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The Implementation of Rough Set on a Group Structure[AH1]

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Abstrak. Diberikan himpunan tak kosong U dan relasi ekuivalensi R pada U. Pasangan berurut (U, R) disebut ruang aproksimasi. Relasi ekuivalensi pada U membentuk kelas-kelas ekuivalensi yang saling asing. Jika $X \subseteq U$, maka dapat dibentuk aproksimasi bawah dan aproksimasi atas dari X. Pada penelitian ini dikonstruksi grup *rough*, subgrup *rough* pada ruang aproksimasi (U, R) terhadap operasi biner yang bersifat komutatif maupun non-komutatif.

Keywords:
lower approximation, upper approximation, rough set, rough group, centralizer.

Abstract. *Let U be a non-empty set and R an equivalence relation on U. Then, (U, R) is an approximation space. The equivalence relation on U forms disjoint equivalence classes. If $X \subseteq U$, we can form a lower approximation and an upper approximation of X. If $X \subseteq U$, then we can form a lower approximation and an upper approximation of X. In this research, rough group and rough subgroups are constructed in the approximation space (U, R) for commutative and non-commutative binary operations.*

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1. Introduction

Zdzislaw Pawlak [1] first introduced the rough set theory in 1982 as a mathematical technique to deal with vagueness and uncertainty problems. Various studies have discussed this theory and the possibility of its applications, for example, in data mining [2] and some algebraic structures. In [3], Biswaz and Nanda introduce the rough group and rough ring. Furthermore, Miao et al. [4] improve definitions of a rough group and rough subgroup and prove their new properties. In [5], Jesmalar investigates the homomorphism and isomorphism of the rough group. Furthermore, in [6], Bagirmaz and Ozcan give the concept of rough semigroups on approximation spaces. Then, Kuroki in [7] gives some results about the rough ideal of semigroups. In [8], Davvaz investigates roughness in the ring, and in [9], Davvaz and Mahdavipour give a roughness in modules. In [10], Isaac and Neelima introduce the concept of the rough ideal. Moreover, in [11], Zhang et al. give some properties of rough modules. Davvaz and Malekzadeh give roughness in modules [12]. They use the notion of reference points. Furthermore, Ozturk and Eren give the multiplicative rough modules [13]. Then, Sinha and Prakash introduce the rough exact sequence of rough modules[14]. They also give the injective module based on rough set theory [15]. In [16], Kazancı and Davvaz give the rough prime in a ring. Jun in [17] investigate the roughness of ideals in BCK-algebras. Moreover, Dubois and Prade [18] define the rough fuzzy sets.

This research focuses on the algebraic aspects by applying a rough set theory to construct a rough group and its subgroups on an approximation space. Moreover, in this research, we discuss the centralizer and the center of a rough group.

2. Preliminaries

In this section, there will be several definitions and theorems that can be helpful for this article. Those definitions are written as follows:

Definition 1 [19] Define $C_G(A) = \{g \in G \mid gag^{-1} = a \text{ for all } a \in A\}$. This subset of G is called the centralizer of A in G. Since $gag^{-1} = a$ if and only if $ga = ag$, $C_G(A)$ is the set of elements of G which commute with every element of A.

Definition 2 [19] Define $Z(G) = \{g \in G \mid gx = xg \text{ for all } x \in G\}$, the set of elements commuting with all the elements of G. This subset of G is called the center of G.

Definition 3 [20] Let R be an equivalence relation on A and $a \in A$. Then the equivalence class of a under R is $[a]_R = \{x : x \in A \text{ and } aRx\}$. In other words, the equivalence class of a under R contains all the elements in A to which a is related by R.

Definition 4 [3] Let (U, R) be an approximation space and X be a subset of U, the sets,

$$\overline{X} = \{x \mid [x]_R \cap X \neq \emptyset\} \quad (1)$$

$$\underline{X} = \{x \mid [x]_R \subseteq X\} \quad (2)$$

are called upper approximation and lower approximation of X.

Definition 5 [1] Let R be an equivalence relation on universe set U, a pair (U, R) is called an approximation space. A subset $X \subseteq U$ can be defined if $\underline{X} = \overline{X}$, in the opposite case, if $\overline{X} - \underline{X} \neq \emptyset$ then X is called a rough set.

Definition 6 [3] Let $K = (U, R)$ be an approximation space and $*$ be a binary operation defined on U. A subset G of universe U is called a rough group if the following properties are satisfied:

i. $\forall x, y \in G, x * y \in \overline{G}$;
ii. Association property holds in $\overline{G}$;
iii. $\exists e \in \overline{G}$ such that $\forall x \in G,\ x * e = e * x = x$; e is called the rough identity element of G;
iv. $\forall x \in G$, $\exists y \in G$ such that $x * y = y * x = e$; y is called the rough inverse element of x in G.

We will give the example of rough group in Section 3.

The following theorem gives the characteristics of a rough group.

Theorem 1. [3] A necessary and sufficient condition for a subset H of rough group G to be a rough subgroup is that:

(i) $\forall x, y \in H, x * y \in \overline{H}$;
(ii) $\forall x \in H, x^{-1} \in H$.

Several steps will be taken to achieve the objectives of this research. Those steps are written as follows:

1. Determine a set U, where $U \neq \emptyset$.
2. Define a relation R on U.
3. Shows that a relation R is the equivalence relation on U.
4. Determine equivalence classes on U.
5. Determine a set G, where $G \subseteq U$ and $G \neq \emptyset$.
6. Determine the approximation space, lower approximation on G $(\underline{G})$, and upper approximation on G $(\overline{G})$.
7. Determine a rough set $Apr(G) = (\underline{G}, \overline{G})$.
8. Determine a binary operation $*$ on the set G.
9. Shows that $\langle G, * \rangle$ is a rough group in the approximation space that has been constructed.
10. Determine a rough subgroup $\langle H, * \rangle$ from a rough group $\langle G, * \rangle$.

3. Rough Group Construction

3.1 Commutative Rough Group Construction

In this section, we will give the construction of commutative rough group.

Example 3.1. Given a non-empty set $U = \{0,1,2,3, \ldots ,99\}$. We define a relation R on the set U, that is, for every $a, b \in U$ apply aRb if and only if $a - b = 7k$ where $k \in \mathbb{Z}$. Furthermore, it can be shown that relation R is reflexive, symmetrical, and transitive. So, relation R is an equivalence relation on U. As a result, relation R produces some disjoint partitions called equivalence classes. The equivalence classes are written as follows:

$$E_1 = [1] = \{1,8,15,22,29,36,43,50,57,64,71,78,85,92,99\};$$
$$E_2 = [2] = \{2,9,16,23,30,37,44,51,58,65,72,79,86,93\};$$
$$E_3 = [3] = \{3,10,17,24,31,38,45,52,59,66,73,80,87,94\};$$
$$E_4 = [4] = \{4,11,18,25,32,39,46,53,60,67,74,81,88,95\};$$
$$E_5 = [5] = \{5,12,19,26,33,40,47,54,61,68,75,82,89,96\};$$

$E_6 = [6] = \{6,13,20,27,34,41,48,55,62,69,76,83,90,97\}$;
$E_7 = [0] = \{0,7,14,21,28,35,42,49,56,63,70,77,84,91,98\}$.

Given a non-empty subset $X \subseteq U$ that is $X = \{10,20,30,40,50,60,70,80,90\}$. Because the set $U \neq \emptyset$ and R is an equivalence relation on U, a pair (U, R) is the approximation space. Furthermore, it can be obtained the lower approximation and upper approximation of X, that is:

$\underline{X} = \emptyset$.
$\overline{X} = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup E_7 = U$.

After determining the lower approximation and upper approximation of X, then given a binary operation $+_{100}$ on X. Here is given Table Cayley of X with the operation $+_{100}$.

Table 1. Table Cayley of X with the operation $+_{100}$

$+_{100}$	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**
10	20	30	40	50	60	70	80	90	0
20	30	40	50	60	70	80	90	0	10
30	40	50	60	70	80	90	0	10	20
40	50	60	70	80	90	0	10	20	30
50	60	70	80	90	0	10	20	30	40
60	70	80	90	0	10	20	30	40	50
70	80	90	0	10	20	30	40	50	60
80	90	0	10	20	30	40	50	60	70
90	0	10	20	30	40	50	60	70	80

i. Based on Table 1, it is proved that for each $x, y \in X$, apply $x(+_{100})y \in \overline{X}$.
ii. For each $x, y, z \in X$, the associative property that is $(x(+_{100})y)(+_{100})z = x(+_{100})(y(+_{100})z)$ holds in $\overline{X}$. The operation $+_{100}$ is associative in $\overline{X}$.
iii. There is a rough identity element $e \in \overline{X}$ that is $0 \in \overline{X}$ such that for each $x \in X$, $x(+_{100})e = e(+_{100})x = x$.

Table 2. Table of element inverse of the set *X*

$\boldsymbol{x}$	10	20	30	40	50	60	70	80	90
$\boldsymbol{x^{-1}}$	90	80	70	60	50	40	30	20	10

iv. For each $x \in X$, there is a rough inverse element of x that is $x^{-1} \in X$ such that $x(+_{100})x^{-1} = x^{-1}(+_{100})x = e$. Based on Table 2, it can be seen that each element x in the set X, then the inverse element x^{-1} is also in X.

Since those four conditions have been satisfied, then $\langle X, +_{100} \rangle$ is a rough group.

3.2 Non-Commutative Rough Group Construction

In this section, we will give the construction of non-commutative rough group.

Example 3.2. Given a permutation group S_3 to the operation of permutation multiplication " $\circ$." For example, take a subgroup $G = \{(1), (12)\}$ of the group S_3. For $x, y \in S_3$, define a relation R that is xRy if and only if $x \circ y^{-1} \in G$. Furthermore, it can be shown that relation R is reflexive, symmetrical, and transitive. So, relation R is an equivalence relation on S_3. As a result, relation R produces some disjoint partitions called equivalence classes. Suppose a is the element in S_3, the equivalence class containing a defined as follows:

$[a]_R = \{x \in S_3 \mid xRa\}$
$\quad\;\; = \{x \in S_3 \mid x \circ a^{-1} \in G\}$

$= \{x \in S_3 \mid x \circ a^{-1} = g,\ g \in G\}$
$= \{x \in S_3 \mid x = g \circ a,\ g \in G\}$
$= \{g \circ a \mid g \in G\}$ (3)

Based on the Equation (3), this is corresponding to the definition of the right coset of G in S_3 that is $Ga = \{g \circ a \mid g \in G\}$. Thus, the right cosets of G in S_3 as follows:

$G \circ (1) = G \circ (12) = \{(1), (1\ 2)\};$
$G \circ (1\ 3) = G \circ (1\ 2\ 3) = \{(1\ 3), (1\ 2\ 3)\};$
$G \circ (2\ 3) = G \circ (1\ 3\ 2) = \{(2\ 3), (1\ 3\ 2)\}.$

Given a non-empty subset $Y \subseteq S_3$ that is $Y = \{(1), (1\ 2), (1\ 2\ 3), (1\ 3\ 2)\}$. Furthermore, it can be obtained the lower approximation and upper approximation of Y, that is:

$\underline{Y} = \{(1), (1\ 2)\}.$
$\overline{Y} = \{(1), (1\ 2)\} \cup \{(1\ 3), (1\ 2\ 3)\} \cup \{(2\ 3), (1\ 3\ 2)\} = S_3.$

After determining the lower approximation and upper approximation of Y, then we give a permutation multiplication "$\circ$" on Y. We give a Table Cayley of Y with the operation of permutation multiplication as follows.

Table 3. Table Cayley of Y with the operation of permutation multiplication

$\circ$	**(1)**	**(1 2)**	**(1 2 3)**	**(1 3 2)**
(1)	(1)	(1 2)	(1 2 3)	(1 3 2)
(1 2)	(1 2)	(1)	(2 3)	(1 3)
(1 2 3)	(1 2 3)	(1 3)	(1 3 2)	(1)
(1 3 2)	(1 3 2)	(2 3)	(1)	(1 2 3)

i. Based on Table 3, it is proved that for each $x, y \in Y$, apply $x \circ y \in \overline{Y}$.
ii. For each $x, y, z \in Y$, the associative property that is $(x \circ y) \circ z = x \circ (y \circ z)$ holds in $\overline{Y}$. The operation $\circ$ is associative in $\overline{Y}$.
iii. There is a rough identity element $e \in \overline{Y}$ that is $(1) \in \overline{Y}$ such that for each $y \in Y$, $y \circ e = e \circ y = y$.

Table 4. Table of inverse element of Y

$\boldsymbol{y}$	(1)	(1 2)	(1 2 3)	(1 3 2)
$\boldsymbol{y^{-1}}$	(1)	(1 2)	(1 3 2)	(1 2 3)

iv. For each $y \in Y$, there is a rough inverse element of y that is $y^{-1} \in Y$ such that $y \circ y^{-1} = y^{-1} \circ y = e$. Based on Table 4, it can be seen that each element y in the set Y, then the inverse element y^{-1} is also in the set Y.
Since those four conditions have been satisfied, then $\langle Y, \circ \rangle$ is a rough group.

4. Subgroup Construction of the Rough Group

After constructing a commutative rough group and a non-commutative rough group, we will construct subgroups of each of the previously constructed rough groups.

4.1 Subgroup Construction of Commutative Rough Group

Before it has been obtained, a commutative rough group X with the operation "$+_{100}$". Furthermore, we will construct several subgroups that can be formed from the rough group X. Based on Theorem 1, we can obtain several subgroups from the rough group X that written as follows:

1. $\langle\{20,30,40,50,60,70,80\}, +_{100}\rangle$;
2. $\langle X, +_{100}\rangle$.

After determining several subgroups from the rough group X that is commutative, then we will determine the centralizer and the center of subgroups in rough group X. Suppose all subgroups of rough group X above are denoted by A. Based on Definition 1, the centralizer A in X is the set where is the element of X is commutative with each element of A. Here is given the table that shows the centralizer of subgroups A in rough group X.

Table 5. Table of the centralizer of subgroups A in rough group X

$\boldsymbol{A}$	$\boldsymbol{C_X(A)}$
$\{20,30,40,50,60,70,80\}$	X
$X = \{10,20,30,40,50,60,70,80,90\}$	X

Since the operation $+_{100}$ of rough group X is commutative, the centralizer of subgroups in rough group X is X itself.

Based on Definition 2, the center of X is the set of elements that is commutative with all elements of X. Because rough group X using commutative operation, the center of rough group X is X itself, or it can be written as $Z(X) = X$.

Using Theorem 1, we will show that the center of rough group X that is $Z(X) = X$ is a rough subgroup of rough group X.

i. Based on Table 1, it is proved that for each $x, y \in Z(X) = X$, apply $x(+_{100})y \in \overline{Z(X)} = \overline{X} = U$.
ii. For each $x \in Z(X) = X$, there is an inverse element of x that is $x^{-1} \in Z(X) = X$. Based on Table 2, it can be seen that if each element x in the set X then the inverse element of x also in the set X.

Two conditions on Theorem 1 have been satisfied, so it is proved that the center of rough group X that is $Z(X) = X$ is a rough subgroup of rough group X.

4.2 Subgroup Construction of Non-Commutative Rough Group

Before it has been obtained a non-commutative rough group Y with the operation of permutation multiplication " $\circ$." Furthermore, we will construct several subgroups that can be formed from the rough group Y. Based on Theorem 1, we can obtain several subgroups from the rough group Y that written as follows:

1. $\langle\{(1)\},\circ\rangle$;
2. $\langle\{(1),(1\ 2)\},\circ\rangle$;
3. $\langle\{(1),(1\ 2\ 3),(1\ 3\ 2)\},\circ\rangle$;
4. $\langle\{(1\ 2),(1\ 2\ 3),(1\ 3\ 2)\},\circ\rangle$;
5. $\langle Y,\circ\rangle$.

After determining several subgroups from the rough set Y that are non-commutative, then we will determine the centralizer and the center of subgroups in rough group Y. Suppose all subgroups of rough group Y above are denoted by B. Based on Definition 1, the centralizer B in Y is the set where is the element of Y is commutative with each element of B. Here is given the table that shows the centralizer of subgroups B in rough group Y.

Table 6. Table of the centralizer of subgroups B in rough group Y

$\boldsymbol{B}$	$\boldsymbol{C_Y(B)}$
$\{(1)\}$	Y
$\{(1),(1\ 2)\}$	$\{(1),(12)\}$
$\{(1),(1\ 2\ 3),(1\ 3\ 2)\}$	$\{(1)\}$
$\{(1\ 2),(1\ 2\ 3),(1\ 3\ 2)\}$	$\{(1)\}$
$Y=\{(1),(1\ 2),(1\ 2\ 3),(1\ 3\ 2)\}$	$\{(1)\}$

Based on Definition 2, the center of Y is the set of elements that is commutative with all elements of Y. From the Definition 2, the center of rough group Y is an identity element, or it can be written as $Z(Y)=\{(1)\}$.

Using Theorem 1, we will show that the center of rough group Y that is $Z(Y)=\{(1)\}$ is a rough subgroup of rough group Y. Previously, determine the upper approximation of $Z(Y)$ that is $\overline{Z(Y)}=\{(1),(1\ 2)\}$.

i. For $(1)\in Z(Y)$, apply $(1)\circ(1)=(1)\in\overline{Z(Y)}$.
ii. For $(1)\in Z(Y)$, there is an inverse element of (1) that is $(1)\in Z(Y)$.

Based on Theorem 1, because the two conditions have been satisfied, it is proved that the center of rough group Y that is $Z(Y)=\{(1)\}$ is a rough subgroup of rough group Y.

5 Conclusions

Based on the results, we construct a rough group, a rough subgroup in the case of the commutative and non-commutative binary operation. Furthermore, the centralizer of a commutative rough subgroup is also a rough group. In comparison, the centralizer of the subgroup of a non-commutative rough group must contain the identity element and the center. The center of each rough group, both commutative and non-commutative, are subgroups of each rough group.

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