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ZERO GRAVITY OF FLOW UNDER A SLUICE GATE

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Abstract

Free surface flow is a classical problem in hydrodynamics, such as flow under a sluice gate. The ratio between the uniform depth at far downstream and the height of the gate, called contraction coefficient, is important in building the sluice gate. In this report, that number is calculated based on a model of potential flow, without involving gravity. A conformal mapping and defining an analytical function can solve the model, and the contraction coefficient gives 0.611, that agrees to the result in reference, i.e., $\pi/(2+\pi)$.

1. Introduction

This paper is concerned with the formulation of an integral equation from a free-surface flow under a sluice gate. Physically, the water occupies a

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dam having a slit at the bottom corner, and the water depth is much higher than the width of the slit. When the slit is opened, the water comes through the slit and it forms stream uniformly far from the slit. This flow can be determined by knowing the free surface profile. Most of works relating to sluice gate flow are for finite depth of the fluid far upstream. The model is an integral equation, and solved by boundary element method. See for example, Frangmeier and Strelkoff [1]; Loroch [2]; Chung [3]; also Asavanant and Vanden Broeck [4]; Vanden Broeck [5]; and Binder and Vanden Broeck [6]. The solutions are characterized by uniform and supercritical flow far downstream, and the flow at upstream can be supercritical or subcritical. However, the calculation is basically relatively close to the gate, so that it produces contraction coefficient less accurate, since the numerical procedure is done by truncating the domain.

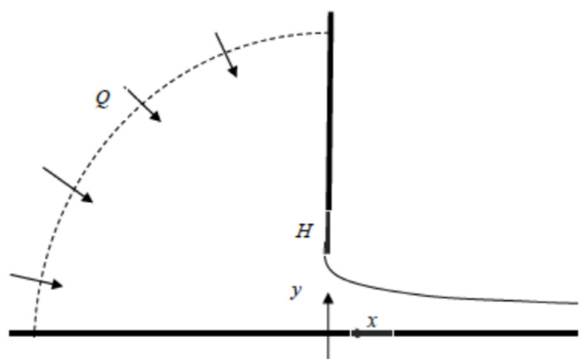

Figure 1. Sketch of the flow and the coordinates.

Alternatively, the sluice gate flow is modeled without involving gravity. It is simpler but we need to construct an analytical function. When it is done the analytical solution can be obtained, and the uniform depth at the downstream can be calculated, representing the contraction coefficient as the problem has been worked in non-dimensional with respect to the height of the gate. Wiryanto [7, 8] have done the zero gravity case for a jet, and also in Wiryanto [9] for the case of flow over a weir. Recently, Wiryanto [10] calculated the contraction coefficient of the flow in Borda Moutpiece.

2. Problem Formulation

The free surface flow that we consider is illustrated in Figure 1. The flow domain is bounded by horizontal and vertical walls, and a free surface. The vertical wall is lifted up above the horizontal wall, forming a slit width *H*, so that the fluid flows through the slit. Since the upstream fluid is relatively deeper than height of the slit, we assume that the volume flux *Q* is crossing a quarter of circle far from the slit. The fluid velocity decreases for larger circle. As the coordinates, we choose Cartesian with the *x* axis along the horizontal wall and the *y* axis at the vertical wall.

The flow domain is between streamline along the horizontal bottom wall and streamline of vertical wall continued as the free surface. So that the flow domain can be expressed in complex potential $f = \phi + i\psi$ of non-dimensional so that it is in strip-line $0 < \psi < 1, -\infty < \phi < \infty.$ Any point in that domain has velocity $\frac{df}{dz} = e^{\Omega},$ where $\Omega = \tau - i\theta,$ the speed related to τ, and the direction corresponds to θ. So, physically the boundary has $\theta = 0$ along the bottom wall and $\theta = -\pi/2$ along the vertical wall. Our task is to determine θ along the free surface. This can be obtained when we know the relation between θ and τ, through an analytical function. Both then satisfy the dynamics condition, i.e. the pressure is zero along the surface, presented by Bernoulli equation.

3. Analytical Details

The mathematical description of the problem is presented in non-dimensional variables, based on unity of the velocity and length at uniform flow downstream. First, the flow domain in *f*-plane is transformed in an artificial domain $\zeta = \xi + i\eta,$ by using $f = \frac{-1}{\pi}\log\zeta,$ the strip line is mapped into half below ζ-plane. This transformation can be seen in zero gravity problems given by Wiryanto [7, 8, 9, 10]. The streamlines $\psi = 0$

and $\psi = 1$ as the boundary of the flow are transformed into the real axis of ζ, where the horizontal bottom wall is mapped into $\eta = 0, \xi > 0,$ the vertical wall is mapped into $\eta = 0, \xi < -1,$ and the free surface is mapped into $\eta = 0, -1 < \xi < 0.$

Along the solid boundary, the value of θ follows the direction of the stream on the walls. Meanwhile the value along the free boundary $-1 < \xi < 0$ is unknown, but θ and τ must satisfy the dynamic condition as representative of the velocity and the surface elevation related by

$$\frac{dz}{d\zeta} = \frac{dz}{df}\frac{df}{d\zeta} = e^{-\Omega}\left(-\frac{1}{\pi\zeta}\right).$$

The real and imaginary parts of that relation give

$$\frac{dx}{d\xi} = \frac{-e^{-\tau}\cos\theta}{\pi\xi},\ \frac{dy}{d\xi} = \frac{-e^{-\tau}\sin\theta}{\pi\xi} \tag{1}$$

for $-1 < \xi < 0.$ These are later used to evaluate the coordinates of the free surface. The dynamic condition is Bernoulli equation

$$\frac{1}{2}e^{2\tau} + gy = \frac{1}{2} + g$$

with flow at downstream as the reference. In case the gravity is neglected, that condition gives $e^{2\tau} = 1,$ on the other hand we have $\tau = 0$ for $-1 < \xi < 0.$

Now, we construct the analytical function to get the relation between τ and θ. Based on the singular points $\zeta = -1$ and $\zeta = 0,$ they represent the separation point at the edge of the vertical wall, and the uniform flow far downstream. At those points, the function has order square root, see in [7, 8, 9, 10]. The appropriate function is

$$\chi(\zeta) = \Omega(\zeta + 1)^{-1/2}\zeta^{-1/2}. \tag{2}$$

The function is then written along the real axis as

$$\chi(\xi) = \begin{cases} \dfrac{\tau + i\pi/2}{\sqrt{\xi(\xi+1)}}, & \xi < -1 \\ \dfrac{-\theta}{\sqrt{-\xi(\xi+1)}}, & -1 < \xi < 0 \\ \dfrac{\tau}{\sqrt{\xi(\xi+1)}}, & \xi > 0 \end{cases}$$

after substituting the condition along the solid boundary for θ and the free boundary $\tau = 0$, from Bernoulli equation. Hence, the unknown function can be obtained by applying (2) to Cauchy integral theorem. The real part of the integral gives

$$\theta(\xi) = \frac{1}{2}\sqrt{-\xi(\xi+1)}\int_{-\infty}^{-1} \frac{ds}{(s-\xi)\sqrt{s(s+1)}} \tag{3}$$

for $-1 < \xi < 0$. From table of integration, we have

$$\int \frac{dx}{\sqrt{x^2 - a^2(x-b)}} = \frac{2\tan^{-1}\left|\sqrt{\frac{(a+b)(x-a)}{(a-b)(x+a)}}\right|}{\sqrt{a^2 - b^2}} + \text{constant},$$

where a and b are constants, so that the integral in (3) gives

$$\theta(\xi) = \frac{-\pi}{2} + \arctan\sqrt{\frac{1+\xi}{-\xi}}. \tag{4}$$

The analytical solution for θ is then used to determine the free surface profile following (1). They are then solved numerically. That x is integrated from $\xi = -1$, where $x(-1) = 0$, and y is integrated from $\xi = 0$, where $y(0) = 1$. Therefore,

$$x(\xi) = \int_{-1}^{\xi} \frac{\cos\theta}{\pi s}\, ds.$$

$$y(\xi) = 1 - \int_{\xi}^{0} \frac{\sin\theta}{\pi s}\, ds$$

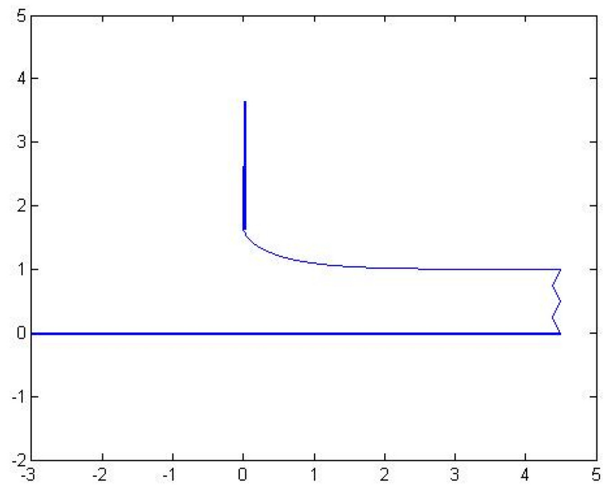

Figure 2. Free surface profile for the case of zero gravity.

Note that θ in both the integrations uses (4). The results obtained are plotted as the coordinates (x, y). We present the free surface profile, as given in Figure 2. We then compare the height of the uniform depth $y(0) = 1$ and the width of the slit $y(-1) = 1.6366$. Our calculation gives $\frac{y(0)}{y(-1)} \sim 0.611$. This number is as contraction coefficient, and agrees with the result in references such as in Batchelor [11], i.e. $\pi/2 + \pi$.

4. Conclusion

Free surface flow under a sluice gate has been solved for the case of zero gravity by constructing an analytical complex function on the artificial domain. This model is able to calculate the contraction coefficient, which agrees with the analytical work.

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