HALAMAN PENGESAHAN

- 1. Judul Artikel : The Locating-Chromatic Number for Certain Operation o Generalized Petersen Graphs sP(4,2)
 - Nama Jurnal : Journal of Physics: Conf. Series Vol 1338 Th. 2019
 - ISSN : 17426588, 17426596
 - : A Irawan, Asmiati, Suharsono, and K Muludi
 - : Jurnal Terindex Scopus Q4
 - https://iopscience.iop.org/article/10.1088/1742-6596/1338/1/012033/pdf

Bandar Lampung, 27 September 2021

Penulis,

Dr. Ir. Kurnia Muludi, M.S.Sc. NIP 19640616 1989021 001

Didik Kurniawan, S.Si., M.T 198004192005011004

Ketua Jurtsan,

2.

3.

4.

5.

6.

Penulis

Jenis Publikasi

Alamat web

Mengetahui,

Dekan Pakultas MIPA, ERSITAS LAL

Dr. Eng. Suripto Dwi Yuwono, S.Si., M.T. NIP. 197407052000031001

Ketua LPPM.

Dr. Lusmeilia Afriani, D.E.A.

NIP 196505101993032008 /

/ENCHIAS LARPUNC *** 10/11/2011 :1 VO TIVE! 27 2171 P JENIS. L1342

Cover

Annual (Physis) The second se	The open access <i>Journal of Physics: Conference Series (JPCS)</i> provides a fast, versatile and cost-effective proceedings publication service.	Latest published conferences	JOURNAL LINKS
		Vol 2020 🗸 Go	Journal home
			Journal Scope
		2021	Information for organizers
		2021	Information for authors
		View forthcoming volumes accepted for	Contact us
ou would like m l if you are inter nference organi	ore detailed information regarding <i>Journal of Physics: Conference</i> ested in publishing a proceedings with IOP Conference Series pl zers can use our <u>online form</u> and we will get in touch with a quo	e Series please visit <u>conferenceseries.iop.org</u> , ease visit our page for <u>conference organizers</u> . te and further details.	Associates
ou would like m d if you are inter nference organi Most read	ore detailed information regarding <i>Journal of Physics: Conference</i> ested in publishing a proceedings with IOP Conference Series pl zers can use our <u>online form</u> and we will get in touch with a quo Open all abstracts	e Series please visit <u>conferenceseries.iop.org</u> , ease visit our page for <u>conference organizers</u> . te and further details.	Associates
ou would like m d if you are inter nference organi Most read Most cited	ore detailed information regarding <i>Journal of Physics: Conference</i> ested in publishing a proceedings with IOP Conference Series pl zers can use our <u>online form</u> and we will get in touch with a quo Open all abstracts OPEN ACCESS	e Series please visit <u>conferenceseries.iop.org</u> , ease visit our page for <u>conference organizers</u> . te and further details.	Associates
ou would like m d if you are inter nference organi Most read Most cited Latest articles	ore detailed information regarding <i>Journal of Physics: Conference</i> ested in publishing a proceedings with IOP Conference Series ple zers can use our <u>online form</u> and we will get in touch with a quo Open all abstracts OPEN ACCESS Preface	e Series please visit <u>conferenceseries.iop.org</u> , ease visit our page for <u>conference organizers</u> . te and further details.	Associates
you would like m di fi you are inter onference organi Most read Most cited Latest articles	ore detailed information regarding <i>Journal of Physics: Conference</i> ested in publishing a proceedings with IOP Conference Series plu zers can use our <u>online form</u> and we will get in touch with a quo Open all abstracts OPEN ACCESS Preface 2021 <i>J. Phys.: Conf. Ser.</i> 1999 011001	e Series please visit <u>conferenceseries.iop.org</u> , ease visit our page for <u>conference organizers</u> . te and further details.	Associates
you would like m d if you are inter inference organi Most read Most cited Latest articles	ore detailed information regarding <i>Journal of Physics: Conference</i> ested in publishing a proceedings with IOP Conference Series plu zers can use our <u>online form</u> and we will get in touch with a quo Open all abstracts OPEN ACCESS Preface 2021 <i>J. Phys.: Conf. Ser.</i> 1999 011001 + Open abstract IView article PDF	e Series please visit <u>conferenceseries.iop.org</u> , ease visit our page for <u>conference organizers</u> . te and further details.	Associates Raman Spectroscopy for Reactio Advancement Monitoring CLICK HERE FOR MORE DETALS JOURNAL HISTORY 2004-present Journal of Physic Conference Series doi:10.1088/issn.1742-6596 Online ISSN: 1742-6598

Daftar Isi



PAPER • OPEN ACCESS

The Locating-Chromatic Number for Certain Operation of Generalized Petersen Graphs sP(4,2)

To cite this article: A Irawan et al 2019 J. Phys.: Conf. Ser. 1338 012033

View the article online for updates and enhancements.



IOP ebooks[™]

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

The Locating-Chromatic Number for Certain Operation of Generalized Petersen Graphs sP(4, 2)

A Irawan^{1,2,a}, Asmiati ^{3,b}, Suharsono^{3,c}, and K Muludi^{4,d}

 ¹ Posgraduate student, Faculty of Mathematics and Natural Sciences Lampung University, Jl. Sumantri Brodjonegoro No.1, Bandar Lampung, Indonesia.
 ² School of Information and Computer Management (STMIK) Pringsewu Lampung. Jl. Wisma Rini No.09, Pringsewu, Lampung, Indonesia
 ³ Departement Mathematics, Faculty of Mathematics and Natural Sciences Lampung University. Jl. Sumantri Brodjonegoro No.1, Bandar Lampung, Indonesia
 ⁴Computer Sciences, Faculty of Mathematics and Natural Sciences Lampung University. Jl. Sumantri Brodjonegoro No.1, Bandar Lampung, Indonesia

^aagusirawan814@gmail.com; ^basmiati.1976@fmipa.unila.ac.id; ^csuharsono.1962@fmipa.unila.ac.id; ^dkurnia.muludi@fmipa.unila.ac.id

Abstract. The locating-chromatic number of a graph combined two graph concept, coloring vertices and partition dimension of a graph. The locating-chromatic number, denoted by $\chi_L(G)$, is the smallest k such that *G* has a locating *k*-coloring. In this paper, we discuss the locating-chromatic number for certain operation of generalized Petersen graphs sP(4,2).

1. Introduction

Chartrand et *al.* [1] in 2002 introduced the locating-chromatic number of a graph, with derived two graph concept, coloring vertices and partition dimension of a graph. Let G = (V, E) be a connected graph and *c* be a proper *k*-coloring of *G* with color 1,2, ..., *k*. Let $\Pi = \{C_1, C_2, ..., C_k\}$ be a partition of *V*(*G*) which is induced by coloring *c*. The color code $c_{\Pi}(v)$ of *v* is the ordered *k*-tuple $(d(v, C_1), d(v, C_2), ..., d(v, C_k))$ where $d(v, C_i) = \min \{d(v, x) | x \in C_i\}$ for any *i*. If all distinct vertices of *G* have distinct color codes, then *c* is called *k*-locating coloring of *G*. The locating-chromatic number, denoted by $\chi_L(G)$, is the smallest *k* such that *G* has a locating *k*-coloring.

In 2003, Chartrand et *al.* [2] successed in constructing $n \ge 5$ tree graphs with locating-chromatic numbers ranging from 3 to *n*, except (n - 1). Behtoe and Omoomi [3] found the locating-chromatic numbers on the Kneser graph. Furthermore, Baskoro and Purwasih [4] found the locating chromatic number for corona product of graphs. Next, Asmiati [5] determined the locating chromatic number of banana tree graph and Asmiati et *al.* [6] for amalgamation of stars graphs. Asmati et *al.* [7] also found the locating chromatic number of firecracker graphs and Syofyan et *al.* [8] for lobster graph.

Specially for non-homogenous tree graph in 2014, Asmiati [9] determined the locating-chromatic number of non-homogeneous amalgamation of stars, then Asmiati [10] for caterpillar graphs and non-homogenous firecracker graphs. In 2017, Asmiati et *a*l. [11] determined some generalized Petersen graphs P(n, 1) having locating-chromatic number 4 for odd $n \ge 3$ or 5 for even $n \ge 4$.

The generalized Petersen graph P(n,k), $n \ge 3$ and $1 \le k \le \lfloor (n-1)/2 \rfloor$, consists of an outer *n*-cycle $u_1, u_2, ..., u_n$, a set of *n* spokes $u_i, v_i, 1 \le i \le n$, and *n* edges $v_i, v_{i+k}, 1 \le i \le n$, with indices taken modulo *n*. The generalized Petersen graph was introduced by Watkins in [12].

To define the generalized Petersen graph sP(4,2), suppose there are sgeneralized Petersen graph P(4,2). Some vertices on the outer cycle u_i , i = 1,2,3,4 for the generalized Petersen graph t^{th} , t = 1,2,...,s, $s \ge 1$ denoted by u_i^t , while some vertices on the inner cycle v_i , i = 1,2,3,4 for the generalized Petersen graph t^{th} , t = 1,2,...,s, $s \ge 1$ denoted by u_i^t , while some vertices on the inner cycle v_i , i = 1,2,3,4 for the generalized Petersen graph t^{th} , t = 1,2,...,s, $s \ge 1$ denoted by v_i^t . Generalized Petersen graph sP(4,2) obtained from $s \ge 1$ graph P(4,2), which every vertices on the outer cycle u_i^t , $i \in [1,4]$, $t \in [1,s]$ connected by a path $(u_i^t u_i^{t+1}) t = 1,2,...,s - 1$, $s \ge 2$.

Some researchers have determined the locating-chromatic number for certain operation. Behtoei and Omoomi [13] obtained locating-chromatic number from the grid, cartesian multiplication for trajectories and complete graphs, and cartesian multiplication of two complete graphs. Furthermore Behtoei and Omoomi [14] determined the locating-chromatic number of the fan graph, wheel and friendship graph for join multiplication of two graphs. Asmiati [15] foundlocating-chromatic number for certain operation of tree. In this paper, we discuss the locating-chromatic number for certain operation of generalized Petersen graphs sP(4,2).

The following theoremsis basic to determine the locating chromatic number of a graph. The set of neighbours of a vertex s in G, denoted by N(s).

Theorem 1.1.Chartrand et al.[1] Let c be a locating coloring in a connected graph G. If r and s are distinct vertices of G such that d(r,w)=d(s,w) for all $w \in V(G) - \{r,s\}$, then $c(r) \neq c(s)$. In particular, if x and y are non-adjacent vertices of G such that $N(x) \neq N(y)$, then $c(x) \neq c(y)$.

Theorem 1.2.Chartrand et *al.*[1] *The locating chromatic number of a cycle* C_n *, is 3 for odd n and 4 for otherwise.*

2. Results and Discussion

In this section we will discuss the locating chromatic number of sP(4,2).

Theorem 2.1. The locating chromatic number of generalized Petersen graph sP(4,2) is 5 for $s \ge 2$.

Proof : First, we determine lower bound of $\chi_L(sP(4,2))$ for $s \ge 2$. Because generalized Petersen graph P(4,2), for $s \ge 2$, contains some even cycles. Then by Theorem 2, $\chi_L(sP(4,2)) \ge 4$. Next, we will show that $\chi_L(sP(4,2)) \ge 5$, for $s \ge 2$. For a contradiction, suppose that c is 4-locating coloring on $sP_{4,1}$ for $s \ge 2$. Consider $c(u_i^1) = i, i = 1,2,3,4$ and $c(v_j^1) = j, j = 1,2,3,4$ such that $c(u_i^1) \neq c(v_j^1)$ for $c(u_i^1)$ adjacent to $c(v_j^1)$. Observe that if we assign color 4 for any vertices in u_i^2 or v_i^2 , then we have two vertices which have color codes. Therefore, c is not locating 4-coloring on sP(4,2). As the result $\chi_L(sP(4,2)) \ge 5$ for $s \ge 2$.

Next, we determine the upper bound of $\chi_L(sP(4,2))$ for $s \ge 2$. Let *c* be a coloring of generalized Petersen graph sP(4,2) for $s \ge 2$. We make the partition of the vertices of V(sP(4,2)):

 $\begin{array}{l} C_1 = \{u_1^t | \text{for odd } s\} \cup \{u_2^t, v_4^t | \text{for even } s\} \\ C_2 = \{u_2^t, u_4^t | \text{for odd } s\} \cup \{u_3^t, v_1^t | \text{for even } s\} \\ C_3 = \{u_3^t, v_1^t, v_2^t | \text{for odd } s\} \cup \{u_4^t, v_2^t, v_3^t | \text{for even } s\} \\ C_4 = \{v_3^t | \text{for odd } s\} \cup \{u_1^t | \text{untuk sgenap}\} \cup \{v_4^t | \text{for odd } s \ge 3\} \\ C_5 = \{v_4^1\} \end{array}$

Therefore the color codes of all the vertices of G are :

(a) $C_1 = \{u_1^t | \text{for odd } s\} \cup \{u_2^t, v_4^t | \text{for even } s\}$

For odd *s*, the color codes of sP(4,2) are:

$$c_{\Pi}(u_1^t) = \begin{cases} 0 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd}, 3^{rd} \text{ and } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For even *s*, the color codes of sP(4,2) are:

	(0	,	for 1 st component
$c_{\Pi}(u_2^t) = -$	{ 1	,	for 2^{nd} , 3^{rd} and 4^{th} component
	(s+1)	,	for 5 th component
	(0	,	for 1 st component
$a(n^{t}) =$	2	,	for 2 nd and 4 th component
$c_{\Pi}(v_4) = \langle$	1	,	for 3 rd component
	s+1	,	for 5 th component

(b) $C_2 = \{u_2^t, u_4^t | \text{for odd } s\} \cup \{u_3^t, v_1^t | \text{for even } s\}$ For odd *s* the color codes of sP(4,2) are:

$$c_{\Pi}(u_2^t) = \begin{cases} 1 & , \text{ for } 1^{st} \text{ and } 3^{rd} \text{ component} \\ 0 & , \text{ for } 2^{nd} \text{ component} \\ 4 & , \text{ for } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \\ 0 & , \text{ for } 1^{st}, 3^{rd} \text{ and } 5^{th} \text{ component} \\ 2 & , \text{ for } 4^{th} \text{ component} \end{cases}$$

For odd $s \ge 3$, the color codes of sP(4,2) are:

color codes of
$$SP(4,2)$$
 are:
 $c_{\Pi}(u_4^t) = \begin{cases} 1 & , \text{ for } 1^{st}, 3^{rd} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 2^{nd} \text{ component} \\ s & , \text{ for } 5^{th} \text{ component} \end{cases}$

For even *s*, the color codes of sP(4,2) are:

$$c_{\Pi}(u_3^t) = \begin{cases} 1 & , \text{ for } 1^{st} \text{ and } 3^{ra} \text{ component} \\ 0 & , \text{ for } 2^{nd} \text{ component} \\ 2 & , \text{ for } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \\ s+1 & , \text{ for } 1^{st} \text{ component} \\ 0 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 3^{rd} \text{ and } 4^{th} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

(c) $C_3 = \{u_3^t, v_1^t, v_2^t | \text{for odd } s \} \cup \{u_4^t, v_2^t, v_3^t | \text{for even } s \}.$ For odd *s*, the color codes of *sP*(4,2) are:

$$c_{\Pi}(u_3^t) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \\ 1 & , \text{ for } 1^{st} \text{ and } 4^{th} \text{ component} \\ 2 & , \text{ for } 1^{st} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

IOP Publishing

$$c_{\Pi}(v_2^1) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ and } 5^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ 3 & , \text{ for } 4^{th} \text{ component} \end{cases}$$

For odd $s \ge 3$ the color codes of sP(4,2) are:

$$c_{\Pi}(v_2^t) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ and } 4^{th} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For even *s* the color codes of sP(4,2) are:

$$c_{\Pi}(u_4^t) = \begin{cases} 1 & , \text{ for } 1^{st}, 2^{nd} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s & , \text{ for } 5^{th} \text{ component} \\ 1 & , \text{ for } 1^{st} \text{ component} \\ 2 & , \text{ for } 1^{st} \text{ component} \\ 2 & , \text{ for } 2^{nd} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s + 2 & , \text{ for } 5^{th} \text{ component} \\ s + 2 & , \text{ for } 5^{th} \text{ component} \\ 1 & , \text{ for } 1^{st} \text{ and } 4^{th} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ component} \\ 1 & , \text{ for } 3^{rd} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s + 2 & , \text{ for } 5^{th} \text{ component} \\ s + 2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

(d) $C_4 = \{v_3^t | \text{for odd } s \} \cup \{u_1^t | \text{for even } s \} \cup \{v_4^t | \text{for odd } s \ge 3 \}$ For odd *s* the color codes of *sP*(4,2) are:

$$c_{\Pi}(v_3^t) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ and } 2^{nd} \text{ component} \\ 1 & , \text{ for } 3^{rd} \text{ component} \\ 0 & , \text{ for } 4^{th} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For odd $s \ge 3$ the color codes of sP(4,2) are:

$$c_{\Pi}(v_4^t) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ and } 3^{rd} \text{ component} \\ 0 & , \text{ for } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For even *s* the color codes of sP(4,2) are:

$$c_{\Pi}(v_4^t) = \begin{cases} 1 & , \text{ for } 1^{st}, 2^{nd} \text{ and } 3^{rd} \text{ component} \\ 0 & , \text{ for } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

(e) $C_5 = \{v_4^1\}$ $c_{\Pi}(v_4^1) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ and } 3^{rd} \text{ component} \\ 3 & , \text{ for } 4^{th} \text{ component} \\ 0 & , \text{ for } 5^{th} \text{ component} \end{cases}$

Since all the vertices have different color codes, *c* is a locating coloring of generalized Petersen graphs sP(4,2), so $\chi_L(sP(4,2)) = 5$, for even $s \ge 2$.

In figure 1 is illustrated a locating coloring of generalized Petersen graphs 4P(4,2) with the locating chromatic number 5.



Figure 1. A minimum locating coloring of 4P(4,2)

3. Conclusion

Based on the results, locating chromatic number of generalized Petersen graph sP(4,2) is 5 for $s \ge 2$.

References

- [1] Chartrand G, Erwin D, Henning M, Slater P and Zhang P 2002 The locating-chromatic number of a graph *Bull. Inst. Combin. Appl.* **36** pp 89–101
- [2] Chartrand G, Erwin D, Henning M, Slater P, and Zhang P 2003 Graph of order n with locatingchromatic number *n*-1 *Discrete Math* **269** 1-3 pp 65–79
- [3] Behtoei A and Omoomi B 2011 On the locating chromatic number of kneser graphs *Discrete Applied Mathematics* **159** pp 2214–2221
- [4] Baskoro E T and Purwasih I 2012 The locating-chromatic number for corona product of graphs Southeast-Asian *J. Of Sciences* **1** pp 124–134
- [5] Asmiati 2017 Locating chromatic number of banana tree *International Mathematical Forum* **12** (1) pp 39–45
- [6] Asmiati, Assiyatun H and Baskoro E T 2011 Locating-chromatic number of amalgamation of stars ITB *J.of Sci.* **43** 1 pp 1–8

- [7] Asmiati, Baskoro E T, Assiyatun H, Suprijanto D, Simanjuntak R, and Uttunggadewa S 2012 The Locating-chromatic number of firecracker graphs *Far East Journal of Mathematical Sciences* 63 1 pp 11 – 23
- [8] Syofyan D K, Baskoro E T, Assiyatun H 2013 The locating-chromatic number of homogeneous lobsters *AKCE Int. J. Graphs Comb.* **10** 3 pp 215 252
- [9] Asmiati 2014 The locating-chromatic number of non-homogeneous amalgamation of stars *Far East Journal of Mathematical Sciences* **93** 1 pp 89 – 96
- [10] Asmiati 2016 On the locating-chromatic numbers of non-homogeneous caterpillars and firecrackers graphs *Far East Journal of Mathematical Sciences* **100** 8 pp 1305 1316
- [11] Asmiati, Wammiliana, Devriyadi and Yulianti L 2017 On some Petersen graphs having locating chromatic number four or five *Far East Journal of Mathematical Sciences* 102 4 pp 769 – 778
- [12] Watkins M E 1969 A theorem on tait colorings with application to the generalized Petersen graphs *Journal of Combinatorial Theory* **6** pp 152-164