The Locating Chromatic Number of a Disjoint Union of Some Double Stars

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Abstract. In this paper we discuss the locating chromatic number of a disconnected graph, namely a disjoint union of some double stars. To determine the locating-chromatic number of a graph, first we construct the upper bound of the number and observe the minimum coloring used. Then, to determine the lower bound of the number, we use the trivial lower bound for some leaves which incident to one vertex. Two main theorems about the locating chromatic number of a disjoint union of some double stars. We obtained some original results of the locating chromatic number of disjoint union of some double stars. Moreover, we can generalize the graph such that the locating chromatic number remains the same with the previous one.

1. Introduction

The locating chromatic number of a graph is a combination between the partition dimension [1] and coloring of a graph. The definition of locating-chromatic number of a disconnected graph is taken from [2] extended based on definition of the locating chromatic number of a connected graph [3].

Let $G=(V,E)$ be a disconnected graph and c be a proper coloring of G using the colors $1,2,\ldots,k$ for some positive integer k. Let $\Pi=\{C_1,C_2,\ldots,C_k\}$ be a partition of $V(G)$, where C_i is the set of colored vertices i (color classes), for $i\in[1,k]$. The color code $c_\Pi(v)$ of vertex v in G is the ordered k-tuple $(d(v,C_1),\ldots,d(v,C_k))$ where $d(v,C_i)=\min\{d(v,x)\mid x\in C_i\}$ for $i\in[1,k]$. If all distinct vertices of G have distinct color codes, then c is called a locating coloring of G using k colors. The locating-chromatic number of G is the smallest k such that G has a locating coloring, denoted by $\chi'_L(G)$, whereas locating chromatic number for a connected graph G, denoted by $\chi_L(G)$.

The following theorem is basic theorem about the locating chromatic number of a graph, proven by Chartrand et al. [3]. The neighborhood of vertex s in a connected graph G, denoted by $N(s)$ is the set of vertices adjacent to s.

Theorem 1.1. *Let c be a locating coloring in a connected graph G. If s and t are distinct vertices of G such that $d(s,u)=d(t,u)$ for all $u\in V(G)-\{s,t\}$, then $c(s)\neq c(t)$. In particular, if s and t are non adjacent vertices of G such that $N(s)=N(t)$, then $c(s)\neq c(t)$.*

Corollary 1.1. *If G is a connected graph and there is a vertex adjacent to k leaves, then $\chi_L(G)\geq k+1$.*

Chartrand et al. [3] in 2002 determined the locating chromatic number for some classes of graph such that paths, stars, complete graph, and double stars. A double stars is a tree which has two vertices x , y which have degree bigger than 1. Let degree of x be $a+1$ (deg (x) = $a+1$)) and deg (y) = $b+1$, then we denoted the double stars $S_{a,b}$. For any a and b , where $1\leq a\leq b$ and $b\geq 2$, $\chi_L(S_{a,b})=b+1$.

Next, many researchers continue research about the locating chromatic number, especially for trees. Asmiati et al. [4, 5] determined the locating chromatic number for amalgamation of stars and firecracker graphs, respectively. Moreover, Des Welyyanti [6] also obtained the locating chromatic number for tree, namely complete n-ary tree. Asmiati [7] successes to determine the locating chromatic number of nonhomogeneous amalgamation of stars, obtaining the general results of Asmiati et al.[4].

Besides that, for characterizing graphs, Chartrand et al. [3] determined graphs having locating chromatic number (n-1) or (n-2), Baskoro and Asmiati [8] characterized all trees with locating chromatic number 3. Next, Asmiati and Baskoro [9] characterized some of graphs containing cycle with locating chromatic number three.

Locating chromatic number of a disconnected graph firstly studied by Des Welyyanti et al. [6]. They determined the locating chromatic number of a uniform linier forest, namely a disjoint union of some paths with the same length.

In this paper, we will discuss the locating chromatic number of a disjoint union of some double stars, partially for $a=b=n$ for m disconnected double stars, we denoted $S^i_{n,n}$ where $i\epsilon[1,m]$ and $m=1,2,\ldots,\left\lceil\frac{n}{2}\right\rceil$. We denote the vertices of $S^i_{n,n}$ a^i and b^i as the vertices of degree $(n+1)$, respectively. The leaves incident to a^i are denoted by x^i_j, while the leaves incident to b^i are denoted by y^i_j, for $i\epsilon[1,m]$ and $j\epsilon[1,n]$. The disjoint union of some double stars where $n\geq 3$, $i\epsilon[1,m]$, $m=1,2,\ldots,\left\lceil\frac{n}{2}\right\rceil$, is denoted by A=$\bigcup_{i=1}^{m}S^i_{n,n}$.

2. Methods

To determine the locating chromatic number of a disjoint union of some double stars $A=\bigcup_{i=1}^{m}S^i_{n,n}$, we construct the upper bound and the lower bound. The trivial lower bound of the locating chromatic number for this graph is derived from Corollary 1.1. Next, we construct the minimum locating coloring to obtain the upper bound. Finally, we have the result desired, the locating chromatic numbe of A, $\chi'_L(A)$. The next step, we subdivide some edges of $S^i_{n,n}$ to get the maximal edges. The new graph obtained is denoted by $S^{i*}_{n,n}$ and we denote D =$\bigcup_{i=1}^{m}S^{i*}_{n,n}$. We do the subdivision such that $\chi'_L(A)=\chi'_L(D)$.

3. Results and discussion.

In this section, we will discuss the locating chromatic number of a disjoint union of some double stars and subdivisions.

Theorem 3.1
Let $S^i_{n,n}$ be a disjoint union of some double stars, where $n\geq 3$, $i\epsilon[1,m]$,

$m = 1,2,\dots,\left\lceil\frac{n}{2}\right\rceil$, and $A = \cup_{i=1}^{m} S_{n,n}^{i}$. Then $\chi'_L(A) = n + 1$.

Proof:

First, we determine the lower bound of $\chi'_L(A)$. Since every vertex on a^m, b^m for $m = 1,2,\dots,\left\lceil\frac{n}{2}\right\rceil$ adjacent to n leaves, then by Corollary 1.1, we have $\chi'_L(A) \geq n + 1$. Next, we determine the upper bound of $\chi'_L(A)$. Let c be a proper coloring of A using $(n + 1)$ colors. We assign the vertices of $V(A)$:

a. Each color of a^i or b^k in $V(A)$ are different, where $i = k$ or $i \neq k$ $, a^i, b^k \in \{1,2,\dots,n+1\}$ and $i, k \in [1, m]$.

b. $\{c(x_j^i)\} = \{1,2,\dots,n+1\} \backslash c(a^i)$, $i \in [1, m]$ and $j \in [1, n]$.

c. $\{c(y_j^i)\} = \{1,2,\dots,n+1\} \backslash c(b^i)$, $i \in [1, m]$ and $j \in [1, n]$.

the coloring c will induces a partition $\prod = \{C_1, C_2, \dots, C_{n+1}\}$ of $V(A)$.
Next, we will show that the color code of every vertex on $V(A)$ is unique. Consider $u, v \in V(A)$ and $c(u) = c(v)$, then we have some cases :

- If $c(a^i) = c(y_j^k)$, where $i = k$ or $i \neq k$, then $c_\Pi(a^i)$ contains at least three components of value 1, whereas $c_\Pi(y_j^k)$ contains exactly one component of value 1. Thus, $c_\Pi(a^i) \neq c_\Pi((y_n^k)$.
- If $c(b^i) = c(x_j^k)$, where $i = k$ or $i \neq k$, then $c_\Pi(b^i)$ contains at least three components of value 1, whereas $c_\Pi((x_j^k)$ contains exactly one component of value 1. Thus, $c_\Pi(b^i) \neq c_\Pi(x_j^k)$.
- If $u = x_j^i$ and $v = y_j^k$. Then $c_\Pi(x_j^i) \neq c_\Pi(y_j^k)$ because $c(x_j) \neq c(x_j)$.

From the cases above, it is clear that every vertex on $V(A)$ has different color codes. Therefore, c is a locating coloring and $\chi'_L(A) \leq n + 1$.

Next we will subdivide some edge of the disjoint union of some double stars. Graph $S_{n,n}^{i*}$ is obtained from graph $S_{n,n}^{i}$ by inserting one vertex between a^i and b^i (we denote the new vertex by c^i). Next we will prove that the locating chromatic number of the new graph is the same with the locating chromatic number of the previous graph. Consider $V(S_{n,n}^{i*}) = \{a^i, b^i, x_{jk}^i, y_{jk}^i \mid i = 1,2,3 \dots, m; j = 1,2,3,\dots n; k = 1,2,3,\dots l\}$.

Theorem 3.2.
Let $S_{n,n}^{i*}$ be a disjoint union of some double stars with subdivisions, where $n \geq 3$, $i \in [1, m]$, $m = 1,2,\dots,\left\lceil\frac{n}{2}\right\rceil$, and $D = \cup_{i=1}^{m} S_{n,n}^{i*}$. Then $\chi'_L(D) = n + 1$.

Proof:
First, we determine the lower bound of $\chi'_L(D)$. Since each vertex a^m and b^m, for $m = 1,2,\dots,\left\lceil\frac{n}{2}\right\rceil$ adjacent to n leaves, then by Corollary 1.1, we have $\chi'_L(D) \geq n + 1$.

Now, let c be a coloring on $V(D)$ using $(n + 1)$ colors. The colors for vertices a^i, b^i, x_j^i, and y_j^i are following the proof of Theorem 3.1. The colors for vertex c^i is one of $\{1,2,\dots,n+1\}$. We assign the alternating colors $c(a^i)$ or $c(x_j^i)$, to the vertices x_{jl}^i, respectively. We do the coloring similarly for vertices y_{jl}^i, we assign alternating colors $c(b^i)$ or $c(y_j^i)$, respectively. Observe that these color codes are unique and for the remaining of the vertices, the color codes are also unique as proven in Theorem 3.1. So, *c* is a locating coloring and $\chi'_L(D) \leq n+1$.

The following figure shows the minimum locating coloring on $S_{3,3}^{2*}$.

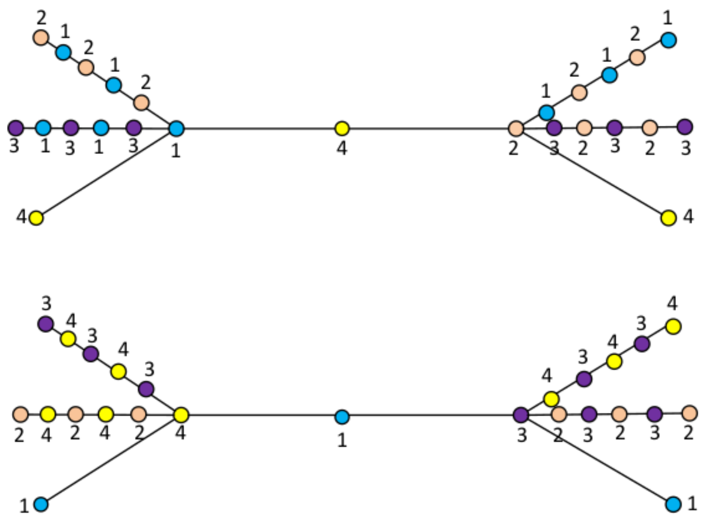

Figure 1. The Minimum locating chromatic number of $S_{3,3}^{2*}$

4. Conclusions

By doing the subdivision to $(n-1)$ pendant edges and adding one vertex to some edge *ab* of A such that we have the maximal edges, we have new graph, denote by D. We prove that the locating chromatic number of A and D are the same.

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