PAPER NAME AUTHOR

The Implementation of Rough Set on a G roup Structure.pdf

Ananto Adi Nugraha

WORD COUNT CHARACTER COUNT

3517 Words 15227 Characters

PAGE COUNT FILE SIZE

8 Pages 260.9KB

SUBMISSION DATE REPORT DATE

Aug 19, 2022 10:00 PM GMT+7 Aug 19, 2022 10:00 PM GMT+7

13% Overall Similarity

The combined total of all matches, including overlapping sources, for each database.

- 9% Internet database
- · Crossref database
- 8% Submitted Works database

- 11% Publications database
- Crossref Posted Content database

Excluded from Similarity Report

- · Bibliographic material
- · Cited material
- Manually excluded sources

- Ouoted material
- Small Matches (Less then 10 words)
- · Manually excluded text blocks

ISSN: 2527-3159 (print) 2527-3167 (online)



The Implementation of Rough Set on a Group Structure

Ananto Adi Nugraha¹, Fitriani^{1*}, Muslim Ansori¹, Ahmad Faisol¹

¹Department of Mathematics, Universitas Lampung, Indonesia

Article history: Received Jul 6, 2021 Revised, May 9, 2022 Accepted, May 31, 2022

Kata Kunci: aproksimasi bawah, aproksimasi atas, himpunan rough, grup

rough, sentralizer

Abstrak. Diberikan himpunan tak kosong *U* dan relasi ekuivalensi R pada U. Pasangan berurut (U,R) disebut ruang aproksimasi. Relasi ekuivalensi pada U membentuk kelas-kelas ekuivalensi yang saling asing. Jika $X \subseteq U$, maka dapat dibentuk aproksimasi bawah dan aproksimasi atas dari X. Pada penelitian ini dikonstruksi grup rough, subgrup rough pada ruang aproksimasi (U,R) terhadap operasi biner yang bersifat komutatif maupun non-komutatif.

Keywords:

lower approximation, upper approximation, rough set, rough group, centralizer.

Abstract. Let U be a non-empty set and R an equivalence relation on U. Then, (U,R) is an approximation space. The equivalence relation on U forms disjoint equivalence classes. If $X \subseteq U$, we can form a lower approximation and an upper approximation of X. If $X \subseteq U$, then we can form a lower approximation and an upper approximation of X. In this research, rough group and rough subgroups are constructed in the approximation space (U,R) for commutative and non-commutative binary operations.

How to cite:

A. A. Nugraha, Fitriani, M. Ansori, and A. Faisol, "The Implementation of Rough Set on a Group Structure", J. Mat. Mantik, vol. 8, no. 1, pp. 19-26, Jun. 2022.



1. Introduction

Zdzislaw Pawlak [1] first introduced the rough set theory in 1982 as a mathematical technique to deal with vagueness and uncertainty problems. Various studies have discussed this theory and the possibility of its applications, for example, in data mining [2] and some algebraic structures. In [3], Biswaz and Nanda introduce the rough group and rough ring. Furthermore, Miao et al. [4] improve definitions of a rough group and rough subgroup and prove their new properties. In [5], Jesmalar investigates the homomorphism and isomorphism of the rough group. Furthermore, in [6], Bagirmaz and Ozcan give the concept of rough semigroups on approximation spaces. Then, Kuroki in [7] gives some results about the rough ideal of semigroups. In [8], Davvaz investigates roughness in the ring, and in [9], Davvaz and Mahdavipour give a roughness in modules. In [10], Isaac and Neelima introduce the concept of the rough ideal. Moreover, in [11], Zhang et al. give some properties of rough modules. Davvaz and Malekzadeh give roughness in modules [12]. They use the notion of reference points. Furthermore, Ozturk and Eren give the multiplicative rough modules [13]. Then, Sinha and Prakash introduce the rough exact sequence of rough modules [14]. They also give the injective module based on rough set theory [15]. In [16], Kazancı and Davvaz give the rough prime in a ring. Jun in [17] investigate the roughness of ideals in BCK-algebras. Moreover, Dubois and Prade [18] define the rough fuzzy sets.

This research focuses on the algebraic aspects by applying a rough set theory to construct a rough group and its subgroups on an approximation space. Moreover, in this research, we discuss the centralizer and the center of a rough group.

2. Preliminaries

In this section, there will be several definitions and theorems that can be helpful for this article. Those definitions are written as follows:

Definition 1 [19] Define $C_G(A) = \{g \in G \mid gag^{-1} = a \text{ for all } a \in A\}$. This subset of G is called the centralizer of A in G. Since $gag^{-1} = a$ if and only if ga = ag, $C_G(A)$ is the set of elements of G which commute with every element of A.

Definition 2 [19] Define $Z(G) = \{g \in G \mid gx = xg \text{ for all } x \in G\}$, the set of elements commuting with all the elements of G. This subset of G is called the center of G.

Definition 3 [20] Let R be an equivalence relation on A and $a \in A$. Then the equivalence class of a under R is $[a]_R = \{x : x \in A \text{ and } aRx\}$. In other words, the equivalence class of a under R contains all the elements in A to which a is related by R.

Definition 4 [3] Let (U, R) be an approximation space and X be a subset of U, the sets,

$$\overline{X} = \{ x \mid [x]_R \cap X \neq \emptyset \} \tag{1}$$

$$X = \{x \mid [x]_R \subseteq X\} \tag{2}$$

are called upper approximation and lower approximation of X.

Definition 5 [1] Let R be an equivalence relation on universe set U, a pair (U, R) is called an approximation space. A subset $X \subseteq U$ can be defined if $\overline{X} = \overline{X}$, in the opposite case, if $\overline{X} - \underline{X} \neq \emptyset$ then X is called a rough set.

Definition 6 Let K = (U, R) be an approximation space and * be a binary operation defined on U. A subset G of universe U is called a rough group if the following properties are satisfied:

- i. $\forall x, y \in G, x * y \in \overline{G}$;
- ii. Association property holds in \overline{G} ;
- iii. $\exists e \in \overline{G}$ such that $\forall x \in G$, x * e = e * x = x; e is called the rough identity element of G;
- iv. $\forall x \in G$, $\exists y \in G$ such that x * y = y * x = e; y is called the rough inverse element of x in G.

We will give the example of rough group in Section 3.

The following theorem gives the characteristics of a rough group.

Theorem 1. [3] A necessary and sufficient condition for a subset H of rough group G to be a rough subgroup is that:

- (i) $\forall x, y \in H, x * y \in \overline{H}$;
- (ii) $\forall x \in H, x^{-1} \in H$.

Several steps will be taken to achieve the objectives of this research. Those steps are written as follows:

- 1. Determine a set U, where $U \neq \emptyset$.
- 2. Define a relation R on U.
- 3. Shows that a relation R is the equivalence relation on U.
- 4. Determine equivalence classes on *U*.
- 5. Determine a set G, where $G \subseteq U$ and $G \neq \emptyset$.
- 6. Determine the approximation space, lower approximation on $G(\underline{G})$, and upper approximation on $G(\overline{G})$.
- 7. Determine a pough set $Apr(G) = (G, \overline{G})$.
- 8. Determine a vinary operation * on the set G.
- 9. Shows that (G,*) is a rough group in the approximation space that has been constructed.
- 10. Determine a rough subgroup $\langle H, * \rangle$ from a rough group $\langle G, * \rangle$.

3. Rough Group Construction

3.1 Commutative Kough Group Construction

In this section, we will give the construction of commutative rough group.

Example 3.1. Given a non-empty set $U = \{0,1,2,3,...,99\}$. We define a relation R on the set U, that is, for every $a, b \in U$ apply aRb if and only if a - b = 7k where $k \in \mathbb{Z}$. Furthermore, it can be shown that relation R is reflexive, symmetrical, and transitive. So, relation R is an equivalence relation on U. As a result, relation R produces some disjoint partitions called equivalence classes. The equivalence classes are written as follows:

```
E_1 = [1] = \{1,8,15,22,29,36,43,50,57,64,71,78,85,92,99\};
E_2 = [2] = \{2,9,16,23,30,37,44,51,58,65,72,79,86,93\};
E_3 = [3] = \{3,10,17,24,31,38,45,52,59,66,73,80,87,94\};
E_4 = [4] = \{4,11,18,25,32,39,46,53,60,67,74,81,88,95\};
E_5 = [5] = \{5,12,19,26,33,40,47,54,61,68,75,82,89,96\};
```

$$E_6 = [6] = \{6,13,20,27,34,41,48,55,62,69,76,83,90,97\};$$

 $E_7 = [0] = \{0,7,14,21,28,35,42,49,56,63,70,77,84,91,98\}.$

Given a non-energy subset $X \subseteq U$ that is $X = \{10,20,30,40,50,60,70,80,90\}$. Because the set $U \neq \emptyset$ and R is an equivalence relation on U, a pair (U,R) is the approximation space. Furthermore, it can be obtained the lower approximation and upper approximation of X, that is:

$$\underline{\underline{X}} = \emptyset.$$

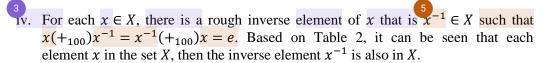
$$\overline{X} = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup E_7 = U.$$

After determining the lower approximation and upper approximation of X, then given a binary operation $+_{100}$ on X. Here is given Table Cayley of X with the operation $+_{100}$.

	,	Table 1.	Table Ca	ayley of	X with tl	ne operat	tion + ₁₀₀)	
+100	10	20	30	40	50	60	70	80	90
10	20	30	40	50	60	70	80	90	0
20	30	40	50	60	70	80	90	0	10
30	40	50	60	70	80	90	0	10	20
40	50	60	70	80	90	0	10	20	30
50	60	70	80	90	0	10	20	30	40
60	70	80	90	0	10	20	30	40	50
70	80	90	0	10	20	30	40	50	60
80	90	0	10	20	30	40	50	60	70
90	0	10	20	30	40	50	60	70	80

- i. Based on the least proved that for each $x, y \in X$, apply $x(+_{100})y \in \overline{X}$. ii. For each $x, y, z \in X$, the associative property that is $(x(+_{100})y)(+_{100})z =$ $x(+_{100})(y(+_{100})z)$ holds in \overline{X} . The operation $+_{100}$ is associative in \overline{X} .
- iii. There is a rough identity element $e \in \overline{X}$ that is $0 \in \overline{X}$ such that for each $x \in X$, $x(+_{100})e = e(+_{100})x = x.$

Table 2. Table of element inverse of the set <i>X</i>									
x	10	20	30	40	50	60	70	80	90
x^{-1}	90	80	70	60	50	40	30	20	10



Since those four conditions have been satisfied, then $(X, +_{100})$ is a rough group.

3.2 Non-Commutative Rough Group Construction

In this section, we will give the construction of non-commutative rough group.

Example 3.2. Given a permutation group S_3 to the operation of permutation multiplication " \circ ." For example, take a begroup $G = \{(1), (12)\}$ of the group S_3 . For $x, y \in S_3$, define a relation R that is xRy if and only if $x \circ y^{-1} \in G$. Furthermore, it can be shown that relation R is reflexive, symmetrical, and transitive. So, relation R is an equivalence relation on S_3 . As a result, relation R produces some disjoint partitions called equivalence classes. Suppose a is the element in S_3 , the equivalence class containing a defined as follows:

$$[a]_R = \{ x \in S_3 \mid xRa \} = \{ x \in S_3 \mid x \circ a^{-1} \in G \}$$

$$= \{x \in S_3 \mid x \circ a^{-1} = g, g \in G\}$$

$$= \{x \in S_3 \mid x = g \circ a, g \in G\}$$

$$= \{g \circ a \mid g \in G\}$$

$$(3)$$

Based on the Equation (3), this is corresponding to the definition of the right coset of G in S_3 that is $Ga = \{g \circ a \mid g \in G\}$. Thus, the right cosets of G in S_3 as follows:

$$G \circ (1) = G \circ (12) = \{(1), (12)\};$$

 $G \circ (13) = G \circ (123) = \{(13), (123)\};$
 $G \circ (23) = G \circ (132) = \{(23), (132)\}.$

Given a non-empty subject $Y \subseteq S_3$ that is $Y = \{(1), (12), (123), (132)\}$. Furthermore, it can be obtained the lower approximation and upper approximation of Y, that is:

$$\frac{\underline{Y}}{\overline{Y}} = \{(1), (1\ 2)\}.$$

$$\overline{Y} = \{(1), (1\ 2)\} \cup \{(1\ 3), (1\ 2\ 3)\} \cup \{(2\ 3), (1\ 3\ 2)\} = S_3.$$

After determining the lower approximation and upper approximation of Y, then we give a permutation multiplication " \circ " on Y. We give a Table Cayley of Y with the operation of permutation multiplication as follows.

Table 3. Table Cayley of *Y* with the operation of permutation multiplication

۰	(1)	(1 2)	(1 2 3)	(1 3 2)
(1)	(1)	(12)	(123)	(1 3 2)
(1 2)	(12)	(1)	(23)	(13)
(1 2 3)	(1 2 3)	(1 3)	(1 3 2)	(1)
(1 3 2)	(1 3 2)	(23)	(1)	(1 2 3)

- i. Based on Table 3, it is proved that for each $x, y \in Y$, apply $x \circ y \in \overline{Y}$.
- ii. For each $x, y, z \in Y$, the associative property that is $(x \circ y) \circ z = x \circ (y \circ z)$ holds in \overline{Y} . The operation \circ is associative in \overline{Y} .
- iii. There is a rough identity element $e \in \overline{Y}$ that is (1) $\in \overline{Y}$ such that for each $y \in Y$, $y \circ e = e \circ y = y$.

 Table 4. Table of inverse element of Y

 y
 (1)
 (1 2)
 (1 2 3)
 (1 3 2)

 y^{-1} (1)
 (1 2)
 (1 3 2)
 (1 2 3)

1v. For each $y \in Y$, there is a rough inverse element of y that is $y^{-1} \in Y$ such that $y \circ y^{-1} = y^{-1} \circ y = e$. Based on Table 4, it can be seen that each element y in the set Y, then the inverse element y^{-1} is also in the set Y.

Since those four conditions have been satisfied, then $\langle Y, \circ \rangle$ is a rough group.

4. Subgroup Construction of the Rough Group

After constructing a commutative rough group and a non-commutative rough group, we will construct subgroups of each of the previously constructed rough groups.

4.1 Subgroup Construction of Commutative Rough Group

Before it has been obtained, a commutative rough group X with the operation " $+_{100}$ ". Furthermore, we will construct several subgroups that can be formed from the rough group X. Based on Theorem 1, we can obtain several subgroups from the rough group X that written as follows:

- 1. $\langle \{20,30,40,50,60,70,80\}, +_{100} \rangle$;
- 2. $\langle X, +_{100} \rangle$.

After determining several subgroups from the rough group X that is commutative, then we will determine the centralizer and the center of subgroups in rough group X. Suppose all subgroups of rough group X above are denoted by A. Based on Definition 1, the centralizer A in X is the set where is the element of X is commutative with each element of A. Here is given the table that shows the centralizer of subgroups A in rough group X.

Table 5. Table of the centralizer of subgroups A in rough group X

A	$C_X(A)$
{20,30,40,50,60,70,80}	X
$X = \{10,20,30,40,50,60,70,80,90\}$	X

Since the operation $+_{100}$ of rough group X is commutative, the centralizer of subgroups in rough group X is X itself.

Based on Definition $\frac{6}{X}$, the center of X is the set of elements that is commutative with all elements of X. Because rough group X using commutative operation, the center of rough group X is X itself, or it can be written as Z(X) = X.

Using Theorem 1, we will show that the center of rough group X that is Z(X) = X is a rough subgroup of rough group X.

- i. Based on Table 1, it is proved that for each $x, y \in Z(X) = X$, apply $x(+_{100})y \in \overline{Z(X)} = \overline{X} = U$.
- ii. For each $x \in Z(X) = X$, there is an inverse element of x that is $x^{-1} \in Z(X) = X$. Based on Table 2, it can be seen that if each element x in the set X then the inverse element of x also in the set X.

Two conditions on Theorem 1 have been satisfied, so it is proved that the center of rough group X that is Z(X) = X is a rough subgroup of rough group X.

4.2 Subgroup Construction of Non-Commutative Rough Group

Before it has been obtained a non-commutative rough group Y with the operation of permutation multiplication " \circ ." Furthermore, we will construct several subgroups that can be formed from the rough group Y. Based on Theorem 1, we can obtain several subgroups from the rough group Y that written as follows:

- ({(1)},∘);
- 2. $\{(1), (12)\}, \circ\};$
- 3. $\langle \{(1), (123), (132)\}, \circ \rangle;$
- 4. $\langle \{(1\ 2), (1\ 2\ 3), (1\ 3\ 2)\}, \circ \rangle;$
- 5. $\langle Y, \circ \rangle$.

After determining several subgroups from the rough set Y that are non-commutative, then we will determine the centralizer and the center of subgroups in rough group Y. Suppose all subgroups of rough group Y above are denoted by B. Based on Definition 1, the centralizer B in Y is the set where is the element of Y is commutative with each element of B. Here is given the table that shows the centralizer of subgroups B in rough group Y.

Table 6. Table of the centralizer of subgroups B in rough group Y

В	$C_Y(B)$
{(1)}	Y
{(1), (1 2)}	{(1), (12)}
{(1), (1 2 3), (1 3 2)}	{(1)}
{(1 2), (1 2 3), (1 3 2)}	{(1)}
$Y = \{(1), (12), (123), (132)\}$	{(1)}

Based on Definition $\stackrel{6}{2}$, the center of Y is the set of elements that is commutative with all elements of Y. From the Definition 2, the center of rough group Y is an identity element, or it can be written as $Z(Y) = \{(1)\}$.

Using Theorem 1, we will show that the center of rough group Y that is $Z(Y) = \{(1)\}$ is a rough subgroup of rough group Y. Previously, determine the upper approximation of Z(Y) that is $\overline{Z(Y)} = \{(1), (12)\}$.

- i. For $(1) \in Z(Y)$, apply $(1) \circ (1) = (1) \in \overline{Z(Y)}$.
- ii. For $(1) \in Z(Y)$, there is an inverse element of (1) that is $(1) \in Z(Y)$.

Based on Theorem 1, because the two conditions have been satisfied, it is proved that the center of rough group Y that is $Z(Y) = \{(1)\}$ is a rough subgroup of rough group Y.

5 Conclusions

Based on the results, we construct a rough group, a rough subgroup in the case of the commutative and non-commutative binary operation. Furthermore, the centralizer of a commutative rough subgroup is also a rough group. In comparison, the centralizer of the subgroup of a non-commutative rough group must contain the identity element and the center. The center of each rough group, both commutative and non-commutative, are subgroups of each rough group.

References

- [1] Z. Pawlak, "Rough Sets," *Int. J. Comput. Inf. Sci.*, vol. 11, no. 5, pp. 341–356, 1982, doi: https://doi.org/10.1007/BF01001956.
- [2] Z. Pawlak, Rough sets-theoretical aspects of reasoning about data. Dordrecht, Kluwer, 1991.
- [3] R. Biswas and S. Nanda, "Rough Groups and Rough Subring," *Bull. Polish Acad. Sci. Math.*, vol. 42, pp. 251–254, 1994.
- [4] D. Miao, S. Han, D. Li, and L. Sun, "Rough group, Rough subgroup, and their properties," in *Lecture Notes in Artificial Intelligence*, 3641, 2005, pp. 104–113.
- [5] L. Jesmalar, "Homomorphism and Isomorphism of Rough Group,", *International Journal of Advance Research. Ideas and Innovations in Technology*, vol. 3, no. 3, pp. 1382–1387, 2017.
- [6] N. Bagirmaz and A. Ozcan, "Rough Semigroups on Approximation Spaces," *Int. J. Algebr.*, vol. 9, no. 7, pp. 339–350, 2015.
- [7] N. Kuroki, "Rough ideals in semigroups," *Inform. Sci.*, vol. 100, pp. 139–163, 1997.
- [8] B. Davvaz, "Roughness in rings," *Inf. Sci.* (Ny)., vol. 164, pp. 147–163, 2004.
- [9] B. Davvaz and M. Mahdavipour, "Roughness in modules," *Inf. Sci.* (*Ny*)., vol. 176, pp. 3658–3674, 2006.
- [10] P. Isaac, Neelima, C.A., "Rough Ideals and Their Properties," Journal of Global

- Research in Mathematical Archives, vol. 1, no. 6, pp 90-98, 2013.
- [11] Q. F. Zhang, A. M. Fu, and S. x. Zhao, "Rough modules and their some properties," in *Proceeding of the Fifth International Conference on Machine Learning and Cybernetics*, 2006.
- [12] B. Davvaz and A. Malekzadeh, "Roughness in Modules by using the Notion of Reference Points," *Iranian Journal of Fuzzy Systems*, vol 10, no. 8, pp 109-124, 2013.
- [13] E. Ozturk and S. Eren, "On Multiplicative Rough Modules," *International Journal of Algebra*, vol.7, no 15, 735-742, 2013.
- [14] A. K. Sinha and A. Prakash, "Rough Exact Sequences of Modules," *Int. J. Appl. Eng. Res.*, 2016.
- [15] A. K. Sinha and A. Prakash, "Injective Module Based on Rough Set Theory," *Cogent Mathematics*, Vol. 2, 2015.
- [16] O. Kazancı and B. Davvaz, "On the structure of Rough prime (primary) ideals and rough fuzzy prime (primary) ideals in commutative rings", *Information Sciences*, vol. 178, no. 5, pp. 1343-1354, 2008.
- [17] Y. B. Jun, "Roughness of ideals in BCK-algebras," *Scientiae Math. Japonica*, vol. 57, no. 1, pp. 165-169, 2003.
- [18] D. Dubois and H. Prade, "Rough Fuzzy Sets and Fuzzy Rough Sets," *Int. J. General Syst.*, vol. 17, pp. 191-209, 1990.
- [19] D. S. Dummit and R. M. Foote, *Abstract Algebra*, Third Edit. John Wiley and Sons, Inc., 2004.
- [20] W. Barnier and N. Feldman, *Introduction to Advanced Mathematics*. New Jersey: Prentice Hall, Inc., 1990.

13% Overall Similarity

Top sources found in the following databases:

- 9% Internet database
- Crossref database
- 8% Submitted Works database

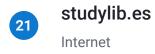
- 11% Publications database
- Crossref Posted Content database

TOP SOURCES

The sources with the highest number of matches within the submission. Overlapping sources will not be displayed.

1	nozdr.ru Internet	3%
2	Higher Education Commission Pakistan on 2010-09-23 Submitted works	1%
3	Ramji Lal. "Algebra 1", Springer Science and Business Media LLC, 2017 Crossref	<1%
4	Osman Kazancı, B. Davvaz. "On the structure of rough prime (primary) i	·<1%
5	University of Wales Swansea on 2015-12-09 Submitted works	<1%
6	arxiv.org Internet	<1%
7	iaeng.org Internet	<1%
8	media.opencurriculum.org Internet	<1%

dtic.mil Internet	<1%
egyankosh.ac.in Internet	<1%
research-publication.com Internet	<1%
mafiadoc.com Internet	<1%
Duoqian Miao, Suqing Han, Daoguo Li, Lijun Sun. "Chapter 11 Rough Crossref	Gr <1%
University of Bristol on 2016-05-04 Submitted works	<1%
Nurettin Bağırmaz, İlhan İçen, Abdullah F. Özcan. "Topological Roug Crossref	h G<1%
XiangPeng Li, Min Dong, Liu Qing. "Axiomatic approaches of the ap	opr <1%
Yuhua H. Qian, Jiye Liang, Chuangyin Dang. "MGRS in Incomplete In	for <1%
bu.edu.eg Internet	<1%
ebin.pub Internet	<1%
pasca.unila.ac.id Internet	<1%



<1%

Excluded from Similarity Report

- Bibliographic material
- Cited material
- Manually excluded sources

- Quoted material
- Small Matches (Less then 10 words)
- · Manually excluded text blocks

EXCLUDED SOURCES

jurnalsaintek.uinsby.ac.id

Internet

EXCLUDED TEXT BLOCKS

of Mathematics, Universitas

jurnalsaintek.uinsby.ac.id

Abstract. Let

jurnalsaintek.uinsby.ac.id

3%