

PAPER NAME

Characteristics of Bayes Estimator in the Geometric Distribution with Prior Beta.p df

AUTHOR

T. Susilo

WORD COUNT

3109 Words

CHARACTER COUNT

12977 Characters

PAGE COUNT

10 Pages

FILE SIZE

608.2KB

SUBMISSION DATE

Oct 17, 2022 9:05 PM GMT+7

REPORT DATE

Oct 17, 2022 9:06 PM GMT+7

● 20% Overall Similarity

The combined total of all matches, including overlapping sources, for each database.

- 11% Internet database
- 10% Publications database
- Crossref database
- Crossref Posted Content database
- 15% Submitted Works database

● Excluded from Similarity Report

- Bibliographic material
- Quoted material
- Cited material
- Small Matches (Less than 10 words)
- Manually excluded sources

PAPER • OPEN ACCESS

2 Characteristics of Bayes Estimator in the Geometric Distribution with Prior Beta

5 To cite this article: T Susilo *et al* 2021 *J. Phys.: Conf. Ser.* **1751** 012020

View the [article online](#) for updates and enhancements.



The banner features a decorative top border with a repeating pattern of red, white, and blue diagonal stripes. On the left, the ECS logo is displayed in green and blue, followed by the text 'The Electrochemical Society' and 'Advancing solid state & electrochemical science & technology'. To the right of this text is a logo for the 18th International Meeting of Chemical Solid State Ionics (IMCS18). The main text of the banner reads '239th ECS Meeting with IMCS18', 'DIGITAL MEETING • May 30-June 3, 2021', and 'Live events daily • Free to register'. On the right side, there is a red button with the text 'Register now!'. The background of the banner is a collage of images including a person's face, a laptop, and abstract digital network patterns.

ECS The Electrochemical Society
Advancing solid state & electrochemical science & technology

239th ECS Meeting with IMCS18

DIGITAL MEETING • May 30-June 3, 2021

Live events daily • Free to register

Register now!

2 Characteristics of Bayes Estimator in the Geometric Distribution with Prior Beta

T Susilo¹, Widiarti^{1,a}, D Kurniasari¹, D Aziz¹

¹Departement of Mathematics, Faculty of Mathematics and Natural Sciences, University of Lampung, Indonesia, Jl. Soemantri Brojonegoro No. 1, Bandar Lampung, Indonesia.

email: widiarti08@gmail.com^a

Abstract. This study aims to examine the unbiased, minimum variance (efficient), and consistent characteristics of Bayes estimator in the Geometric distribution with prior Beta. Based on the results of simulation studies it is found that the Bayes estimator in the Geometric distribution with prior Beta are symptomatically unbiased estimator for values $\theta < 0,5$ and is biased for others, are efficient for the number of samples sizes large and values $\theta \leq 0,6$ and not efficient for others and consistent when value $\theta \leq 0,5$ and inconsistent for other.

Keyword: Bayes estimator, unbiased, efficient, consistent, geometric distribution.

1. Introduction

Bayes method is a method that combines the likelihood function and prior distribution of parameters to get the posterior distribution which is the basis in the estimation parameters [1]. Estimation of parameters using the Bayes method has been widely applied in various fields of research, namely spatial analysis, small area estimation, survival analysis and others such as those carried out by Widiarti et al., Rizki et al., Hartono et al., Yanuar et al., Fulop and Li, Kinyanjui and Korir, Puspongoro and Rachmawati, Aw and Cabral [2-10]. In the Bayes method, parameter estimation is done by looking at unknown parameters as random variables that have an initial distribution of these parameters, this distribution is called a prior distribution[11].

Prior distributions are usually obtained based on the subjective beliefs of the researchers themselves. However, for the distribution of samples from the Exponential distribution family, one way is to use the prior conjugate [11]. Geometric distribution is a probability distribution with a success event symbolized by θ and a failure event is denoted by $(1 - \theta)$ and is a distribution that comes from the family of Exponential distribution. The random variable of this distribution is the amount of effort needed to get the first success. One of the prior conjugates for the Exponential distribution family with parameters stating probability is the Beta distribution. The Beta distribution is a continuous distribution at interval $(0,1)$ and has two parameters namely α and β [12].

Characteristics of estimators need to be examined in order to see the quality of an estimator. Estimator is called a good estimator if it meets certain characteristics. Some characteristics of estimators are



unbiased, efficient, consistent, sufficient statistics and complete statistics. An estimator is consistent for its parameters if the sample size gets larger then the estimator will approach the parameter and if the sample size becomes infinite then the estimator will be equal to the parameter or the Mean Square Error (MSE) of the estimator is equal to zero [13]. Thus, this study will examine the quality of Bayes estimators in the Geometric distribution with prior Beta. The characteristics studied are unbiased, efficient and consistent using MSE.

2. Method

2.1. Research Data

The data used in this are simulation data generated using R-Studio Software version 1.2.1.335. The data generated by scenario $\theta \sim \text{Beta}(\alpha, \beta)$ and $y \sim \text{Geometrik}(\theta)$ as well as with a varying number of sample sizes is 20, 50, 100, 500 and 1000. Paired values (α, β) are set as many as nine pairs is (2,18), (2,8), (2,5), (2,3), (2,2), (3,2), (5,2), (8,2) and (18,2) which results θ value is 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9.

2.2. Bayes Method

Bayes method is a method that combines the likelihood function and prior distribution of parameters to get the posterior distribution which is the basis in the estimation parameters [1]. If B_1, B_2, \dots, B_k is an exclusive and complete event, then for each occurrence of B_i conditional A is:

$$P(B_i|A) = \frac{P(B_i \cap A)}{\sum_{i=1}^k P(B_i \cap A)} = \frac{P(B_i)P(A|B_i)}{\sum_{i=1}^k P(B_i)P(A|B_i)} \tag{1}$$

Equation 1 is Bayes rule [14].

2.3. Unbiased

The estimator $\hat{\theta}$ is said to be unbiased for θ if the expected value is the same as the parameter of the probability distribution parameter.

$$E(\hat{\theta}) = \theta \tag{2}$$

and it is said to be asymptotically biased if the estimator is biased but when the sample size is greater the expected value is the same as the parameter

$$\lim_{n \rightarrow \infty} E(\hat{\theta}) = \theta \tag{3}$$

or

$$\lim_{n \rightarrow \infty} \text{Biased}(\hat{\theta}) = 0 \tag{4}$$

with

$$\text{Biased}(\hat{\theta}) = E(\hat{\theta}) - \theta \tag{5}$$

and n is the number of sizes sample [13].

2.4. Efficient

The estimator $\hat{\theta}$ is said to be efficient for θ parameter if the estimator $\hat{\theta}$ has the smallest variance. If $\hat{\theta}_1$ and $\hat{\theta}_2$ are estimator for θ , then estimator $\hat{\theta}_1$ is said to be more efficient than estimator $\hat{\theta}_2$ if:

$$\text{Var}(\hat{\theta}_1) \leq \text{Var}(\hat{\theta}_2) \tag{6}$$

for $\hat{\theta}_1$ and $\hat{\theta}_2$ is unbiased estimator for θ [14]. If $\hat{\theta}$ is an unbiased estimator for θ , then the estimator $\hat{\theta}$ is said to be efficient if and only if the variance of the estimator $\hat{\theta}$ reaches the Rao-Cramer lower bound.

$$\text{Var}(\hat{\theta}) \geq \frac{1}{nI(\theta)} \tag{7}$$

with $\frac{1}{nI(\theta)}$ is the Rao-Cramer lower bound and $I(\theta)$ Fisher's information [15].

$$I(\theta) = E \left\{ \left[\frac{\partial \ln f(x; \theta)}{\partial \theta} \right]^2 \right\} \tag{8}$$

or

$$I(\theta) = -E \left\{ \left[\frac{\partial^2 \ln f(x; \theta)}{\partial \theta^2} \right] \right\}$$

2.5. Consistent

The estimator $\hat{\theta}$ is said to be consistent for θ parameter if the estimator is convergent in probability θ or if the sample size gets larger then the estimator will approach the parameter and if the sample size becomes infinite then the estimator will be equal to the parameter.

$$\lim_{n \rightarrow \infty} P(|\hat{\theta}_n - \theta| \geq \varepsilon) = 0 \tag{9}$$

atau

$$\lim_{n \rightarrow \infty} E((\hat{\theta} - \theta)^2) = 0$$

for every $\varepsilon > 0$ [16].

The consistency of an estimator also can be evaluated based on the value of MSE. if $\hat{\theta}$ is an estimator of parameter θ , then the MSE for $\hat{\theta}$ is defined as follows:

$$\begin{aligned} MSE(\hat{\theta}) &= E[\hat{\theta} - E(\hat{\theta})]^2 + [E(\hat{\theta}) - \theta]^2 \\ &= Var(\hat{\theta}) + [Bias(\hat{\theta})]^2 \end{aligned} \tag{11}$$

If $\hat{\theta}$ is the an unbiased estimator then MSE will be equal to the variance $\hat{\theta}$ [13].

3. Result and Discussion

3.1 Bayes Estimator

In the Bayes Estimator, the probability function $f(y_i|\theta)$ is expressed by the conditional probability function $f(y_i|\theta)$, so that for Y_1, \dots, Y_n a random sample of a population with a Geometric distribution with parameter θ can be written as follows:

$$Y_i \sim Geo(\theta) \Leftrightarrow f(y_i|p) = \begin{cases} \theta(1-\theta)^{y-1} & ; y_i = 1, 2, 3, \dots; i = 1, 2, \dots, n; 0 \leq \theta \leq 1 \\ 0 & ; y_i \text{ lainnya} \end{cases} \tag{12}$$

The prior distribution for $Y_i \sim Geo(\theta)$, is $\theta \sim Beta(\alpha, \beta)$. So the probability function of the prior distribution is

$$f(\theta) = \begin{cases} \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1} & ; 0 < \theta < 1 \\ 0 & ; \theta \text{ lainnya} \end{cases} \tag{13}$$

The likelihood function of $Y_i \sim Geo(\theta)$:

$$f(y_1, y_2, \dots, y_n | \theta) = \prod_{i=1}^n \theta(1-\theta)^{y_i-1} = \theta^n (1-\theta)^{\sum_{i=1}^n y_i - n} \tag{14}$$

Joint probability density function of Y_1, \dots, Y_n and θ is:

$$\begin{aligned}
 f(y_1, y_2, \dots, y_n; \theta) &= f(y_1, y_2, \dots, y_n | \theta) f(\theta) \\
 &= \theta^n (1 - \theta)^{\sum_{i=1}^n y_i - n} \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1} (1 - \theta)^{\beta-1} \\
 &= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha+n-1} (1 - \theta)^{\beta + \sum_{i=1}^n y_i - n - 1}
 \end{aligned} \tag{15}$$

Marginal function of Y_i is:

$$\begin{aligned}
 m(y_1, y_2, \dots, y_n) &= \int_{-\infty}^{\infty} f(y_1, y_2, \dots, y_n; \theta) d\theta \\
 &= \int_0^1 \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha+n-1} (1 - \theta)^{\beta + \sum_{i=1}^n y_i - n - 1} \\
 &= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha + n)\Gamma(\beta + \sum_{i=1}^n y_i - n)}{\Gamma(\alpha + \beta + \sum_{i=1}^n y_i)}
 \end{aligned} \tag{16}$$

The posterior distribution is:

$$\begin{aligned}
 f(\theta | y_1, y_2, \dots, y_n) &= \frac{f(y_1, y_2, \dots, y_n; \theta)}{m(y_1, y_2, \dots, y_n)} \\
 &= \frac{\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha+n-1} (1 - \theta)^{\beta + \sum_{i=1}^n y_i - n - 1}}{\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha + n)\Gamma(\beta + \sum_{i=1}^n y_i - n)}{\Gamma(\alpha + \beta + \sum_{i=1}^n y_i)}} \\
 &= \frac{\Gamma(\alpha + \beta + \sum_{i=1}^n y_i)}{\Gamma(\alpha + n)\Gamma(\beta + \sum_{i=1}^n y_i - n)} \theta^{\alpha+n-1} (1 - \theta)^{\beta + \sum_{i=1}^n y_i - n - 1}
 \end{aligned} \tag{17}$$

So, the Bayes estimator for θ is:

$$\begin{aligned}
 \hat{\theta}^B &= E(f(\theta | y_1, y_2, \dots, y_n)) = \int_0^1 \theta f(\theta | y_1, y_2, \dots, y_n) d\theta \\
 &= \int_0^1 \theta \frac{\Gamma(\alpha + \beta + \sum_{i=1}^n y_i)}{\Gamma(\alpha + n)\Gamma(\beta + \sum_{i=1}^n y_i - n)} \theta^{\alpha+n-1} (1 - \theta)^{\beta + \sum_{i=1}^n y_i - n - 1} d\theta \\
 &= \frac{\alpha + n}{\alpha + \beta + \sum_{i=1}^n y_i}
 \end{aligned} \tag{18}$$

3.2 Unbiased

The estimator $\hat{\theta}^B$ is said to be unbiased for θ if $E(\hat{\theta}^B) = \theta$.

$$E(\hat{\theta}^B) = E\left(\frac{\alpha + n}{\alpha + \beta + \sum_{i=1}^n Y_i}\right)$$

$$= \frac{(\alpha + n)\theta}{\alpha\theta + \beta\theta + n} \tag{19}$$

Because $E(\hat{\theta}^B) \neq \theta$, then $\hat{\theta}^B$ is biased estimator for θ . But, $\hat{\theta}^B$ is asymptotically unbiased because:

$$\begin{aligned} \lim_{n \rightarrow \infty} E(\hat{\theta}^B) &= \lim_{n \rightarrow \infty} \frac{\alpha\theta + n\theta}{\alpha\theta + \beta\theta + n} \\ &= \lim_{n \rightarrow \infty} \frac{\frac{\alpha\theta}{n} + \frac{n\theta}{n}}{\frac{\alpha\theta}{n} + \frac{\beta\theta}{n} + \frac{n}{n}} \\ &= \frac{0 + \theta}{0 + 0 + 1} = \theta \end{aligned} \tag{20}$$

3.3 Efficient

The estimator $\hat{\theta}^B$ is efficient for θ parameter if the estimator $\hat{\theta}^B$ has the smallest variance.

$$\begin{aligned} Var(\hat{\theta}^B) &= Var\left(\frac{\alpha + n}{\alpha + \beta + \sum_{i=1}^n Y_i}\right) \\ &= (\alpha + n)^2 Var\left(\frac{1}{\alpha + \beta + \sum_{i=1}^n Y_i}\right) \end{aligned} \tag{21}$$

Because $Var\left(\frac{1}{\alpha + \beta + \sum_{i=1}^n Y_i}\right)$ cannot be solved analytically, the value of variance will be shown by simulation using R-Studio software version 1.2.1335.

3.4 Consistent

The estimator $\hat{\theta}^B$ is consistent for θ if $\lim_{n \rightarrow \infty} E((\hat{\theta} - \theta)^2) = \lim_{n \rightarrow \infty} MSE(\hat{\theta}^B) = 0$.

$$\begin{aligned} \lim_{n \rightarrow \infty} E((\hat{\theta}^B - \theta)^2) &= \lim_{n \rightarrow \infty} MSE(\hat{\theta}^B) \\ &= \lim_{n \rightarrow \infty} \{Var(\hat{\theta}^B) + [bias(\hat{\theta}^B)]^2\} \end{aligned} \tag{22}$$

Because $Var(\hat{\theta}^B)$ cannot be solved analytically, the value of MSE will be shown by simulation using R-Studio software version 1.2.1335

3.5 Simulation

Data simulation using R-Studio Software version 1.2.1.335 with several pairs of (α, β) and varying number of sample sizes yields the bias, variance and MSE values of the Bayes estimator for the Geometric distribution. Tables 1 to 3 display the simulation results for these values.

Table 1. The bias value of the Bayes estimator

α	β	θ	n	Bias $\hat{\theta}^B$	α	β	θ	n	Bias $\hat{\theta}^B$	α	β	θ	n	Bias $\hat{\theta}^B$
			20	0,015606				20	0,270064				20	1,056814
2	18	0,1	50	0,013049	2	3	0,4	50	0,269691	5	2	0,7	50	1,460895
			100	0,012367				100	0,265345				100	1,620098

	500	0,011222		500	0,263759		500	1,789975
	1000	0,011024		1000	0,261983		1000	1,805650
	20	0,055389		20	0,487611		20	1,121733
	50	0,054096		50	0,504101		50	1,887139
2	8	0,2	2	2	0,5	8	2	0,8
	100	0,051252		100	0,500914		100	2,404464
	500	0,049785		500	0,503232		500	3,044202
	1000	0,049275		1000	0,500697		1000	3,140313
	20	0,119766		20	0,748747		20	0,816880
	50	0,118141		50	0,855411		50	1,793407
2	5	0,3	3	2	0,6	18	2	0,9
	100	0,113565		100	0,881539		100	2,959372
	500	0,110819		500	0,911317		500	6,086961
	1000	0,110019		1000	0,911573		1000	6,975545

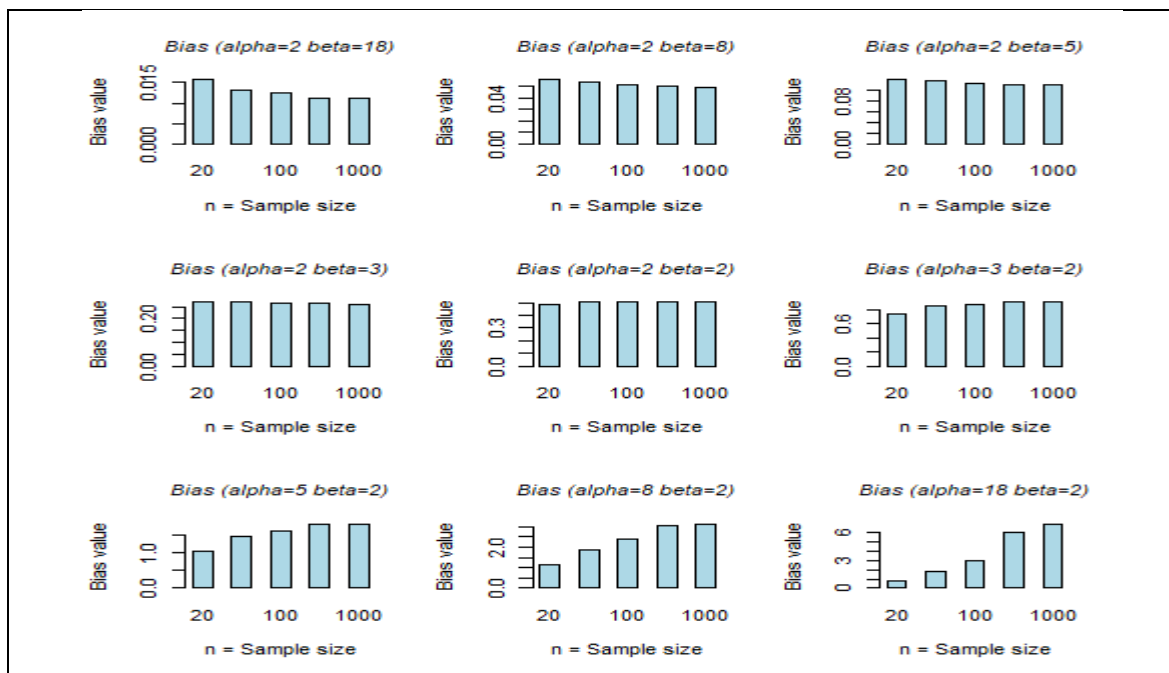


Figure 1. Histogram of the bias value.

Based on Table 1 and Figure 1, it can be seen that the bias value of Bayes estimator for $\theta < 0,5$ gets smaller when the number of sample sizes (n) gets bigger and vice versa for $\theta > 0,5$ and fluctuative for $\theta = 0.5$. So it can be concluded that the Bayes estimator is asymptotically unbiased estimator for $\theta < 0,5$ and biased for the others.

Table 2. The variance value of the Bayes estimator

α	β	θ	n	$\text{Var } \hat{\theta}^B$	α	β	θ	n	$\text{Var } \hat{\theta}^B$	α	β	θ	n	$\text{Var } \hat{\theta}^B$
			20	0,000676				20	0,035284				20	0,171253
2	18	0,1	50	0,000281	2	3	0,4	50	0,015158	5	2	0,7	50	0,234915
			100	0,000142				100	0,006887				100	0,144501

	500	0,000027		500	0,001305		500	0,038901
	1000	0,000013		1000	0,000708		1000	0,021432
	20	0,003512		20	0,087257		20	0,100130
	50	0,001535		50	0,039709		50	0,235316
2	8	0,2	2	2	0,5	8	2	0,8
	100	0,000749		100	0,017910		100	0,294061
	500	0,000145		500	0,003702		500	0,121428
	1000	0,000074		1000	0,001906		1000	0,072042
	20	0,010244		20	0,152678		20	0,012953
	50	0,004663		50	0,090424		50	0,074497
2	5	0,3	3	2	0,6	18	2	0,9
	100	0,002203		100	0,048039		100	0,210522
	500	0,000442		500	0,010725		500	0,571526
	1000	0,000215		1000	0,005551		1000	0,447854

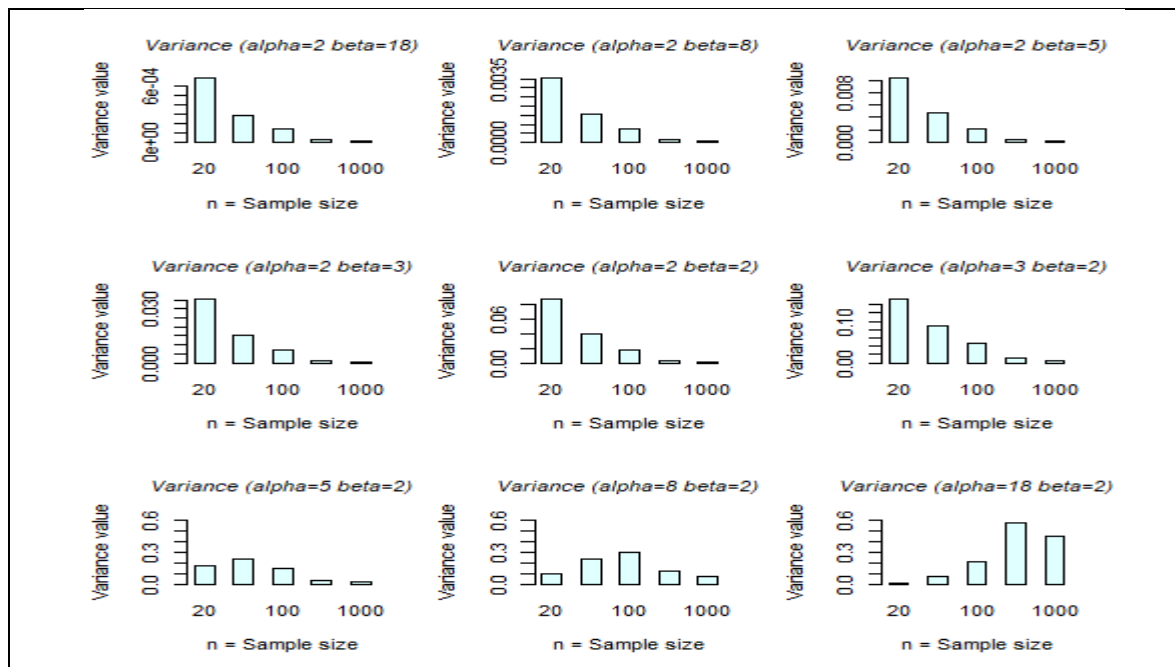


Figure 2. Histogram of the variance value.

Based on Table 2 and Figure 2, it can be seen that the variance value for $\theta \leq 0,6$ gets smaller when the number of sample sizes (n) gets bigger and fluctuative for others. The Bayes estimator is efficient when number of samples sizes large and values $\theta \leq 0,6$ and inefficient for the others.

Table 3. The MSE value of the Bayes estimator

α	β	θ	n	MSE $\hat{\theta}^B$	α	β	θ	n	MSE $\hat{\theta}^B$	α	β	θ	n	MSE $\hat{\theta}^B$
			20	0,000919				20	0,108219				20	1,288108
2	18	0,1	50	0,000451	2	3	0,4	50	0,087891	5	2	0,7	50	2,369129
			100	0,000295				100	0,077295				100	2,769218

	500	0,000153		500	0,070874		500	3,242913
	1000	0,000135		1000	0,069343		1000	3,281805
	20	0,006580		20	0,325021		20	1,358415
	50	0,004462		50	0,293827		50	3,796610
2	8	0,2	2	2	0,5	8	2	0,8
	100	0,003376		100	0,268825		100	6,075506
	500	0,002624		500	0,256944		500	9,388595
	1000	0,002502		1000	0,252603		1000	9,933610
	20	0,024588		20	0,713299		20	0,680247
	50	0,018620		50	0,822152		50	3,290806
2	5	0,3	3	2	0,6	18	2	0,9
	100	0,015100		100	0,825149		100	8,968404
	500	0,012723		500	0,841224		500	37,622619
	1000	0,012319		1000	0,836517		1000	49,106082

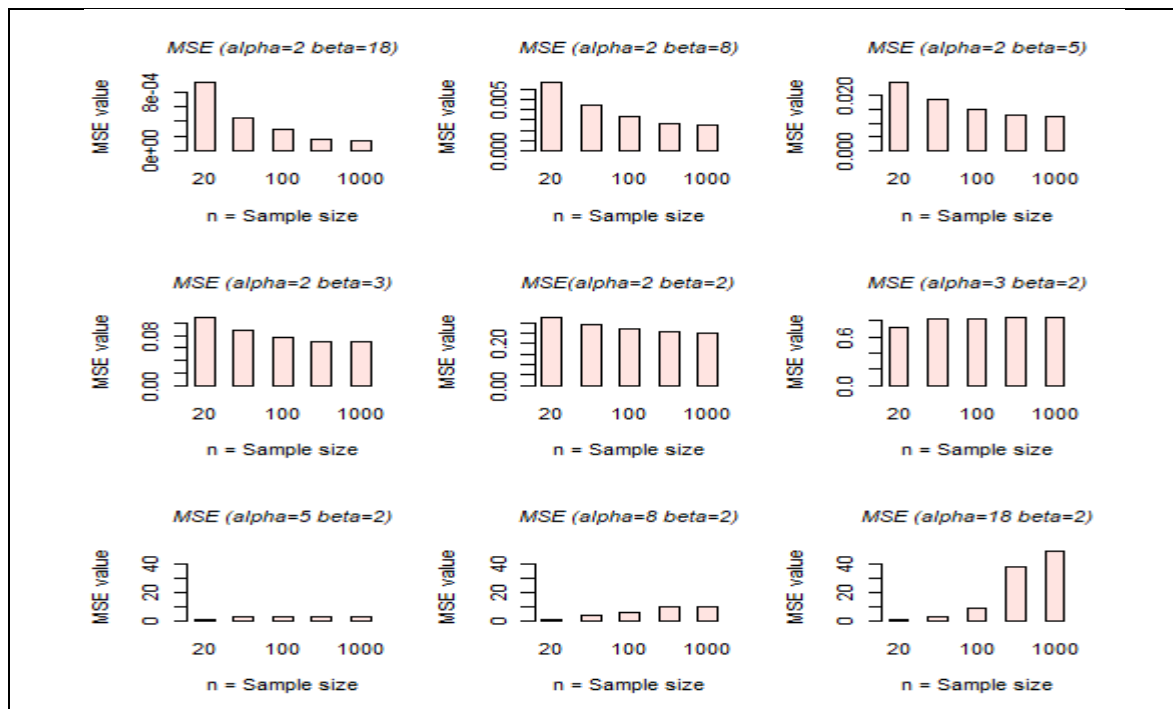


Figure 3. Histogram of the MSE value.

Based on Table 3 and Figure 3, it can be seen that MSE value of the Bayes estimator for $\theta \leq 0,5$ gets smaller when the number of sample sizes (n) gets bigger and vice versa for other. So, the Bayes estimator is consistent when value $\theta \leq 0,5$ and inconsistent for other.

4. Conclusion

Based on the result and discussion it can be concluded that the Bayes estimator on the Geometric distribution with prior Beta are asymptotically unbiased estimator for $\theta < 0,5$ and biased for the others, efficient when number of samples sizes large and values $\theta \leq 0,6$ and inefficient for others and consistent when value $\theta \leq 0,5$ and inconsistent for other.

References

- [1] Casella G and Berger R 2002 *Statistical Inference Second Edition* (California: Duxbury)
- [2] Widiarti, Adityawati N and Nusyirwan 2019 *Journal of Physics: Conf. Series* 1338012039
- [3] Widiarti, D D Oktafiani, M Usman, and D Kurniasari 2020 *J. Physics: Conf. Series* **1567** 022082
- [4] Rizki S W, Mara M N and Sulistianingsih E 2017 *Journal of Physics: Conf. Series* 855012036
- [5] Hartono B, Kurnia A and Indahwati 2017 *International Journal of Computer Science and Network* **6**(6) 2277-5420
- [6] Yanuar F, Putri N C E, and Yozza H 2019 *Journal of Physics: Conf. Series* 1317012006
- [7] Fulop A and li J 2019 *Journal of Econometrics* **209** 114-138
- [8] Kinyanjui J K and Korir B C 2020 *International Journal of Statistics and Probability* **9**(2)
- [9] Puspongoro N H and Rachmawati N R 2018 *Procedia Computer Science* **135** 712-718
- [10] Aw A and Cabral E N 2019 *Laboratory of Mathematics* (Ziguinchor: Assane Seck University of Ziguinchor)
- [11] Bolstad W M 2007 *Introduction to Bayesian Statistical Second Edition* (USA: John Willey and Sons)
- [12] Spiegel M R, Schiller J J and R A Srinivasan 2009 *Probability and Statistics Third Edition* (New York: The McGraw-Hill Companies Inc)
- [13] Montgomery D C and Runger G C 2014 *Applied Statistics and Probability for Engineers sixth Edition* (USA: John Wiley and Sons)
- [14] Bain L J and Engelhardt 1992 *Introduction to Probability and Mathematical Statistical Second Edition* (California: Duxbury)
- [15] Hogg RV, Mckean J W, and Craig A T 2005 *Introduction to Mathematical Statistics Sixth Edition* (USA: Pearson Education Inc)
- [16] Sahoo P 2008 *Probability and Mathematical Statistics* (Louisville: Department of Mathematics University of Louisville)

● 20% Overall Similarity

Top sources found in the following databases:

- 11% Internet database
- 10% Publications database
- Crossref database
- Crossref Posted Content database
- 15% Submitted Works database

TOP SOURCES

The sources with the highest number of matches within the submission. Overlapping sources will not be displayed.

1	R N Putri, M Usman, Warsono, Widiarti, E Virginia. "Modeling Autoregr...	3%
	Crossref	
2	irep.iium.edu.my	3%
	Internet	
3	repo.ur.krakow.pl	2%
	Internet	
4	University of Edinburgh on 2018-10-26	1%
	Submitted works	
5	lppm.ibrahimy.ac.id	1%
	Internet	
6	Laura M. Chihara, Tim C. Hesterberg. "Estimation", Wiley, 2018	1%
	Crossref	
7	University of New South Wales on 2017-05-03	<1%
	Submitted works	
8	School of Business and Management ITB on 2021-03-20	<1%
	Submitted works	

9	IUBH - Internationale Hochschule Bad Honnef-Bonn on 2022-05-11	<1%
	Submitted works	
10	University of Nottingham on 2021-05-18	<1%
	Submitted works	
11	mdc.edu	<1%
	Internet	
12	pt.scribd.com	<1%
	Internet	
13	Johns Hopkins University on 2022-06-27	<1%
	Submitted works	
14	University of Babylon on 2015-08-05	<1%
	Submitted works	
15	University of Leeds on 2018-09-28	<1%
	Submitted works	
16	University of New South Wales on 2017-05-03	<1%
	Submitted works	
17	ejournal.undip.ac.id	<1%
	Internet	
18	Doraiswami, . "Estimation Theory", Identification of Physical Systems, ...	<1%
	Crossref	
19	City University on 2020-08-20	<1%
	Submitted works	
20	Fitri Wijarini, Muhammad Abrar Putra Siregar, Aidil Adhani, Nur Elisia, S...	<1%
	Crossref	

21

Open University of Mauritius on 2019-10-02

<1%

Submitted works

22

University of New South Wales on 2018-05-03

<1%

Submitted works

● Excluded from Similarity Report

- Bibliographic material
- Cited material
- Manually excluded sources
- Quoted material
- Small Matches (Less than 10 words)

EXCLUDED SOURCES

repository.lppm.unila.ac.id	67%
Internet	
T Susilo, Widiarti, D Kurniasari, D Aziz. "Characteristics of Bayes Estimator in ...	67%
Crossref	
E G Simanjuntak, Widiarti, D Kurniasari, Amanto. "Asymptotically Unbiased, E...	13%
Crossref	
iopscience.iop.org	5%
Internet	
digilib.unila.ac.id	4%
Internet	