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Abstract

In ring and module theory, the concept of an exact sequence is commonly employed. The exact sequence is generalized into the U-exact sequence and the V-coexact sequence. Rough set theory has also been applied to a variety of algebraic structures, including groups, rings, modules, and others. In this study, we investigated characteristics of a rough V-coexact sequence in rough groups.

Keywords: *exact sequence, V-coexact sequence, rough group.*

1. INTRODUCTION

One of the most fundamental concepts in algebraic structures is the exact sequence [1]. The concept of an exact sequence is developed in module theory into U-exact sequences, V -exact sequences, and X -sub-exact sequences. The generalization of the exact sequence is the U-exact sequence [2]. The V-coexact sequence is a dualization of the U-exact sequence. Anvanriyeh and Davvaz show how U-split sequences and projective modules are related in [3]. Generalization of Schanuel's Lemma and the relationship between quasi-exact sequences and their submodules can be obtained using the generalization of an exact sequences [4]. The generalization of Snake's Lemma and Five's Lemma was then studied in [5]. The X-sub-exact sequence is a generalization of the exact sequence [6]. The concept of an exact sequence is used to define an X-sublinearly independent set [7]. In 2018, the U-generator concept was introduced based on V-coexact sequences [8]. The concept of a U_v-generator and an X-sublinear independent module family were utilized to develop by (X, V)-basis and U-free modules in the same year [9].

Rough set theory is a mathematical concept initially introduced in 1982 [10]. Several concepts of algebraic structure in the rough set have been studied, including homomorphisms on rough sets [11], rough groups [12], rough subgroups [13], application of rough sets to computers [14], projective modules on rough sets [15], anti-homomorphism on rough prime ideals [16], and rough homomorphisms on rough set, rough group, and rough semigroups in approximation space [17]. Furthermore, Sinha gives a rough exact sequence of rough modules over rough rings [18].

Many researchers discuss the application of rough set theory in several aspects of science, including data mining and algebraic elements. In this research, we will give the properties of a rough V-coexact sequence in a rough group.

2. RESEARCH METHODS

The research methods rely on the upper and lower approximation spaces, the rough group, the exact sequence, the V-coexact sequence, and literature. We first define the rough set using its binary operation and define the rough V-coexact sequence of the rough groups. We also investigate the properties of the rough group and use the finite set to construct an example of the rough V-coexact sequence of the rough groups. Finally, we investigate the properties of the rough V-coexact sequence of rough groups.

The following are the stages of the research.
1. We define the rough V-coexact sequence of the rough groups.
2. We analyze the properties of the rough V-coexact sequence.
3. We construct the examples of the rough group, rough group homomorphisms, and rough V-coexact sequences by using the finite set.

3. RESULTS AND DISCUSSION

3.1. Rough V-Coexact Sequence in Rough Group

Motivated by the definition of the V-coexact sequence of the R-modules, we define the rough V-coexact sequence of the rough groups as follows.

Definition 1. Let (U, θ) be an approximation space, A, B, C the rough groups in (U, θ), and V the rough subgroups of A in (U, θ). If $f(\overline{V}) = \ker(g)$, this sequence
$$\overline{A} \xrightarrow{f} \overline{B} \xrightarrow{g} \overline{C}$$

is called rough V-coexact in A.

Next, we give the construction of a rough subgroup in an approximation space.

Example 1 Let $\mathbb{Z}_{16} = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}, \overline{8}, \overline{9}, \overline{10}, \overline{11}, \overline{12}, \overline{13}, \overline{14}, \overline{15}\}$. We define $a\theta b$ if only if $a - b = 4k$ for some $k \in \mathbb{Z}$. From this equivalence relation, we have four equivalence classes in the following table.

Tabel 1. The Equivalence Classes of $\mathbb{Z}_{16}$

The Equivalence Class	The Element of the Class
E_1	$\{\overline{1}, \overline{5}, \overline{9}, \overline{13}\}$
E_2	$\{\overline{2}, \overline{6}, \overline{10}, \overline{14}\}$
E_3	$\{\overline{3}, \overline{7}, \overline{11}, \overline{15}\}$
E_4	$\{\overline{0}, \overline{4}, \overline{8}, \overline{12}\}$

Furthermore,we give three rough groups to form a rough V-coexact sequence of rough groups.
Let $X_1 = \{\overline{1}, \overline{2}, \overline{8}, \overline{14}, \overline{15}\}$. We have $\overline{X_1} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_{16}$.

Next, we will prove that X_1 is a rough group.

Tabel 2. Cayley Table for X_1

$+_{16}$	$\overline{1}$	$\overline{2}$	$\overline{8}$	$\overline{14}$	$\overline{15}$
$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{9}$	$\overline{15}$	$\overline{0}$
$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{10}$	$\overline{0}$	$\overline{1}$
$\overline{8}$	$\overline{9}$	$\overline{10}$	$\overline{0}$	$\overline{6}$	$\overline{7}$
$\overline{14}$	$\overline{15}$	$\overline{0}$	$\overline{6}$	$\overline{10}$	$\overline{13}$

1. Table 2 shows that $x(+_{16})y \in \overline{X_1}$ for every $x, y \in X_1$,
2. the associative property is satisfied in $\overline{X_1}$,
3. there exist $\overline{0} \in \overline{X_1}$, such that $x(+_{16})\overline{0} = \overline{0}(+_{16})x = x$ for every $x \in \overline{X_1}$,
4. for every $x \in X_1$, there exist $y \in X_1$ such that $x(+_{16})y = \overline{0}$,

Tabel 3. Inverse Table for X_1

$x \in X_1$	Inverse of x
$\overline{1}$	$\overline{15}$
$\overline{2}$	$\overline{14}$
$\overline{8}$	$\overline{8}$

Based on Table 3, we can see that every element of X_1 has a rough inverse in X_1. So, it proves that X_1 is a rough group on $\mathbb{Z}_{16}$.

Let $X_2 = \{\overline{5}, \overline{6}, \overline{8}, \overline{10}, \overline{11}\}$, $\overline{X_2} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_{16}$. We will prove that X_2 is a rough group.

Tabel 4. Cayley Table for X_2

$+_{16}$	$\overline{5}$	$\overline{6}$	$\overline{8}$	$\overline{10}$	$\overline{11}$
$\overline{5}$	$\overline{10}$	$\overline{11}$	$\overline{13}$	$\overline{15}$	$\overline{0}$
$\overline{6}$	$\overline{11}$	$\overline{12}$	$\overline{14}$	$\overline{0}$	$\overline{1}$
$\overline{8}$	$\overline{13}$	$\overline{14}$	$\overline{0}$	$\overline{2}$	$\overline{3}$
$\overline{10}$	$\overline{15}$	$\overline{0}$	$\overline{2}$	$\overline{4}$	$\overline{5}$
$\overline{11}$	$\overline{0}$	$\overline{2}$	$\overline{3}$	$\overline{5}$	$\overline{6}$

1. Table 4, shows that $x(+_{16})y \in \overline{X_2}$ for every $x, y \in X_2$,
2. the associative property is satisfied in $\overline{X_2}$,
3. there exist $\overline{0} \in \overline{X_2}$, such that $x(+_{16})\overline{0} = \overline{0}(+_{16})x = x$, for every $x \in \overline{X_2}$,
4. for every $x \in X_2$, there exist $y \in X_2$ such that $x(+_{16})y = \overline{0}$,

Tabel 5. Inverse Table for X_2

$x \in X_2$	Inverse of x
$\overline{5}$	$\overline{11}$
$\overline{6}$	$\overline{10}$
$\overline{8}$	$\overline{8}$

Based on Table 5, we can see that every element of X_2 has a rough inverse in X_2. Hence, it proves that X_2 is a rough group on $\mathbb{Z}_{16}$.

Let $X_3 = \{\overline{0}, \overline{6}, \overline{8}, \overline{10}\}, \overline{X_3} = E_2 \cup E_4 = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}, \overline{12}, \overline{14}\}$. Next, we will prove that X_3 is a rough group.

Tabel 6. Cayley Table for X_3

$+_{16}$	$\overline{0}$	$\overline{6}$	$\overline{8}$	$\overline{10}$	$+_{16}$
$\overline{0}$	$\overline{0}$	$\overline{6}$	$\overline{8}$	$\overline{10}$	$\overline{0}$
$\overline{6}$	$\overline{6}$	$\overline{12}$	$\overline{14}$	$\overline{0}$	$\overline{6}$
$\overline{8}$	$\overline{8}$	$\overline{14}$	$\overline{0}$	$\overline{2}$	$\overline{8}$
$\overline{10}$	$\overline{10}$	$\overline{0}$	$\overline{2}$	$\overline{4}$	$\overline{10}$

1. Table 6 shows that $x(+_{16})y \in \overline{X_3}$ for every $x, y \in X_3$,
2. the associative property is satisfied in $\overline{X_3}$,
3. there exist $\overline{0} \in \overline{X_3}$, such that $x(+_{16})\overline{0} = \overline{0}(+_{16})x = x$ for every $x \in \overline{X_3}$,
4. for every $x \in X_3$, there exist $y \in X_3$ such that $x(+_{16})y = \overline{0}$,

Tabel 7. Inverse Table for X_3

$x \in X_3$	Inverse of x
0	$\overline{0}$
$\overline{6}$	$\overline{10}$
$\overline{8}$	$\overline{8}$

Based on Table 7, we can see that every element of X_3 has a rough inverse in X_3. Hence, it proves that X_3 is a rough group on $\mathbb{Z}_{16}$.

Then $V \subseteq X_1$, let $V = \{\overline{2}, \overline{8}, \overline{14}\}, \overline{V} = E_4 = E_2 \cup E_4 = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}, \overline{12}, \overline{14}\}$ is a rough subgroup of X_1. We can see that $\overline{2}(+_{16})\overline{14} = \overline{0} \in \overline{V}$ and $(\overline{2})^{-1} = \overline{14}$. After that, we form a sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ with f is identity function, and $g(a) = 2a$, for every $a \in \overline{X_2}$. We have $V \subseteq X_1$. We will show the sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is a rough V-coexact sequence. Since $f(\overline{V}) = \ker(g) = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}, \overline{12}, \overline{14}\}$, we have $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is rough V-exact sequence.

Next, we will give the properties of the rough V-coexact sequence.

Proposition 1 Let $\overline{A} \xrightarrow{f} \overline{B} \xrightarrow{g} \overline{C}$ is a rough exact sequence. If A' rough subgroup of A, B' rough subgroup of B, C' rough subgroup of c, and $\overline{A} = \overline{A'}, \overline{B} = \overline{B'}, \overline{C} = \overline{C'}$ then $\overline{A'} \xrightarrow{f} \overline{B'} \xrightarrow{g} \overline{C'}$ is is a rough exact sequence.

Proof. We know $\overline{A} \xrightarrow{f} \overline{B} \xrightarrow{g} \overline{C}$ is a rough exact sequence, then $\text{im}(f) = \ker(g)$. Next, with homomorphism rough f and g in the same sequence, we have $\overline{A'} \xrightarrow{f} \overline{B'} \xrightarrow{g} \overline{C'}$ is is a rough exact sequence.

Moreover, we give a illustration of Proposition 1

Example 2 Let $\mathbb{Z}_{16} = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}, \overline{10}, \overline{11}, \overline{12}, \overline{13}, \overline{14}, \overline{15}\}$. We define $a\theta b$ if only if $a - b = 4k$ for some $k \in \mathbb{Z}$. From this equivalence relation, we have four equivalence classes as follows:
$E_1 = \{\bar{1}, \bar{5}, \bar{9}, \overline{13}\}$,
$E_2 = \{\bar{2}, \bar{6}, \overline{10}, \overline{14}\}$,
$E_3 = \{\bar{3}, \bar{7}, \overline{11}, \overline{15}\}$,
$E_4 = \{\bar{0}, \bar{4}, \bar{8}, \overline{12}\}$.

Furthermore, we give three rough groups to form a rough exact sequence of rough groups

Let $X_1 = \{\bar{0}, \bar{1}, \bar{2}, \bar{8}, \overline{14}, \overline{15}\}$. We have $\overline{X_1} = \mathbb{Z}_{16}$.
1. for every $x, y \in X_1$, $x(+_{16})y \in \overline{X_1}$;
2. the associative property is satisfied in $\overline{X_1}$;
3. there exists $\bar{0} \in \overline{X_1}$, such that for every $\bar{x} \in \overline{X_1}$, $\bar{x}(+_{16})\bar{0} = \bar{0}(+_{16})\bar{x} = \bar{x}$;
4. for every $\bar{x} \in X_1$, there exists $\bar{y} \in X_1$ such that $\bar{x}(+_{16})\bar{y} = \bar{0}$ or $\bar{y} = (\bar{x})^{-1}$, that is
   $(\bar{0})^{-1} = \bar{0} \in X_1, (\bar{1})^{-1} = \overline{15} \in X_1, (\bar{2})^{-1} = \overline{14} \in X_1, (\bar{8})^{-1} = \bar{8} \in X_1, (\overline{15})^{-1} = \bar{1} \in X_1,$.
   $(\overline{14})^{-1} = \bar{2} \in X_1$.
So, X_1 is a rough group.

Let $X_2 = \{\bar{0}, \bar{5}, \bar{6}, \bar{8}, \overline{10}, \overline{11}\}$, then $\overline{X_2} = \mathbb{Z}_{16}$.
1. for every $x, y \in X_2$, $x(+_{16})y \in \overline{X_2}$;
2. the associative property is satisfied in in $\overline{X_2}$;
3. there exists $\bar{0} \in \overline{X_2}$, such that for every $\bar{x} \in \overline{X_2}$, $\bar{x}(+_{16})\bar{0} = \bar{0}(+_{16})\bar{x} = \bar{x}$;
4. for every $\bar{x} \in X_2$, there exist $\bar{y} \in X_2$ such that $\bar{x}(+_{16})\bar{y} = \bar{0}$ or $\bar{y} = (\bar{x})^{-1}$, that is
   $(\bar{0})^{-1} = \bar{0} \in X_2, (\bar{5})^{-1} = \overline{11} \in X_2, (\bar{6})^{-1} = \overline{10} \in X_2, (\bar{8})^{-1} = \bar{8} \in X_2, (\overline{10})^{-1} = \bar{6} \in X_2,$.
   $(\overline{11})^{-1} = \bar{5} \in X_2$.
So, X_2 is a rough group.

Let $X_3 = \{\bar{0}, \bar{3}, \bar{6}, \bar{8}, \overline{10}, \overline{13}\}$, then $\overline{X_3} = \mathbb{Z}_{16}$.
1. for every $x, y \in X_3$, $x(+_{16})y \in \overline{X_3}$;
2. the associative property is satisfied in $\overline{X_3}$;
3. there exists $\bar{0} \in \overline{X_3}$, such that for every $\bar{x} \in \overline{X_3}$, $\bar{x}(+_{16})\bar{0} = \bar{0}(+_{16})\bar{x} = \bar{x}$;
4. for every $\bar{x} \in X_3$, there exist $\bar{y} \in X_3$ such that $\bar{x}(+_{16})\bar{y} = \bar{0}$ or $\bar{y} = (\bar{x})^{-1}$, that is
   $(\bar{0})^{-1} = \bar{0} \in X_3, (\bar{3})^{-1} = \overline{13} \in X_3, (\bar{6})^{-1} = \overline{10} \in X_3, (\bar{8})^{-1} = \bar{8} \in X_3, (\overline{10})^{-1} = \bar{6} \in X_3,$.
   $(\overline{13})^{-1} = \overline{13} \in X_3$.
So, X_3 is a rough group.

Next, we form a sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$, where $f(a) = a \bmod 16$, for every $a \in \overline{X_1}$ and g is an identity function. We have $\text{im}(f) = \ker(g) = \mathbb{Z}_{16}$. Hence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is rough exact sequence.

After that, we give $X_1' \subseteq X_1$.
Let $X_1' = \{\bar{1}, \bar{2}, \bar{8}, \overline{14}, \overline{15}\}$, then $\overline{X_1'} = \mathbb{Z}_{16}$. We can see X_1' is a subgroup rough of X, because every element in X_1' has element rough inverse in X_1', and every $x, y \in X_1'$, $x(+_{16})y \in \overline{X_1'}$.

Next, we give $X_2' \subseteq X_2$.
Let $X_2' = \{\bar{5}, \bar{6}, \bar{8}, \overline{10}, \overline{11}\}$, then $\overline{X_2'} = \mathbb{Z}_{16}$. We can see X_2' is a subgroup rough of X, because every element in X_2' has element rough inverse in X_2', and every $x, y \in X_2'$, $x(+_{16})y \in \overline{X_2'}$.

Furthermore, we give $X_3' \subseteq X_3$.

Let $X_3' = \{\overline{3}, \overline{6}, \overline{8}, \overline{10}, \overline{13}\}$, then $\overline{X_3'} = \mathbb{Z}_{16}$. We can see X_3' is a subgroup rough of X, because every element in X_3' has element rough inverse in X_3', and every $x, y \in X_3', x(+_{16})y \in \overline{X_3'}$.

Next, we form a sequence $\overline{X_1'} \xrightarrow{f} \overline{X_2'} \xrightarrow{g} \overline{X_3'}$ with f, g homomorphism rough group is $f: a \bmod 16$, and g: identity function. Since $\overline{X_1'} \xrightarrow{f} \overline{X_2'}$ with $f: a \bmod 16$, and $\overline{X_2'} \xrightarrow{g} \overline{X_3'}$ with g: identity function, we can have $\mathrm{im}(f) = \ker(g) = \mathbb{Z}_{16}$. Hence $\overline{X_1'} \xrightarrow{f} \overline{X_2'} \xrightarrow{g} \overline{X_3'}$ is rough exact sequence.

After we construct rough V-coexact sequence, next we define the properties of rough group in approximation spaces with finite sets

3.2. The Properties in a Rough Groups

Proposition 2 *Given an approximation space (U, θ), V the rough group in the approximation space (U, θ), and $X_1, X_2, X_3, \dots, X_n$ a subgroup of rough group V. If $\overline{X_1} \cap \overline{X_2} \cap \dots \cap \overline{X_n} = \overline{X_1 \cap X_2 \cap \dots \cap X_n}$, then $X_1 \cap X_2 \cap \dots \cap X_n$ is a rough subgroup of V of in approximation space (U, θ).*

Proof. Given a rough group $V, X_1, X_2, X_3, \dots, X_n$ a rough subgroup of V. We can show that $X_1 \cap X_2 \cap \dots \cap X_n$

is a rough subgroup V if $\overline{X_1} \cap \overline{X_2} \cap \dots \cap \overline{X_n} = \overline{X_1 \cap X_2 \cap \dots \cap X_n}$ as follows.

1. We have $X_1 \cap X_2 \cap \dots \cap X_n \neq \emptyset$.
2. For every $x, y \in X_1 \cap X_2 \cap \dots \cap X_n$, we have $x - y \in \overline{X_1}$, $x - y \in \overline{X_2}$, ...and $x - y \in \overline{X_n}$.

So, $X_1 \cap X_2 \cap \dots \cap X_n$ is a rough subgroup of V of in approximation space (U, θ).   □

Furthermore, we give an example of rough subgroup V- of rough groups using the finite set as follows.

Example 3 Let $\mathbb{Z}_{50} = \{\overline{0}, \overline{1}, \overline{2}, .., \overline{49}\}$, we define $a\theta b$ if only if $a - b = 4k$ with $k \in \mathbb{Z}$, for every $a, b \in U$. We know that θ is an equivalence relation on U. From this equivalence relation, we have four equivalence classes in the following table.

Tabel 8. The Equivalence Classes of $\mathbb{Z}_{50}$

The Equivalence Class	The Element of the Class
E_1	$\{\overline{0}, \overline{4}, \overline{8}, \overline{12}, \overline{16}, \overline{20}, \overline{24}, \overline{28}, \overline{32}, \overline{36}, \overline{40}, \overline{44}, \overline{48}\}$
E_2	$\{\overline{1}, \overline{5}, \overline{9}, \overline{13}, \overline{17}, \overline{21}, \overline{23}, \overline{25}, \overline{29}, \overline{33}, \overline{37}, \overline{41}, \overline{45}, \overline{49}\}$
E_3	$\{\overline{2}, \overline{6}, \overline{10}, \overline{14}, \overline{18}, \overline{22}, \overline{26}, \overline{30}, \overline{34}, \overline{38}, \overline{42}, \overline{46}\}$
E_4	$\{\overline{3}, \overline{7}, \overline{11}, \overline{15}, \overline{19}, \overline{23}, \overline{27}, \overline{31}, \overline{35}, \overline{39}, \overline{43}, \overline{47}\}$

Give $X = \{\overline{4}, \overline{5}, \overline{6}, \overline{8}, \overline{10}, \overline{13}, \overline{15}, \overline{21}, \overline{29}, \overline{35}, \overline{37}, \overline{40}, \overline{42}, \overline{44}, \overline{45}, \overline{46}\} \subseteq \mathbb{Z}_{50}$. Then $\underline{X} = \emptyset$, $\overline{X} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_{50}$. Hence the rough set is $\mathrm{Apr}(X) = (\underline{X}, \overline{X}) = (\{\}, \{\overline{0}, \overline{1}, \overline{2}, .., \overline{49}\})$. Next, we define the binary operation $(+_{50})$ on $\mathbb{Z}_{50}$. We will show that X is a rough group.

1. For every $a, b \in X, a(+_{50})b \in \overline{X}$,
2. Association property holds in $\overline{X}$,
3. There exist $\overline{0} \in \overline{X}$ such that for every $x \in X, x(+_{50})\overline{0} = \overline{0}(+_{50})x = x$,
4. In the following table, we can show that every element of X has a rough inverse in X.

Tabel 9. Inverse Table for X

$x \in X$	Inverse of x
$\overline{4}$	$\overline{46}$
$\overline{5}$	$\overline{45}$

$\overline{6}$	$\overline{44}$
$\overline{8}$	$\overline{42}$
$\overline{10}$	$\overline{40}$
$\overline{13}$	$\overline{37}$
$\overline{15}$	$\overline{35}$
$\overline{21}$	$\overline{29}$

Basic in Table 9, every element of X has an inverse in X. So, it proves that X is a rough group on $\mathbb{Z}_{50}$. If we choose a subset of X that is $X_1 = \{\overline{4}, \overline{8}, \overline{10}, \overline{40}, \overline{42}, \overline{46}\}$, we have $\underline{X_1} = \emptyset$ and $\overline{X_1} = E_1 \cup E_3$, so X_1 is a rough set. Then, $X_2 = \{\overline{4}, \overline{8}, \overline{12}, \overline{38}, \overline{42}, \overline{46}\}$, we have $\underline{X_2} = \emptyset$ and $\overline{X_2} = E_1 \cup E_3$, so X_2 is a rough set. We can see that every element of X_1 has a rough inverse in X_1, and for every $x, y \in X_1, x, y \in \overline{X_1}$ so that X_2. Hence X_1 and X_2 are rough groups in $(\mathbb{Z}_{50}, \theta)$, so that it can be said to be a rough group. Furthermore, we will show that $X_1 \cap X_2$ is a rough group in approximation space $\mathbb{Z}_{50}$. Based on the two examples above, we can see $\overline{X_1} \cap \overline{X_2} = \overline{X_1 \cap X_2} = E_1 \cup E_3$. So, $X_1 \cap X_2$ is rough subgroup X.

Moreover, we give proposition about cross product in rough group.

Proposition 3 *If M_1, M_2 are a rough group in the approximation space (U, θ), then $M_1 \times M_2$ ar a rough group in approximation space $(U \times U, \theta^2)$ with $(a_1, a_2)\theta^2(b_1, b_2)$ if only if $(a_1\theta b_1, a_2\theta b_2)$ for every $(a_1, a_2), (b_1, b_2) \in M_1 \times M_2$.*

Proof First, we show that θ^2 is an equivalence relation in $U \times U$.
1. Given $(a, b) \in U \times U$, then $(a, b)\theta^2(a, b)$. So, θ^2 is reflective,
2. given $(a, b), (c, d) \in U \times U$ with $(a, b)\theta^2(c, d)$, then $(c, d)\theta^2(a, b)$. So, θ^2 is symmetrical,
3. given $(a, b), (c, d), (e, f) \in U \times U$ with $(a, b)\theta^2(c, d)$ and $(c, d)\theta^2(e, f)$, then $(a, b)\theta^2(e, f)$. So, θ^2 is transitive.

So, it proves θ^2 is an equivalence relation in $U \times U$.

Next, we will show $M_1 \times M_2$ is a rough group in approximation space $(U \times U, \theta^2)$.
We know $\langle M_1, *_1 \rangle$ and $\langle M_2, *_2 \rangle$ are rough group defined by binary operation $M_1 \times M_2$ is $(a_1, b_1) * (a_2, b_2) = (a_1 *_1 a_2, b_1 *_2 b_2)$.
1. For every $(a_1, b_1), (a_2, b_2) \in M_1 \times M_2, (a_1 *_1 a_2, b_1 *_2 b_2) \in M_1 \times M_2$.
2. Association property holds in $M_1 \times M_2$.
3. There exist $(e_1, e_2) \in \overline{M_1 \times M_2}$ such that for every $a_1 \in M_1$ and $a_2 \in M_2$, $e_1 *_1 a_1 = a_1 *_1 e_1 = a_1$ and $e_2 *_2 a_2 = a_2 *_2 e_2 = a_2$.
4. For every $(a_1, a_2) \in M_1 \times M_2$ has rough invers $(a_1^{-1}, a_2^{-1}) \in M_1 \times M_2$, such that $(a_1, a_2) * (a_1^{-1}, a_2^{-1}) = (a_1 *_1 a_1^{-1}, a_2 *_2 a_2^{-1}) = (e_1, e_2)$.

So, it proves $M_1 \times M_2$ is rough group rough in approximation space $(U \times U, \theta^2)$.   □

Next, we give the illustration of Proposition 3.

Example 4 Given a non-empty set $\mathbb{Z}_6 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$, by definition $\mathbb{Z}_6$ is a relation on expressed as $a\theta b$ with a, b $\in \mathbb{Z}_6$ if and only if $a - b = 2k$ with $k \in \mathbb{Z}$. We have equivalence classes of approximation space $(\mathbb{Z}_6, \theta)$ as follows:
$E_1 = \{\overline{0}, \overline{2}, \overline{4}\}$
$E_2 = \{\overline{1}, \overline{3}, \overline{5}\}$.

Moreover, given rough group $G = \{\overline{1}, \overline{2}, \overline{4}, \overline{5}\}$, and $\overline{G} = \mathbb{Z}_6$. G is called rough group satisfies the following conditions:
1. $a(+_6)b \in \overline{G}$, for every $a, b \in G$,
2. association property hold in G, i.e. $(a + b) + c = a + (b + c)$, for every $a, b, c \in G$,
3. there is a rough identity element $0 \in \overline{G}$ such that $0(+_6)x = x(+_6)0 = x$, for every $x \in G$,
4. every element x of G has rough invers y in G such that $x(+_6)y = y(+_6)x = 0$.

Next, we have a non-empty set $G \times G$ is a rough group in approximation space $(\mathbb{Z}_6 \times \mathbb{Z}_6, \theta^2)$.

$$\mathbb{Z}_6 \times \mathbb{Z}_6 = \{(\bar{0}, \bar{0}), (\bar{0}, \bar{1}), \dots, (\bar{0}, \bar{5}), (\bar{1}, \bar{0}), (\bar{1}, \bar{1}), \dots, (\bar{1}, \bar{5}), (\bar{2}, \bar{0}), (\bar{2}, \bar{1}), \dots, (\bar{2}, \bar{5}),$$
$$= (\bar{3}, \bar{0}), (\bar{3}, \bar{1}), \dots, (\bar{3}, \bar{5}), (\bar{4}, \bar{0}), (\bar{4}, \bar{1}), \dots, (\bar{4}, \bar{5}), (\bar{5}, \bar{0}), (\bar{5}, \bar{1}), \dots, (\bar{5}, \bar{5})\}$$

Relation of θ^2 has equivalence classes as follows:

$E_1 = \{(\bar{0}, \bar{0}), (\bar{0}, \bar{2}), (\bar{0}, \bar{4}), (\bar{2}, \bar{0}), (\bar{2}, \bar{2}), (\bar{2}, \bar{4}), (\bar{4}, \bar{0}), (\bar{4}, \bar{2}), (\bar{4}, \bar{4})\}$,

$E_2 = \{(\bar{0}, \bar{1}), (\bar{0}, \bar{3}), (\bar{0}, \bar{5}), (\bar{2}, \bar{1}), (\bar{2}, \bar{3}), (\bar{2}, \bar{5}), (\bar{4}, \bar{1}), (\bar{4}, \bar{3}), (\bar{4}, \bar{5})\}$,

$E_3 = \{(\bar{1}, \bar{0}), (\bar{1}, \bar{2}), (\bar{1}, \bar{4}), (\bar{3}, \bar{0}), (\bar{3}, \bar{2}), (\bar{3}, \bar{4}), (\bar{5}, \bar{0}), (\bar{5}, \bar{2}), (\bar{5}, \bar{4})\}$,

$E_4 = \{(\bar{1}, \bar{1}), (\bar{1}, \bar{3}), (\bar{1}, \bar{5}), (\bar{3}, \bar{1}), (\bar{3}, \bar{3}), (\bar{3}, \bar{5}), (\bar{5}, \bar{1}), (\bar{5}, \bar{3}), (\bar{5}, \bar{5})\}$.

Furthermore, given $G = \{\bar{1}, \bar{2}, \bar{4}, \bar{5}\}$, we have

$$G \times G = \{(\bar{1}, \bar{1}), (\bar{1}, \bar{2}), (\bar{1}, \bar{4}), (\bar{1}, \bar{5}), (\bar{2}, \bar{1}), (\bar{2}, \bar{2}), (\bar{2}, \bar{4}), (\bar{2}, \bar{5}),$$
$$= (\bar{4}, \bar{1}), (\bar{4}, \bar{2}), (\bar{4}, \bar{4}), (\bar{4}, \bar{5}), (\bar{5}, \bar{1}), (\bar{5}, \bar{2}), (\bar{5}, \bar{4}), (\bar{5}, \bar{5})\},$$

$$\overline{G \times G} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_6 \times \mathbb{Z}_6.$$

We can say $G \times G$ is a rough group, because it satisfies all the properties of rough groups i.e. for every $(a, b) \in G \times G$, then $(a + b) \in \overline{G \times G}$. The association property holds in $\overline{G \times G}$. There exist $(0,0) \in \overline{G \times G}$ such that for every $(x, y) \in G \times G, (x, y)(+_{50})(0,0) = (0,0)(+_{50})(x, y) = (x, y)$. Every element in $G \times G$ has a rough inverse in $G \times G$. Hence, $G \times G$ is a rough group.

4. CONCLUSIONS

The rough V-coexact sequence of the rough groups is the generalization of the rough exact sequence of rough groups. If (U, θ) is an approximation space, A, B, C are the rough groups in (U, θ), and V is the rough subgroups of A in (U, θ), then the sequence $\overline{A} \xrightarrow{f} \overline{B} \xrightarrow{g} \overline{C}$ is a rough V-coexact if $f(\overline{V}) = \ker(g)$. If M_1, M_2 are a rough group in the approximation space (U, θ), then $M_1 \times M_2$ ar a rough group in approximation space $(U \times U, \theta^2)$ with $(a_1, a_2)\theta^2(b_1, b_2)$ if only if $(a_1 \theta b_1, a_2 \theta b_2)$ for every $(a_1, a_2), (b_1, b_2) \in M_1 \times M_2$.

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