: Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \to \{1, 2, \ldots, k\}$ where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$ where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$ where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.
The following theorem is a basic theorem proved by Chartrand et al. in [8]. The neighborhood of vertex s in a connected graph G, denoted by $N(s)$, is the set of vertices adjacent to s.

Page 2, the text after Corollary 1.1 until Theorem 1.2. rewrite by the following way: There are some interesting results related to the determination of the

locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand, et al. in [8] have determined all graphs of order n with locating chromatic number n, namely a complete multipartite graphs of n vertices. Moreover, Chartrand et al. [9] have succeeded in constructing trees on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [6] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [1] obtained the locating chromatic number of the generalized Petersen graph $P(n,1)$ for $n \geq 3$. Baskoro and Asmiati [5] have characterized all trees with locating-chromatic number 3. In [Syofyan, D.K., Baskoro, E.T., Assiyatun, H., Trees with Certain Locating-Chromatic Number, J. Math. Fund. Sci. 48(1) (2016), pp. 39-47] were characterized all trees of order n with locating chromatic number $n-t$, for any integers n and t, where $n > t+3$ and $2 \leq t < \frac{n}{2}$. Asmiati et al. in [4] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [2] for firecracker graphs. Next, Wellyyanti et al. [11] determined the locating chromatic number for complete n-ary trees.
The generalized Petersen graph $P(n,m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [14]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.
Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Page 2 and several times later: The generalized Petersen graph defined by Watkins has notation $P(n,m)$. Therefore change "$P_{n,1}$" for "$P(n,1)$" or use notation $D_n = P_n \square P_2$ as for prism.

Page 3, line 13: write "of the generalized Petersen graph $P(n,1)$" instead of "of generalized Petersen graphs $P_{n,1}$"

Page 3, Theorem 2.1. rewrite as follows: Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.
Theorem 2.1. Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n+1$.

Page 3, lines -10 and -11: The sentence "Next, suppose that ..." replace by "Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction."

Page 3, lines -2, -3 and -4: The labeling $c(v_i)$ and also all other labelings write by the following way

$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \le i \le n-1 \\ n+1, & \text{otherwise.} \end{cases}$$

Page 4 lines from -1 to -4 and on page 5 lines from 1 to 5 replace as follows:
Proof Let $B_{P(n,1)}$, $n \ge 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \le i \le n\}$ and the edge set $E(B_{P(n,1)}) = \{u_i u_{i+1}, u_{n+i}u_{n+i+1}, w_i w_{i+1}, w_{n+i}w_{n+i+1} : 1 \le i \le n-1\} \cup \{u_n u_1, u_{2n}u_{n+1}, w_n w_1, w_{2n}w_{n+1}\} \cup \{u_i u_{n+i}, w_i w_{n+i} : 1 \le i \le n\} \cup \{u_n w_n\}$.
Let us distinguish two cases.
Case 1, n odd. According to Theorem 1.3 for n odd we have $\chi_L(B_{P(n,1)}) \ge 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

Page 6, lines from -8 to -12 rewrite by the following way:
Case 2, n even. In view of the lower bound from Theorem 1.3 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \to \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n \ge 4$, we describe the locating coloring as follows:

Page 8, on the line 7 change "even" for "odd" and on the line 8 change "odd" for "even". It means

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component, } i \le \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component, } i \le \frac{n}{2} \\ & \text{for } 3^{th} \text{ component, } i \le \frac{n}{2} - 1 \\ n-i, & \text{for } 4^{th} \text{ component, } i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component, } i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component, } \frac{n}{2} \le i \le n-1 \\ 0, & \text{for } 1^{st} \text{ component, } i \text{ odd, } i \le n-3 \\ & \text{for } 2^{nd} \text{ component, } i \text{ even, } i \le n-2 \\ 2, & \text{for } 1^{st} \text{ component, } i = n-1 \\ & \text{for } 2^{nd} \text{ component, } i = n \\ 1, & \text{otherwise.} \end{cases}$$

Page 9: insert the reference
Syofyan, D.K., Baskoro, E.T., Assiyatun, H., Trees with certain locating-chromatic number, J. Math. Fund. Sci. 48(1) (2016), pp. 39-47.

Response to Referee's Report on the paper 5327504

We are thankful for the referee's comments. We have revised the manuscript based on suggestions in referee's report, except for Corollary 2.2. The statement in the corollary is correct, that for case $n, m \geq 3$ and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is $max\,\{n, m\}$. The following figure is a counter example for the case.

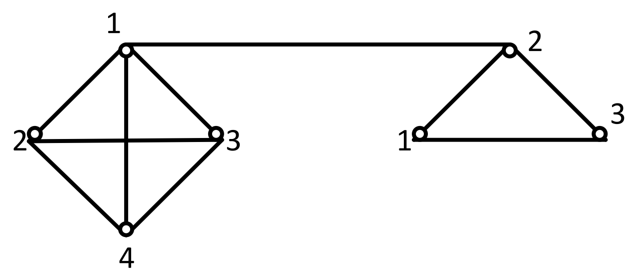

Figure 1. A minimum locating coloring of barbell graph $B_{4,3}$

Let G be a connected graph and c a proper coloring of G. For $i = 1, 2, \ldots, k$ define the color class C_i as the set of vertices receiving color i. The color code $c_{\Pi}(v)$ of a vertex v in is the ordered k-tuple $(d(v, C_1), \ldots, d(v, C_k))$ where $(d(v, C_1)$ is the distance of v to C_i. If all distinct vertices of G have distinct color codes, then c is called a locating-coloring of G . The locating-chromatic number of graph G, denoted by $\chi_L(G)$ is the smallest k such that G has a locating coloring with k colors. Let $\{u_1, u_2, \ldots, u_n\}$ be some vertices on the outer cycle and $\{v_1, v_2, \ldots, v_n\}$ be some vertices on the inner cycle, for $n \geq 3$. The Petersen graph, denoted by $P_{n,k}$, $n \geq 3$, $1 \leq k \leq \lfloor \frac{n-1}{2} \rfloor$, $1 \leq i \leq n$ is a graph that has $2n$ vertices $\{u_i\} \cup \{v_i\}$, and edges $\{u_i u_{i+1}\}$, $\{v_i v_{i+k}\}$, and $\{u_i v_i\}$. We determined that the locating chromatic number of Petersen Graphs $P_{n,1}$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$. In this paper, we discuss the locating-chromatic number for certain operation of s Petersen Graphs $P_{n,1}$.

Response to Referees Report on the paper 5327504

We are thankful for the referees comments. We have revised the manuscript based on suggestions in referees report.

Page 1, abstract replaced by : The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In this paper we investigate the locating chromatic number for two families of barbell graphs.

Page 1, from 1 to 6 and on page 2 lines from 1 up to 7, replaced by : Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \to \{1, 2, \ldots, k\}$ where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$ where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$ where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [6]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Page 2, the text after Corollary 1.1 until Theorem 1.2., replaced by: There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand, et al. in [6] have determined all graphs of order n with locating chromatic number n, namely a complete multipartite graph of n vertices. Moreover, Chartrand et al. [7] have succeeded in constructing tree on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [5] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [3] obtained the locating chromatic number of the generalized Petersen graph $P(n, 1)$ for $n \geq 3$. Baskoro and Asmiati [4] have characterized all trees with locating-chromatic number 3. In [12] were characterized all trees of order n with locating chromatic number $n - t$,

for any integers n and t, where $n > t+3$ and $2 \leq t < \frac{n}{2}$. Asmiati et al. in [1] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [2] for firecracker graphs. Next, Wellyyanti et al. [14] determined the locating chromatic number for complete n-ary trees.
The generalized Petersen graph $P(n,m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [13]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Page 2 and several times later: Generalized Petersen graph $P_{n.1}$ is replaced by $P(n,1)$.

Page 3, Theorem 2.1. written by :Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.
Theorem 2.1 Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n+1$. .

Page 3, lines -10 and -11, replaced by:Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n+1$.

Page 3, lines -2, -3 and -4, replaced by: The labeling $c(v_i)$ and also all other labelings write by the following way

$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases}$$

Page 4 lines from -1 to -4 and on page 5 lines from 1 to 5, replaced by : Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i,$
$w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_i u_{i+1}, u_{n+i} u_{n+i+1}, w_i w_{i+1}, w_{n+i} w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1, u_{2n} u_{n+1}, w_n w_1, w_{2n} w_{n+1}\} \cup \{u_i u_{n+i}, w_i w_{n+i} : 1 \leq i \leq n\} \cup \{u_n w_n\}$.

Let us distinguish two cases.

Case 1, n odd. According to Theorem 1.3 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

Page 6, lines from -8 to -12, replaced by : *Case 2, n even.* In view of the lower bound from Theorem 1.3 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \to \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n \geq 4$, we describe the locating coloring in the following way:

Page 8, on the line 7, replaced by :

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, \ i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component}, \ i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component}, \ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 4^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component}, \ \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component}, \ i \text{ odd}, \ i \leq n-3 \\ & \text{for } 2^{nd} \text{ component}, \ i \text{ even}, \ i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n-1 \\ & \text{for } 2^{nd} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases}$$

Page 9: we have revised references.

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On the locating chromatic number of certain barbell graphs

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Abstract

The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion.

In this paper we investigate the locating chromatic number for two families of barbell graphs.

Keywords: locating chromatic number, barbell graph, complete graph, generalized Petersen graph

1 Introduction

The partition dimension was introduced by Chartrand et al. [8] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [11], the optimization of threat detecting sensors [10] and chemical data classification [9]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [6]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \rightarrow \{1, 2, \ldots, k\}$ where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$ where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G

is defined as the k-vector $(d(v,C_1), d(v,C_2), \ldots, d(v,C_k))$ where $d(v,C_i) = \min\{d(v,x) : x \in C_i\}$ for $1 \le i \le k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [6]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Theorem 1.1. [6] *Let c be a locating coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u,t) = d(v,t)$ for all $t \in V(G) - \{u,v\}$, then $c(u) \ne c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \ne c(v)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph G.

Corollary 1.1. [6] *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \ge k+1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand, et al. in [6] have determined all graphs of order n with locating chromatic number n, namely a complete multipartite graph of n vertices. Moreover, Chartrand et al. [7] have succeeded in constructing tree on n vertices, $n \ge 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [5] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [3] obtained the locating chromatic number of the generalized Petersen graph $P(n,1)$ for $n \ge 3$. Baskoro and Asmiati [4] have characterized all trees with locating-chromatic number 3. In [12] were characterized all trees of order n with locating chromatic number $n-t$, for any integers n and t, where $n > t+3$ and $2 \le t < \frac{n}{2}$. Asmiati et al. in [1] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [2] for firecracker graphs. Next, Wellyyanti et al. [14] determined the locating chromatic number for complete n-ary trees.

The generalized Petersen graph $P(n,m)$, $n \ge 3$ and $1 \le m \le \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \le i \le n$, and n edges $x_i x_{i+m}$, $1 \le i \le n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [13]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Theorem 1.2. [7] *For $n \ge 2$, the locating chromatic number of complete graph K_n is n.*

Theorem 1.3. [3] *The locating chromatic number of generalized Petersen Graph $P(n,1)$ is 4 for odd $n \ge 3$ or 5 for even $n \ge 4$.*

The *barbell graph* is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number for barbell graph $B_{m,n}$ for $m,n \ge 3$, where G and H are complete graphs on m and n vertices, respectively. Secondly, we determine the locating chromatic number of barbell graph $B_{P(n,1)}$ for $n \ge 3$, where G and H are two isomorphic copies of the generalized Petersen graph $P(n,1)$.

2 Results and Discussion

Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.

Theorem 2.1. *Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n + 1$.*

Proof Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1} \{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains two isomorphic copies of a complete graph K_n, then with respect to Theorem 1.2 we have that $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n + 1$.

To show that $n+1$ is an upper bound for the locating chromatic number of barbell graph $B_{n,n}$ it suffices to prove the existence of an optimal locating coloring $c : V(B_{n,n}) \to \{1, 2, \ldots, n+1\}$. For $n \geq 3$ we construct the function c in the following way:

$$c(u_i) = i, \qquad 1 \leq i \leq n$$

$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases}$$

By using the coloring c, we obtain the color codes of $V(B_{n,n})$ as follows:

$$c_\Pi(u_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 1 \leq i \leq n \\ 2, & \text{for } (n+1)^{th} \text{ component}, \ 1 \leq i \leq n-1 \\ 1, & \text{otherwise,} \end{cases}$$

$$c_\Pi(v_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 2 \leq i \leq n-1 \\ & \text{for } n^{th} \text{ component}, \ i = 1, \text{and} \\ & \text{for } (n+1)^{th} \text{ component}, \ i = n, \\ 3, & \text{for } 1^{st} \text{ component}, \ 1 \leq i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases}$$

Since all vertices in $V(B_{n,n})$ have distinct color codes, then the coloring c is desired locating coloring. Thus, $\chi_L(B_{n,n}) = n + 1$. □

Corollary 2.1. *For $n, m \geq 3$ and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is*

$$\chi_L(B_{m,n}) = \max\{m, n\}.$$

Next theorem provides the exact value of the locating chromatic number for barbell graph $B_{P(n,1)}$.

Theorem 2.2. *Let $B_{P(n,1)}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{P(n,1)}$ is*

$$\chi_L(B_{P(n,1)}) = \begin{cases} 4, & \text{for odd } n \\ 5, & \text{for even } n. \end{cases}$$

Proof Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_i u_{i+1}, u_{n+i} u_{n+i+1}, w_i w_{i+1}, w_{n+i} w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1, u_{2n} u_{n+1}, w_n w_1, w_{2n} w_{n+1}\} \cup \{u_i u_{n+i}, w_i w_{n+i} : 1 \leq i \leq n\} \cup \{u_n w_n\}$.

Let us distinguish two cases.

Case 1, n odd. According to Theorem 1.3 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

$$c(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i, \ i \geq 2 \\ 4, & \text{for odd } i, \ i \geq 3. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i, \ i \geq 3 \\ 4, & \text{for even } i, \ i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for odd } i, \ i \leq n-2 \\ 2, & \text{for even } i, \ i \leq n-1 \\ 3, & \text{for } i = n. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i, \ i \leq n-1 \\ 2, & \text{for odd } i, \ i \leq n-2 \\ 4, & \text{for } i = n. \end{cases}$$

For n odd the color codes of $V(B_{P(n,1)})$ are:

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ component}, \ i \leq \frac{n+1}{2} \\ i-1, & \text{for } 1^{st} \text{ component}, \ i \leq \frac{n+1}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component}, \ i > \frac{n+1}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component}, \ i > \frac{n+1}{2} \\ 0, & \text{for } 3^{th} \text{ component}, \ i \text{ even}, \ i \geq 2 \\ & \text{for } 4^{th} \text{ component}, \ i \text{ odd}, \ i \geq 3 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component}, \ i \le \frac{n+1}{2} \\ i-1, & \text{for } 2^{nd} \text{ component}, \ i \le \frac{n+1}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ component}, \ i > \frac{n+1}{2} \\ n-i+2, & \text{for } 1^{st} \text{ component}, \ i > \frac{n+1}{2} \\ 0, & \text{for } 4^{th} \text{ component}, \ i \text{ even}, \ i \ge 2 \\ & \text{for } 3^{th} \text{ component}, \ i \text{ odd}, \ i \ge 3 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 3^{th} \text{ component}, \ i \le \frac{n-1}{2} \\ i+1, & \text{for } 4^{th} \text{ component}, \ i \le \frac{n-1}{2} \\ n-i, & \text{for } 3^{th} \text{ component}, \ i \ge \frac{n+1}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component}, \ i \ge \frac{n+1}{2} \\ 0, & \text{for } 2^{nd} \text{ component}, \ i \text{ even}, \ i \le n-1 \\ & \text{for } 1^{st} \text{ component}, \ i \text{ odd}, \ i \le n-2 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, \ i \le \frac{n-1}{2} \\ i+1, & \text{for } 3^{th} \text{ component}, \ i \le \frac{n-1}{2} \\ n-i, & \text{for } 4^{th} \text{ component}, \ i \ge \frac{n+1}{2} \\ n-i+1, & \text{for } 3^{th} \text{ component}, \ i \ge \frac{n+1}{2} \\ 0, & \text{for } 1^{st} \text{ component}, \ i \text{ even}, \ i \le n-1 \\ & \text{for } 2^{nd} \text{ component}, \ i \text{ odd}, \ i \le n-2 \\ 1, & \text{otherwise.} \end{cases}$$

Since all vertices in $B_{P(n,1)}$ have distinct color codes, then the coloring c with 4 colors is an optimal locating coloring and it proves that $\chi_L(B_{P(n,1)}) \le 4$.

Case 2, n even. In view of the lower bound from Theorem 2.2 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \to \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n \ge 4$, we describe the locating coloring in the following way:

$$c(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i, \ 2 \le i \le n-2 \\ 4, & \text{for odd } i, \ 3 \le i \le n-1 \\ 5, & \text{for } i = n. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i, \ i \ge 3 \\ 4, & \text{for even } i, \ i \ge 2. \end{cases}$$

$$c(w_i)=\begin{cases}1, & \text{for odd } i,\ i\le n-3\\ 2, & \text{for even } i,\ i\le n-2\\ 3, & \text{for } i=n-1\\ 4, & \text{for } i=n.\end{cases}$$

$$c(w_{n+i})=\begin{cases}1, & \text{for even } i,\ i\le n-2\\ 2, & \text{for odd } i,\ i\le n-1\\ 5, & \text{for } i=n.\end{cases}$$

In fact, our locating coloring of $B_{P(n,1)}$, n even, has been chosen in such a way that the color codes are:

$$c_\Pi(u_i)=\begin{cases}i, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i\le\frac{n}{2}\\ i-1, & \text{for } 1^{st} \text{ component},\ i\le\frac{n}{2}\\ n-i, & \text{for } 5^{th} \text{ component},\ i>\frac{n}{2}\\ n-i+1, & \text{for } 1^{st} \text{ component},\ i>\frac{n}{2}\\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i>\frac{n}{2}\\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ 2\le i\le n-2\\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ 3\le i\le n-1\\ 2, & \text{for } 4^{th} \text{ component},\ i=1\\ & \text{for } 3^{th} \text{ component},\ i=n\\ 1, & \text{otherwise.}\end{cases}$$

$$c_\Pi(u_{n+i})=\begin{cases}i, & \text{for } 1^{st} \text{ component},\ i\le\frac{n}{2}\\ i-1, & \text{for } 2^{nd} \text{ component},\ i\le\frac{n}{2}\\ n+i, & \text{for } 5^{th} \text{ component},\ i\le\frac{n}{2}\\ n-i+1, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i>\frac{n}{2}\\ n-i+2, & \text{for } 1^{th} \text{ component},\ i>\frac{n}{2}\\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ 3\le i\le n-1\\ & \text{for } 4^{th} \text{ component},\ i \text{ even},\ 2\le i\le n\\ 2, & \text{for } 3^{th} \text{ component},\ i=1\\ 1, & \text{otherwise.}\end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, \ i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component}, \ i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component}, \ i \leq \frac{n}{2}-1 \\ n-i, & \text{for } 4^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component}, \ \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component}, \ i \text{ odd}, \ i \leq n-3 \\ & \text{for } 2^{nd} \text{ component}, \ i \text{ even}, \ i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n-1 \\ & \text{for } 2^{nd} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i, & \text{for } 5^{th} \text{ component}, \ i \leq \frac{n}{2} \\ i+1, & \text{for } 4^{th} \text{ component}, \ i \leq \frac{n}{2} \\ i+2 & \text{for } 3^{th} \text{ component}, \ i \leq \frac{n}{2}-1 \\ n-i, & \text{for } 3^{th} \text{ component}, \ \frac{n}{2} \leq i \leq n-1 \\ & \text{for } 5^{th} \text{ component}, \ i > \frac{n}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component}, \ i > \frac{n}{2} \\ 0, & \text{for } 1^{st} \text{ component}, \ i \text{ even}, \ i \leq n-2 \\ & \text{for } 2^{nd} \text{ component}, \ i \text{ odd}, \ i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ and } 3^{th} \text{ components}, \ i = n \\ 1, & \text{otherwise.} \end{cases}$$

Since for n even all vertices of $B_{P(n,1)}$ have distinct color codes then our locating coloring has the required properties and $\chi_L(B_{P(n,1)}) \leq 5$. This concludes the proof. □

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Research Article

On the Locating Chromatic Number of Certain Barbell Graphs

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The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In this paper we investigate the locating chromatic number for two families of barbell graphs.

1

1. Introduction

The partition dimension was introduced by Chartrand et al. [1] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [2], the optimization of threat detecting sensors [3], and chemical data classification [4]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [5]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \longrightarrow \{1, 2, \ldots, k\}$, where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$, where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$, where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [5]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Theorem 1 (see [5]). *Let c be a locating coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u, t) = d(v, t)$ for all $t \in V(G) - \{u, v\}$, then $c(u) \neq c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph G.

Corollary 2 (see [5]). *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \geq k + 1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand et al. in [5] have determined all graphs of order n with locating chromatic number n, namely, a complete multipartite graph of n vertices. Moreover, Chartrand et al.

[6] have succeeded in constructing tree on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [7] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [8] obtained the locating chromatic number of the generalized Petersen graph $P(n, 1)$ for $n \geq 3$. Baskoro and Asmiati [9] have characterized all trees with locating chromatic number 3. In [10] were characterized all trees of
2 order n with locating chromatic number $n-t$, for any integers n and t, where $n > t + 3$ and $2 \leq t < n/2$. Asmiati et al. in [11] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [12] for firecracker graphs. Next, Welyyanti et al. [13] determined the locating chromatic number for complete n-ary trees.

The generalized Petersen graph $P(n, m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [14]. Let us note that the generalized Petersen graph $P(n, 1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n, 1)$.

Theorem 3 (see [6]). *For $n \geq 2$, the locating chromatic number of complete graph K_n is n.*

Theorem 4 (see [8]). *The locating chromatic number of generalized Petersen graph $P(n, 1)$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.*

The *barbell graph* is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number for barbell graph $B_{m,n}$ for $m, n \geq 3$, where G and H are complete graphs on m and n vertices, respectively. Secondly, we determine the locating chromatic number of barbell graph $B_{P(n,1)}$ for $n \geq 3$, where G and H are two isomorphic copies of the generalized Petersen graph $P(n, 1)$.

2. Results and Discussion

Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.

Theorem 5. *Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n + 1$.*

Proof. Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1} \{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains two isomorphic copies of a complete graph K_n, then with respect to Theorem 3 we have $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n + 1$.

To show that $n + 1$ is an upper bound for the locating chromatic number of barbell graph $B_{n,n}$ it suffices to prove the existence of an optimal locating coloring $c : V(B_{n,n}) \longrightarrow \{1, 2, \ldots, n+1\}$. For $n \geq 3$ we construct the function c in the following way:

$$c(u_i) = i, \quad 1 \leq i \leq n$$
$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases} \tag{1}$$

By using the coloring c, we obtain the color codes of $V(B_{n,n})$ as follows:

$$c_\Pi(u_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 1 \leq i \leq n \\ 2, & \text{for } (n+1)^{th} \text{ component}, \ 1 \leq i \leq n-1 \\ 1, & \text{otherwise,} \end{cases}$$

$$c_\Pi(v_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 2 \leq i \leq n-1 \\ & \text{for } n^{th} \text{ component}, \ i = 1, \text{ and} \\ & \text{for } (n+1)^{th} \text{ component}, \ i = n, \\ 3, & \text{for } 1^{st} \text{ component}, \ 1 \leq i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases} \tag{2}$$

Since all vertices in $V(B_{n,n})$ have distinct color codes, then the coloring c is desired locating coloring. Thus, $\chi_L(B_{n,n}) = n + 1$. □

Corollary 6. *For $n, m \geq 3$, and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is*

$$\chi_L(B_{m,n}) = \max\{m, n\}. \tag{3}$$

Next theorem provides the exact value of the locating chromatic number for barbell graph $B_{P(n,1)}$.

Theorem 7. *Let $B_{P(n,1)}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{P(n,1)}$ is*

$$\chi_L(B_{P(n,1)}) = \begin{cases} 4, & \text{for odd } n \\ 5, & \text{for even } n. \end{cases} \tag{4}$$

Proof. Let $B_{P(n,1)}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)}) = \{u_i, u_{n+i}, w_i, w_{n+i} : 1 \leq i \leq n\}$ and the edge set $E(B_{P(n,1)}) = \{u_i u_{i+1}, u_{n+i} u_{n+i+1}, w_i w_{i+1}, w_{n+i} w_{n+i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1, u_{2n} u_{n+1}, w_n w_1, w_{2n} w_{n+1}\} \cup \{u_i u_{n+i}, w_i w_{n+i} : 1 \leq i \leq n\} \cup \{u_n w_n\}$.

Let us distinguish two cases.

Case 1 (n odd). According to Theorem 4 for n odd we have $\chi_L(B_{P(n,1)}) \geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

$$c(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i,\ i \geq 2 \\ 4, & \text{for odd } i,\ i \geq 3. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i,\ i \geq 3 \\ 4, & \text{for even } i,\ i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for odd } i,\ i \leq n-2 \\ 2, & \text{for even } i,\ i \leq n-1 \\ 3, & \text{for } i = n. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i,\ i \leq n-1 \\ 2, & \text{for odd } i,\ i \leq n-2 \\ 4, & \text{for } i = n. \end{cases} \tag{5}$$

For n odd the color codes of $V(B_{P(n,1)})$ are

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ component},\ i \leq \frac{n+1}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n+1}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \frac{n+1}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \frac{n+1}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ i \geq 2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ i \geq 3 \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n+1}{2} \\ i-1, & \text{for } 2^{nd} \text{ component},\ i \leq \frac{n+1}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ component},\ i > \frac{n+1}{2} \\ n-i+2, & \text{for } 1^{st} \text{ component},\ i > \frac{n+1}{2} \\ 0, & \text{for } 4^{th} \text{ component},\ i \text{ even},\ i \geq 2 \\ & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ i \geq 3 \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 3^{th} \text{ component},\ i \leq \frac{n-1}{2} \\ i+1, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n-1}{2} \\ n-i, & \text{for } 3^{th} \text{ component},\ i \geq \frac{n+1}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component},\ i \geq \frac{n+1}{2} \\ 0, & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i \leq n-1 \\ & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i \leq n-2 \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n-1}{2} \\ i+1, & \text{for } 3^{th} \text{ component},\ i \leq \frac{n-1}{2} \\ n-i, & \text{for } 4^{th} \text{ component},\ i \geq \frac{n+1}{2} \\ n-i+1, & \text{for } 3^{th} \text{ component},\ i \geq \frac{n+1}{2} \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i \leq n-1 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i \leq n-2 \\ 1, & \text{otherwise}. \end{cases} \tag{6}$$

Since all vertices in $B_{P(n,1)}$ have distinct color codes, then the coloring c with 4 colors is an optimal locating coloring and it proves that $\chi_L(B_{P(n,1)}) \leq 4$.

Case 2 (n even). In view of the lower bound from Theorem 7 it suffices to prove the existence of a locating coloring $c : V(B_{P(n,1)}) \longrightarrow \{1, 2, \ldots, 5\}$ such that all vertices in $B_{P(n,1)}$

have distinct color codes. For n even, $n \geq 4$, we describe the locating coloring in the following way:

$$c(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 3, & \text{for even } i,\ 2 \leq i \leq n-2 \\ 4, & \text{for odd } i,\ 3 \leq i \leq n-1 \\ 5, & \text{for } i = n. \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i,\ i \geq 3 \\ 4, & \text{for even } i,\ i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for odd } i,\ i \leq n-3 \\ 2, & \text{for even } i,\ i \leq n-2 \\ 3, & \text{for } i = n-1 \\ 4, & \text{for } i = n. \end{cases} \quad (7)$$

$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i,\ i \leq n-2 \\ 2, & \text{for odd } i,\ i \leq n-1 \\ 5, & \text{for } i = n. \end{cases}$$

In fact, our locating coloring of $B_{P(n,1)}$, n even, has been chosen in such a way that the color codes are

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i \leq \frac{n}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n}{2} \\ n-i, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ 2 \leq i \leq n-2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ 3 \leq i \leq n-1 \\ 2, & \text{for } 4^{th} \text{ component},\ i = 1 \\ & \text{for } 3^{th} \text{ component},\ i = n \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n}{2} \\ i-1, & \text{for } 2^{nd} \text{ component},\ i \leq \frac{n}{2} \\ n+i, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 1^{th} \text{ component}, i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ 3 \leq i \leq n-1 \\ & \text{for } 4^{th} \text{ component},\ i \text{ even},\ 2 \leq i \leq n \\ 2, & \text{for } 3^{th} \text{ component},\ i = 1 \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component},\ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 4^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component},\ \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i \leq n-3 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component},\ i = n-1 \\ & \text{for } 2^{nd} \text{ component},\ i = n \\ 1, & \text{otherwise}. \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+1, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+2 & \text{for } 3^{th} \text{ component},\ i \leq \frac{n}{2} - 1 \\ n-i, & \text{for } 3^{th} \text{ component},\ \frac{n}{2} \leq i \leq n-1 \\ & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i \leq n-2 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ and } 3^{th} \text{ components},\ i = n \\ 1, & \text{otherwise}. \end{cases} \quad (8)$$

Since for n even all vertices of $B_{P(n,1)}$ have distinct color codes then our locating coloring has the required properties and $\chi_L(B_{P(n,1)}) \leq 5$. This concludes the proof. □

Data Availability

The data used to support the findings of this study are
3 available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

On the Locating Chromatic Number of Certain Barbell Graphs

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The locating chromatic number of a graph G is defined as the cardinality of a minimum resolving partition of the vertex set $V(G)$ such that all vertices have distinct coordinates with respect to this partition and every two adjacent vertices in G are not contained in the same partition class. In this case, the coordinate of a vertex v in G is expressed in terms of the distances of v to all partition classes. This concept is a special case of the graph partition dimension notion. In this paper we investigate the locating chromatic number for two families of barbell graphs.

1. Introduction

The partition dimension was introduced by Chartrand et al. [1] as the development of the concept of metric dimension. The application of metric dimension plays a role in robotic navigation [2], the optimization of threat detecting sensors [3], and chemical data classification [4]. The concept of locating chromatic number is a marriage between the partition dimension and coloring of a graph, first introduced by Chartrand et al in 2002 [5]. The locating chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating chromatic number of any graph.

Let $G = (V, E)$ be a connected graph. We define the *distance* as the minimum length of path connecting vertices u and v in G, denoted by $d(u, v)$. A k-coloring of G is a function $c : V(G) \longrightarrow \{1, 2, \ldots, k\}$, where $c(u) \neq c(v)$ for any two adjacent vertices u and v in G. Thus, the coloring c induces a partition Π of $V(G)$ into k color classes (independent sets) $C_1, C_2, \ldots, C_k$, where C_i is the set of all vertices colored by the color i for $1 \leq i \leq k$. The *color code* $c_\Pi(v)$ of a vertex v in G is defined as the k-vector $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$, where $d(v, C_i) = \min\{d(v, x) : x \in C_i\}$ for $1 \leq i \leq k$. The k-coloring c of G such that all vertices have different color codes is called a *locating coloring* of G. The *locating chromatic number* of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring.

The following theorem is a basic theorem proved by Chartrand et al. [5]. The neighborhood of vertex u in a connected graph G, denoted by $N(u)$, is the set of vertices adjacent to u.

Theorem 1 (see [5]). *Let c be a locating coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u, t) = d(v, t)$ for all $t \in V(G) - \{u, v\}$, then $c(u) \neq c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

The following corollary gives the lower bound of the locating chromatic number for every connected graph G.

Corollary 2 (see [5]). *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G) \geq k + 1$.*

There are some interesting results related to the determination of the locating chromatic number of some graphs. The results are obtained by focusing on certain families of graphs. Chartrand et al. in [5] have determined all graphs of order n with locating chromatic number n, namely, a complete multipartite graph of n vertices. Moreover, Chartrand et al. [6] have succeeded in constructing tree on n vertices, $n \geq 5$, with locating chromatic numbers varying from 3 to n, except for $(n-1)$. Then Behtoei and Omoomi [7] have obtained the locating chromatic number of the Kneser graphs. Recently, Asmiati et al. [8] obtained the locating chromatic number of the generalized Petersen graph $P(n,1)$ for $n \geq 3$. Baskoro and Asmiati [9] have characterized all trees with locating chromatic number 3. In [10] all trees of order n with locating chromatic number $n-1$ were characterized, for any integers n and t, where $n > t+3$ and $2 \leq t < n/2$. Asmiati et al. in [11] have succeeded in determining the locating chromatic number of homogeneous amalgamation of stars and their monotonicity properties and in [12] for firecracker graphs. Next, Wellyyanti et al. [13] determined the locating chromatic number for complete n-ary trees.

The generalized Petersen graph $P(n,m)$, $n \geq 3$ and $1 \leq m \leq \lfloor (n-1)/2 \rfloor$, consists of an outer n-cycle $y_1, y_2, \ldots, y_n$, a set of n spokes $y_i x_i$, $1 \leq i \leq n$, and n edges $x_i x_{i+m}$, $1 \leq i \leq n$, with indices taken modulo n. The generalized Petersen graph was introduced by Watkins in [14]. Let us note that the generalized Petersen graph $P(n,1)$ is a prism defined as Cartesian product of a cycle C_n and a path P_2.

Next theorems give the locating chromatic numbers for complete graph K_n and generalized Petersen graph $P(n,1)$.

Theorem 3 (see [6]). *For $n \geq 2$, the locating chromatic number of complete graph K_n is n.*

Theorem 4 (see [8]). *The locating chromatic number of generalized Petersen graph $P(n,1)$ is 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.*

The *barbell graph* is constructed by connecting two arbitrary connected graphs G and H by a bridge. In this paper, firstly we discuss the locating chromatic number for barbell graph $B_{m,n}$ for $m,n \geq 3$, where G and H are complete graphs on m and n vertices, respectively. Secondly, we determine the locating chromatic number of barbell graph $B_{P(n,1)}$ for $n \geq 3$, where G and H are two isomorphic copies of the generalized Petersen graph $P(n,1)$.

2. Results and Discussion

Next theorem proves the exact value of the locating chromatic number for barbell graph $B_{n,n}$.

Theorem 5. *Let $B_{n,n}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{n,n}$ is $\chi_L(B_{n,n}) = n+1$.*

Proof. Let $B_{n,n}$, $n \geq 3$, be the barbell graph with the vertex set $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $E(B_{n,n}) = \bigcup_{i=1}^{n-1} \{u_i u_{i+j} : 1 \leq j \leq n-i\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+j} : 1 \leq j \leq n-i\} \cup \{u_n v_n\}$.

First, we determine the lower bound of the locating chromatic number for barbell graph $B_{n,n}$ for $n \geq 3$. Since the barbell graph $B_{n,n}$ contains two isomorphic copies of a complete graph K_n, then with respect to Theorem 3 we have $\chi_L(B_{n,n}) \geq n$. Next, suppose that c is a locating coloring using n colors. It is easy to see that the barbell graph $B_{n,n}$ contains two vertices with the same color codes, which is a contradiction. Thus, we have that $\chi_L(B_{n,n}) \geq n+1$.

To show that $n+1$ is an upper bound for the locating chromatic number of barbell graph $B_{n,n}$ it suffices to prove the existence of an optimal locating coloring $c : V(B_{n,n}) \longrightarrow \{1, 2, \ldots, n+1\}$. For $n \geq 3$ we construct the function c in the following way:

$$c(u_i) = i, \quad 1 \leq i \leq n$$
$$c(v_i) = \begin{cases} n, & \text{for } i = 1 \\ i, & \text{for } 2 \leq i \leq n-1 \\ n+1, & \text{otherwise.} \end{cases} \tag{1}$$

By using the coloring c, we obtain the color codes of $V(B_{n,n})$ as follows:

$$c_\Pi(u_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 1 \leq i \leq n \\ 2, & \text{for } (n+1)^{th} \text{ component}, \ 1 \leq i \leq n-1 \\ 1, & \text{otherwise,} \end{cases}$$
$$c_\Pi(v_i) = \begin{cases} 0, & \text{for } i^{th} \text{ component}, \ 2 \leq i \leq n-1 \\ & \text{for } n^{th} \text{ component}, \ i = 1, \text{ and} \\ & \text{for } (n+1)^{th} \text{ component}, \ i = n, \\ 3, & \text{for } 1^{st} \text{ component}, \ 1 \leq i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ component}, \ i = n \\ 1, & \text{otherwise.} \end{cases} \tag{2}$$

Since all vertices in $V(B_{n,n})$ have distinct color codes, then the coloring c is desired locating coloring. Thus, $\chi_L(B_{n,n}) = n+1$. □

Corollary 6. *For $n,m \geq 3$, and $m \neq n$, the locating chromatic number of barbell graph $B_{m,n}$ is*

$$\chi_L(B_{m,n}) = \max\{m, n\}. \tag{3}$$

Next theorem provides the exact value of the locating chromatic number for barbell graph $B_{P(n,1)}$.

Theorem 7. *Let $B_{P(n,1)}$ be a barbell graph for $n \geq 3$. Then the locating chromatic number of $B_{P(n,1)}$ is*

$$\chi_L\left(B_{P(n,1)}\right)=\begin{cases}4, & \textit{for odd } n\\ 5, & \textit{for even } n.\end{cases} \tag{4}$$

Proof. Let $B_{P(n,1)}$, $n\geq 3$, be the barbell graph with the vertex set $V(B_{P(n,1)})=\{u_i,u_{n+i},w_i,w_{n+i}:1\leq i\leq n\}$ and the edge set $E(B_{P(n,1)})=\{u_iu_{i+1},u_{n+i}u_{n+i+1},w_iw_{i+1},w_{n+i}w_{n+i+1}:1\leq i\leq n-1\}\cup\{u_nu_1,u_{2n}u_{n+1},w_nw_1,w_{2n}w_{n+1}\}\cup\{u_iu_{n+i},w_iw_{n+i}:1\leq i\leq n\}\cup\{u_nw_n\}$.

Let us distinguish two cases.

Case 1 (n odd). According to Theorem 4 for n odd we have $\chi_L(B_{P(n,1)})\geq 4$. To show that 4 is an upper bound for the locating chromatic number of the barbell graph $B_{P(n,1)}$ we describe an locating coloring c using 4 colors as follows:

$$\begin{gathered}
c\left(u_i\right)=\begin{cases}1, & \text{for } i=1\\ 3, & \text{for even } i,\ i\geq 2\\ 4, & \text{for odd } i,\ i\geq 3.\end{cases}\\
c\left(u_{n+i}\right)=\begin{cases}2, & \text{for } i=1\\ 3, & \text{for odd } i,\ i\geq 3\\ 4, & \text{for even } i,\ i\geq 2.\end{cases}\\
c\left(w_i\right)=\begin{cases}1, & \text{for odd } i,\ i\leq n-2\\ 2, & \text{for even } i,\ i\leq n-1\\ 3, & \text{for } i=n.\end{cases}\\
c\left(w_{n+i}\right)=\begin{cases}1, & \text{for even } i,\ i\leq n-1\\ 2, & \text{for odd } i,\ i\leq n-2\\ 4, & \text{for } i=n.\end{cases}
\end{gathered} \tag{5}$$

For n odd the color codes of $V(B_{P(n,1)})$ are

$$\begin{gathered}
c_\Pi\left(u_i\right)\\
=\begin{cases}i, & \text{for } 2^{nd} \text{ component},\ i\leq \dfrac{n+1}{2}\\ i-1, & \text{for } 1^{st} \text{ component},\ i\leq \dfrac{n+1}{2}\\ n-i+1, & \text{for } 1^{st} \text{ component},\ i> \dfrac{n+1}{2}\\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i> \dfrac{n+1}{2}\\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ i\geq 2\\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ i\geq 3\\ 1, & \text{otherwise}.\end{cases}\\
c_\Pi\left(u_{n+i}\right)\\
=\begin{cases}i, & \text{for } 1^{st} \text{ component},\ i\leq \dfrac{n+1}{2}\\ i-1, & \text{for } 2^{nd} \text{ component},\ i\leq \dfrac{n+1}{2}\\ n-i+1, & \text{for } 2^{nd} \text{ component},\ i> \dfrac{n+1}{2}\\ n-i+2, & \text{for } 1^{st} \text{ component},\ i> \dfrac{n+1}{2}\\ 0, & \text{for } 4^{th} \text{ component},\ i \text{ even},\ i\geq 2\\ & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ i\geq 3\\ 1, & \text{otherwise}.\end{cases}\\
c_\Pi\left(w_i\right)\\
=\begin{cases}i, & \text{for } 3^{th} \text{ component},\ i\leq \dfrac{n-1}{2}\\ i+1, & \text{for } 4^{th} \text{ component},\ i\leq \dfrac{n-1}{2}\\ n-i, & \text{for } 3^{th} \text{ component},\ i\geq \dfrac{n+1}{2}\\ n-i+1, & \text{for } 4^{th} \text{ component},\ i\geq \dfrac{n+1}{2}\\ 0, & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i\leq n-1\\ & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i\leq n-2\\ 1, & \text{otherwise}.\end{cases}\\
c_\Pi\left(w_{n+i}\right)\\
=\begin{cases}i, & \text{for } 4^{th} \text{ component},\ i\leq \dfrac{n-1}{2}\\ i+1, & \text{for } 3^{th} \text{ component},\ i\leq \dfrac{n-1}{2}\\ n-i, & \text{for } 4^{th} \text{ component},\ i\geq \dfrac{n+1}{2}\\ n-i+1, & \text{for } 3^{th} \text{ component},\ i\geq \dfrac{n+1}{2}\\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i\leq n-1\\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i\leq n-2\\ 1, & \text{otherwise}.\end{cases}
\end{gathered} \tag{6}$$

Since all vertices in $B_{P(n,1)}$ have distinct color codes, then the coloring c with 4 colors is an optimal locating coloring and it proves that $\chi_L(B_{P(n,1)})\leq 4$.

Case 2 (n even). In view of the lower bound from Theorem 7 it suffices to prove the existence of a locating coloring $c: V(B_{P(n,1)})\longrightarrow\{1,2,\ldots,5\}$ such that all vertices in $B_{P(n,1)}$ have distinct color codes. For n even, $n\geq 4$, we describe the locating coloring in the following way:

$$c\left(u_i\right)=\begin{cases}1, & \text{for } i=1\\ 3, & \text{for even } i,\ 2\leq i\leq n-2\\ 4, & \text{for odd } i,\ 3\leq i\leq n-1\\ 5, & \text{for } i=n.\end{cases}$$

$$c(u_{n+i}) = \begin{cases} 2, & \text{for } i = 1 \\ 3, & \text{for odd } i,\ i \geq 3 \\ 4, & \text{for even } i,\ i \geq 2. \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for odd } i,\ i \leq n-3 \\ 2, & \text{for even } i,\ i \leq n-2 \\ 3, & \text{for } i = n-1 \\ 4, & \text{for } i = n. \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 1, & \text{for even } i,\ i \leq n-2 \\ 2, & \text{for odd } i,\ i \leq n-1 \\ 5, & \text{for } i = n. \end{cases} \tag{7}$$

In fact, our locating coloring of $B_{P(n,1)}$, n even, has been chosen in such a way that the color codes are

$$c_\Pi(u_i) = \begin{cases} i, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i \leq \frac{n}{2} \\ i-1, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n}{2} \\ n-i, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 2^{nd} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ even},\ 2 \leq i \leq n-2 \\ & \text{for } 4^{th} \text{ component},\ i \text{ odd},\ 3 \leq i \leq n-1 \\ 2, & \text{for } 4^{th} \text{ component},\ i = 1 \\ & \text{for } 3^{th} \text{ component},\ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component},\ i \leq \frac{n}{2} \\ i-1, & \text{for } 2^{nd} \text{ component},\ i \leq \frac{n}{2} \\ n+i, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ n-i+1, & \text{for } 2^{nd} \text{ and } 5^{th} \text{ components},\ i > \frac{n}{2} \\ n-i+2, & \text{for } 1^{th} \text{ component}, i > \frac{n}{2} \\ 0, & \text{for } 3^{th} \text{ component},\ i \text{ odd},\ 3 \leq i \leq n-1 \\ & \text{for } 4^{th} \text{ component},\ i \text{ even},\ 2 \leq i \leq n \\ 2, & \text{for } 3^{th} \text{ component},\ i = 1 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+1, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ & \text{for } 3^{th} \text{ component},\ i \leq \frac{n}{2}-1 \\ n-i, & \text{for } 4^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i-1, & \text{for } 3^{th} \text{ component},\ \frac{n}{2} \leq i \leq n-1 \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ odd},\ i \leq n-3 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ even},\ i \leq n-2 \\ 2, & \text{for } 1^{st} \text{ component},\ i = n-1 \\ & \text{for } 2^{nd} \text{ component},\ i = n \\ 1, & \text{otherwise.} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} i, & \text{for } 5^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+1, & \text{for } 4^{th} \text{ component},\ i \leq \frac{n}{2} \\ i+2 & \text{for } 3^{th} \text{ component},\ i \leq \frac{n}{2}-1 \\ n-i, & \text{for } 3^{th} \text{ component},\ \frac{n}{2} \leq i \leq n-1 \\ & \text{for } 5^{th} \text{ component},\ i > \frac{n}{2} \\ n-i+1, & \text{for } 4^{th} \text{ component},\ i > \frac{n}{2} \\ 0, & \text{for } 1^{st} \text{ component},\ i \text{ even},\ i \leq n-2 \\ & \text{for } 2^{nd} \text{ component},\ i \text{ odd},\ i \leq n-1 \\ 2, & \text{for } 1^{st} \text{ and } 3^{th} \text{ components},\ i = n \\ 1, & \text{otherwise.} \end{cases} \tag{8}$$

Since for n even all vertices of $B_{P(n,1)}$ have distinct color codes then our locating coloring has the required properties and $\chi_L(B_{P(n,1)}) \leq 5$. This concludes the proof. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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