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The Ring Homomorphisms of Matrix Rings over Skew Generalized Power Series Rings

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ABSTRACT

Let $M_n(R_1[[S_1,\leq_1,\omega_1]])$ and $M_n(R_2[[S_2,\leq_2,\omega_2]])$ be a matrix rings over skew generalized power series rings, where R_1,R_2 are commutative rings with an identity element, $(S_1,\leq_1),(S_2,\leq_2)$ are strictly ordered monoids, $\omega_1\colon S_1\to End(R_1)$, $\omega_2\colon S_2\to End(R_2)$ are monoid homomorphisms. In this research, we define a mapping τ from $M_n(R_1[[S_1,\leq_1,\omega_1]])$ to $M_n(R_2[[S_2,\leq_2,\omega_2]])$ by using a strictly ordered monoid homomorphism $\delta\colon(S_1,\leq_1)\to(S_2,\leq_2)$, and ring homomorphisms $\mu\colon R_1\to R_2$ and $\sigma\colon R_1[[S_1,\leq_1,\omega_1]]\to R_2[[S_2,\leq_2,\omega_2]]$. Furthermore, we prove that τ is a ring homomorphism, and also we give the sufficient conditions for τ to be a monomorphism, epimorphism, and isomorphism.

Keywords: matrix rings; homomorphisms; skew generalized power series rings.

INTRODUCTION

In [1], it has been explained that a matrix is an arrangement of mathematical objects in rectangular rows and columns enclosed by square brackets or regular brackets. These mathematical objects are commonly called entries. If the matrix entries are members of a ring, the matrix is called the matrix over the ring [2]. A ring is a non-empty set with two binary operations and satisfies several axioms [3]. The skew generalized power series rings (SGPSR) $R[[S,\leq,\omega]]$ is one example of a ring [4]. This ring is defined as the set of all functions f from a strictly ordered monoid $(S,\leq)$ to a ring R with an identity element, that $\mathrm{supp}(f)$ is Artinian and narrow, with pointwise addition operation and convolution multiplication operation using a monoid homomorphism $\omega\colon S\to\mathrm{End}(R)$. Some research related to the properties of SGPSR $R[[S,\leq,\omega]]$, can be seen in Mazurek et al. [5]-[10] and Faisol et al. [11]-[16].

A set of matrices over a ring that forms a ring under matrix addition and matrix multiplication is called a matrix ring [17]. Furthermore, the set of all $n\times n$ matrices with entries in ring R is a matrix ring denoted by $M_n(R)$. In 2021, Rugayah et al. [18] have constructed the set of all matrices over SGPSR $R[[S,\leq,\omega]]$, denoted by $M_n(R[[S,\leq,\omega]])$. Moreover, they have defined the ideal of matrix ring over SGPSR $R[[S,\leq,\omega]]$ and studied its ideal properties.

One of the essential concepts in the ring structure is a ring homomorphism, a mapping from ring to ring that preserves binary operations on these rings. In [19], the matrix ring homomorphism from $M_n(R_1)$ to $M_n(R_2)$ defined by $\sigma([a_{ij}])=[\mu(a_{ij})]$ for

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every $a_{ij} \in R_1$ where $\mu: R_1 \to R_2$ is a ring homomorphism has constructed. Several studies related to matrix ring homomorphism can be seen in [20],[21]. This construction motivates us to study the ring homomorphism on the ring matrix over SGPSR $R[[S, \leq, \omega]]$. Therefore, in this research, matrix rings over the SGPSR $R[[S, \leq, \omega]]$ were constructed, i.e., $M_n(R_1[[S_1, \leq_1, \omega_1]])$ and $M_n(R_2[[S_2, \leq_2, \omega_2]])$ where R_1, R_2 are rings, $(S_1, \leq_1), (S_2, \leq_2)$ are strictly ordered monoids, and $\omega_1: S_1 \to End(R_1)$, $\omega_2: S_2 \to End(R_2)$ are monoid homomorphisms. Next, the maping τ from $M_n(R_1[[S_1, \leq_1, \omega_1]])$ to $M_n(R_2[[S_2, \leq_2, \omega_2]])$ is defined by using a strictly ordered monoid homomorphism $\delta: (S_1, \leq_1) \to (S_2, \leq_2)$, and ring homomorphisms $\mu: R_1 \to R_2$ and $\sigma: R_1[[S_1, \leq_1, \omega_1]] \to R_2[[S_2, \leq_2, \omega_2]]$. Furthermore, it is proved that τ is a matrix ring homomorphism, and the sufficient conditions for τ to be a monomorphism, epimorphism, and isomorphism are also given.

METHODS

The method used in this research is a literature study from books and scientific journals. The following steps can be obtained in the results. We construct the matrix rings over SGPSR $M_n(R_1[[S_1, \leq_1, \omega_1]])$ and $M_n(R_2[[S_2, \leq_2, \omega_2]])$, where R_1, R_2 are given rings, strictly ordered monoid $(S_1, \leq_1), (S_2, \leq_2)$, strictly ordered monoid homomorphism $\delta: (S_1, \leq_1) \to (S_2, \leq_2)$, and monoid homomorphisms $\omega_1: S_1 \to \mathrm{End}(R_1)$, $\omega_2: S_2 \to \mathrm{End}(R_2)$. Next, we define a mapping τ from $\mathrm{M_n(R_1[[S_1, \leq_1, \omega_1]])}$ to $\mathrm{M_n(R_2[[S_2, \leq_2, \omega_2]])}$, by using a strictly ordered monoid homomorphism $\delta: (S_1, \leq_1) \to (S_2, \leq_2)$, ring homomorphisms $\mu: R_1 \to R_2$ and $\sigma: R_1[[S_1, \leq_1, \omega_1]] \to R_2[[S_2, \leq_2, \omega_2]]$. Furthermore, we prove that τ is a ring homomorphism. Finally, we give sufficient conditions for τ to be a monomorphism, epimorphism, and isomorphism.

RESULTS AND DISCUSSION

Mazurek and Ziembowski [4] give the structure of skew generalized power series rings (SGPSR) as follows.

Let R_1, R_2 are rings, $(S_1, \leq_1), (S_2, \leq_2)$ are strictly ordered monoids, and $\omega_1: S_1 \to End(R_1)$, $\omega_2: S_2 \to End(R_2)$ are monoid homomorphisms. Homomorphic image of ω_1 and ω_2 are denoted by ω_1^s and ω_2^u for all $s \in S_1$ and $\in S_2$. Therefore,

$$\omega_1^{s+t} = \omega_1(s+t) = \omega_1(s) + \omega_1(t) = \omega_1^s + \omega_1^t, \tag{1}$$

and

$$\omega_2^{u+v} = \omega_2(u+v) = \omega_2(u) + \omega_2(v) = \omega_2^u + \omega_2^v, \tag{2}$$

for every all $s, t \in S_1$ and $u, v \in S_2$.

Next, let $R_1[[S_1, \leq_1, \omega_1]] = \{f: S_1 \to R_1 | \mathrm{supp}(f) \text{ Artinian and narrow}\}$ and $R_2[[S_2, \leq_2, \omega_2]] = \{\alpha: S_2 \to R_2 | \mathrm{supp}(\alpha) \text{ Artinian and narrow}\}$, where $\mathrm{supp}(f) = \{s \in S_1 | f(s) \neq 0\}$ and $\mathrm{supp}(\alpha) = \{u \in S_2 | \alpha(u) \neq 0\}$. Under pointwise addition and convolution multiplication defined by

$$(f+g)(s) = f(s) + g(s), \tag{3}$$

$$(\alpha+\beta)(u) = \alpha(u) + \beta(u), \tag{4}$$

and

$$(fg)(s) = \textstyle\sum_{x+y=s} f(x)\omega_1^x\big(g(y)\big), \tag{5}$$

$$(\alpha\beta)(u) = \textstyle\sum_{p+q=u} \alpha(p)\omega_2^p\big(\beta(q)\big), \tag{6}$$

$R_1[[S_1, \leq_1, \omega_1]]$ and $R_2[[S_2, \leq_2, \omega_2]]$ be a skew generalized power series rings, for every $s \in S_1$, $f, g \in R_1[[S_1, \leq_1, \omega_1]]$, and $u \in S_2$, $\alpha, \beta \in R_2[[S_2, \leq_2, \omega_2]]$.

According to [22], a strictly ordered monoid homomorphism $\delta\colon (S_1, \leq_1) \to (S_2, \leq_2)$ is a monoid homomorphism such that if $s <_1 t$, then $\delta(s) <_2 \delta(t)$ for every $s, t \in S_1$. Now, let δ be a monomorphism such that for any Artinian and narrow subset T of S_1, $\delta(T)$ is an Artinian and narrow subset of S_2, and $\mu\colon R_1 \to R_2$ is a ring homomorphism such that for every $s \in S_1$ the following diagram is commutative:

$$\begin{tikzcd} S_1 \arrow[r, "\delta"] \arrow[d, "f"'] & S_2 \arrow[d, "\alpha"] \\ R_1 \arrow[r, "\mu"] \arrow[d, "\omega_1^s"'] \arrow[dr, phantom, "\circlearrowright"] & R_2 \arrow[d, "\omega_2^{\delta(s)}"] \\ R_1 \arrow[r, "\mu"'] & R_2 \end{tikzcd}$$

Figure 1. Commutative diagram $\omega_2^{\delta(s)} \circ \mu = \mu \circ \omega_1^s$

For $f \in R_1[[S_1, \leq_1, \omega_1]]$, let $\alpha\colon S_2 \to R_2$ be the map defined as follows:

$$\alpha(t) = \begin{cases} \mu \circ f \circ \delta^{-1}(t) & \text{if } t \in \delta(S_1) \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Since $\text{supp}(\alpha) \subseteq \delta\big(\text{supp}(f)\big)$, based on [23](1.(a)), $\alpha \in R_2[[S_2, \leq_2, \omega_2]]$. Therefore, we can define a map $\sigma\colon R_1[[S_1, \leq_1, \omega_1]] \to R_2[[S_2, \leq_2, \omega_2]]$ by putting $\sigma(f) = \alpha$ in (7). According to [24](Lemma 8.1.6), the map $\sigma\colon R_1[[S_1, \leq_1, \omega_1]] \to R_2[[S_2, \leq_2, \omega_2]]$ is a ring homomorphism, and $\text{Ker}(\sigma) = \big(\text{Ker}(\mu)\big)[[S_1, \leq_1, \omega_1]]$.

Now, we construct the matrix rings $M_n(R_1[[S_1, \leq_1, \omega_1]])$ and $M_n(R_2[[S_2, \leq_2, \omega_2]])$, that are the sets of all matrices over SGPSR $R_1[[S_1, \leq_1, \omega_1]]$ and $R_2[[S_2, \leq_2, \omega_2]]$ defined by

$$M_n(R_1[[S_1, \leq_1, \omega_1]]) = \big\{[f_{ij}] \big| f_{ij} \in R_1[[S_1, \leq_1, \omega_1]]; i, j = 1, 2, \cdots, n\big\}, \tag{8}$$

and

$$M_n(R_2[[S_2, \leq_2, \omega_2]]) = \big\{[\alpha_{ij}] \big| \alpha_{ij} \in R_2[[S_2, \leq_2, \omega_2]]; i, j = 1, 2, \cdots, n\big\}, \tag{9}$$

with addition matrix operation

$$[f_{ij}] + [g_{ij}] = [f_{ij} + g_{ij}]\,, \tag{10}$$

$$[\alpha_{ij}] + [\beta_{ij}] = [\alpha_{ij} + \beta_{ij}]\,, \tag{11}$$

and multiplication matrix operation

$$[f_{ij}][g_{ij}] = [h_{ij}]\,, \tag{12}$$

$$[\alpha_{ij}][\beta_{ij}] = [\gamma_{ij}]\,, \tag{13}$$

where $h_{ij} = \sum_{k=1}^{n} f_{ik} g_{kj}$ dan $\gamma_{ij} = \sum_{k=1}^{n} \alpha_{ik} \beta_{kj}$, for every $[f_{ij}], [g_{ij}] \in M_n(R_1[[S_1, \leq_1, \omega_1]])$ and $[\alpha_{ij}], [\beta_{ij}] \in M_n(R_2 R_2[[S_2, \leq_2, \omega_2]])$.

For every $[f_{ij}] \in M_n(R_1[[S_1, \leq_1, \omega_1]])$, we define the map $\tau\colon M_n(R_1[[S_1, \leq_1, \omega_1]]) \to M_n(R_2[[S_2, \leq_2, \omega_2]])$ by

$$\tau([f_{ij}]) = [\sigma(f_{ij})], \tag{14}$$

for every $[f_{ij}] \in M_n(R_1[[S_1, \leq_1, \omega_1]])$. The following theorem shows that τ is a ring homomorphism.

Proposition 1 *Let $M_n(R_1[[S_1, \leq_1, \omega_1]])$ and $M_n(R_2[[S_2, \leq_2, \omega_2]])$ be matrix rings over SGPSR. The mapping τ that is defined in* (14) *is a ring homomorphism.*

Proof Based on [24](Lemma 8.1.6), for $\mathrm{i}, \mathrm{j} = 1, 2, \cdots, \mathrm{n}$, there is $\alpha_{ij} = \sigma(f_{ij}) \in \mathrm{R}_2[[S_2, \leq_2, \omega_2]]$ for every $f_{ij} \in R_1[[S_1, \leq_1, \omega_1]]$. Therefore, $\tau([f_{ij}]) = [\sigma(f_{ij})] = [\alpha_{ij}]$ is well-defined.
For any $t \in S_2$, $f_{ij}, g_{ij} \in R_1[[S_1, \leq_1, \omega_1]]$, we have

$$\begin{aligned}
\mu \circ (f_{ij} + g_{ij}) \circ \delta^{-1}(t) &= \mu\left((f_{ij} + g_{ij})(\delta^{-1}(t))\right) \\
&= \mu\left(f_{ij}(\delta^{-1}(t)) + g_{ij}(\delta^{-1}(t))\right) \\
&= \mu\left(f_{ij}(\delta^{-1}(t))\right) + \mu\left(g_{ij}(\delta^{-1}(t))\right) \\
&= (\mu \circ f_{ij} \circ \delta^{-1})(t) + (\mu \circ g_{ij} \circ \delta^{-1})(t),
\end{aligned}$$

and

$$\begin{aligned}
\mu \circ (f_{ij} g_{ij}) \circ \delta^{-1}(t) &= \mu\left((f_{ij} g_{ij})(\delta^{-1}(t))\right) \\
&= \mu\left(\sum_{x+y=\delta^{-1}(t)} f_{ij}(x)\omega_1^x\left(g_{ij}(y)\right)\right) \\
&= \sum_{x+y=\delta^{-1}(t)} \mu\left(f_{ij}(x)\omega_1^x\left(g_{ij}(y)\right)\right) \\
&= \sum_{x+y=\delta^{-1}(t)} \mu\left(f_{ij}(x)\right)\mu\left(\omega_1^x\left(g_{ij}(y)\right)\right) \\
&= \sum_{\substack{x+y=\delta^{-1}(t) \\ \delta(x)+\delta(y)=t}} \mu\left(f_{ij}(x)\right)\omega_2^{\delta(x)}\left(\mu\left(g_{ij}(y)\right)\right) \\
&= \sum_{u+v=t} \mu\left(f_{ij}(\delta^{-1}(u))\right)\omega_2^u\left(\mu\left(g_{ij}(\delta^{-1}(v))\right)\right) \\
&= \sum_{u+v=t} (\mu \circ f_{ij} \circ \delta^{-1})(u)\omega_2^u\left((\mu \circ g_{ij} \circ \delta^{-1})(v)\right) \\
&= (\mu \circ f_{ij} \circ \delta^{-1})(\mu \circ g_{ij} \circ \delta^{-1})(t).
\end{aligned}$$

In other words, $\mu \circ (f_{ij} + g_{ij}) \circ \delta^{-1} = (\mu \circ f_{ij} \circ \delta^{-1}) + (\mu \circ g_{ij} \circ \delta^{-1})$ and $\mu \circ (f_{ij} g_{ij}) \circ \delta^{-1} = (\mu \circ f_{ij} \circ \delta^{-1})(\mu \circ g_{ij} \circ \delta^{-1})$ for every $f_{\mathrm{ij}}, g_{\mathrm{ij}} \in R_1[[S_1, \leq_1, \omega_1]]$.

Now, we prove that τ is a ring homomorphism. For any $[f_{ij}], [g_{ij}] \in \mathrm{M_n}(R_1[[S_1, \leq_1, \omega_1]])$, we obtain:

(i) $$\begin{aligned}
\tau([f_{ij}] + [g_{ij}]) &= \tau([f_{ij} + g_{ij}]) \\
&= [\sigma(f_{ij} + g_{ij})] \\
&= [\mu \circ (f_{ij} + g_{ij}) \circ \delta^{-1}] \\
&= [(\mu \circ f_{ij} \circ \delta^{-1}) + (\mu \circ g_{ij} \circ \delta^{-1})] \\
&= [\mu \circ f_{ij} \circ \delta^{-1}] + [\mu \circ g_{ij} \circ \delta^{-1}] \\
&= [\sigma(f_{ij})] + [\sigma(g_{ij})] \\
&= \tau([f_{ij}]) + \tau([g_{ij}]).
\end{aligned}$$

(ii) $$\begin{aligned}\tau([f_{ij}][g_{ij}]) &= \tau([\textstyle\sum_{k=1}^{n} f_{ik}g_{kj}]) \\ &= \left[\sigma\left(\sum_{k=1}^{n} f_{ik}g_{kj}\right)\right] \\ &= \left[\mu\circ\left(\sum_{k=1}^{n} f_{ik}g_{kj}\right)\circ\delta^{-1}\right] \\ &= \left[\sum_{k=1}^{n}\mu\circ(f_{ik}g_{kj})\circ\delta^{-1}\right] \\ &= \left[\sum_{k=1}^{n}(\mu\circ f_{ik}\circ\delta^{-1})(\mu\circ g_{kj}\circ\delta^{-1})\right] \\ &= [\mu\circ f_{ij}\circ\delta^{-1}][\mu\circ g_{ij}\circ\delta^{-1}] \\ &= [\sigma(f_{ij})][\sigma(g_{ij})] \\ &= \tau([f_{ij}])\tau([g_{ij}])\end{aligned}$$

According to (i) and (ii), it is proved that τ is a ring homomorphism. ■

The following proposition shows that $\mathrm{Ker}(\tau) = M_n\left((\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]\right)$.

Proposition 2 *Let* $M_n(R_1[[S_1,\leq_1,\omega_1]])$ *and* $M_n(R_2[[S_2,\leq_2,\omega_2]])$ *be matrix rings over SGPSR. Let* $\tau: M_n(R_1[[S_1,\leq_1,\omega_1]]) \to M_n(R_2[[S_2,\leq_2,\omega_2]])$ *is the map that is defined in* (14)*. Then,* $\mathrm{Ker}(\tau) = M_n\left((\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]\right)$.

Proof For any $[f_{ij}] \in \mathrm{Ker}(\tau)$, we have $\tau([f_{ij}]) = [\sigma(f_{ij})] = [0]$. Therefore, for $\mathrm{i,j} = 1,2,\cdots,\mathrm{n}$, $\sigma(f_{ij}) = 0$. So, $f_{ij} \in \mathrm{Ker}(\sigma)$. Based on [24](Lemma 8.1.6), $\mathrm{Ker}(\sigma) = (\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]$. Therefore, $f_{ij} \in (\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]$ for all $\mathrm{i,j} = 1,2,\cdots,\mathrm{n}$. So, $[f_{ij}] \in M_n\left((\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]\right)$. Then, we get $\mathrm{Ker}(\tau) \subset M_n\left((\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]\right)$.

On the other side, for any $[f_{ij}] \in M_n\left((\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]\right)$, we have $f_{ij} \in (\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]$ for all $\mathrm{i,j} = 1,2,\cdots,\mathrm{n}$. According to [24](Lemma 8.1.6), $\mathrm{Ker}(\sigma) = (\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]$. Therefore, $f_{ij} \in \mathrm{Ker}(\sigma)$. Then, $\sigma(f_{ij}) = 0$ for all $\mathrm{i,j} = 1,2,\cdots,\mathrm{n}$. So, we get $[\sigma(f_{ij})] = [0] = \tau([f_{ij}])$. In other words, $[f_{ij}] \in \mathrm{Ker}(\tau)$. Hence, $M_n\left((\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]\right) \subset \mathrm{Ker}(\tau)$.

So, it is proved that $\mathrm{Ker}(\tau) = M_n\left((\mathrm{Ker}(\mu))[[S_1,\leq_1,\omega_1]]\right)$ ■

Next, we give sufficient conditions for τ to be a monomorphism, epimorphism, and isomorphism.

Proposition 3 *Let* $M_n(R_1[[S_1,\leq_1,\omega_1]])$ *and* $M_n(R_2[[S_2,\leq_2,\omega_2]])$ *be matrix rings over SGPSR. Let* $\tau: M_n(R_1[[S_1,\leq_1,\omega_1]]) \to M_n(R_2[[S_2,\leq_2,\omega_2]])$ *is the map that is defined in* (14)*. If* δ *is an isomorphism and* μ *is a monomorphism, then* τ *is a monomorphism.*

Proof Based on Proposition 1, it is clear that τ is a ring homomorphism. So, we only have to show that τ is injective. If $\tau([f_{ij}]) = \tau([g_{ij}])$, then $[\sigma(f_{ij})] = [\sigma(g_{ij})]$. Hence, $[\mu\circ f_{ij}\circ\delta^{-1}] = [\mu\circ g_{ij}\circ\delta^{-1}]$. Therefore, we get $\mu\circ f_{ij}\circ\delta^{-1} = \mu\circ g_{ij}\circ\delta^{-1}$ for all $\mathrm{i,j} =$

$1, 2, \cdots, \mathrm{n}$. In other words, for any $\mathrm{t} \in \mathrm{S}_2$, we have $\mu\left(f_{ij}(\delta^{-1}(\mathrm{t}))\right) = \mu\left(g_{ij}(\delta^{-1}(\mathrm{t}))\right)$. Since μ is a monomorphism, $f_{ij}(\delta^{-1}(\mathrm{t})) = g_{ij}(\delta^{-1}(\mathrm{t}))$. Since δ is an isomorphism, $f_{ij}(\mathrm{s}) = g_{ij}(\mathrm{s})$ for every $\mathrm{s} \in \mathrm{S}_1$. So, $f_{ij} = g_{ij}$ for all $i, j = 1, 2, \cdots, \mathrm{n}$. Therefore $[f_{ij}] = [g_{ij}]$. So, it is proved that if $\tau([f_{ij}]) = \tau([g_{ij}])$, then $[f_{ij}] = [g_{ij}]$. Hence, τ is injective. ■

Proposition 4 *Let $M_n(R_1[[S_1, \leq_1, \omega_1]])$ and $M_n(R_2[[S_2, \leq_2, \omega_2]])$ be matrix rings over SGPSR. Let $\tau: M_n(R_1[[S_1, \leq_1, \omega_1]]) \to M_n(R_2[[S_2, \leq_2, \omega_2]])$ is the map that is defined in* (14)*. If σ is an epimorphism, then τ is an epimorphism.*

Proof Based on Proposition 1, τ is a ring homomorphism. So, we only have to show that τ is surjective. In other words, we have to prove that $\mathrm{Im}(\tau) = M_n(R_2[[S_2, \leq_2, \omega_2]])$. It is clear that $\mathrm{Im}(\tau) \subset M_n(R_2[[S_2, \leq_2, \omega_2]])$, so it suffices to show that $M_n(R_2[[S_2, \leq_2, \omega_2]]) \subset \mathrm{Im}(\tau)$.

For any $[\alpha_{\mathrm{ij}}] \in \mathrm{M_n}(R_2[[S_2, \leq_2, \omega_2]])$, then $\alpha_{ij} \in R_2[[S_2, \leq_2, \omega_2]]$ for all $i, j = 1, 2, \cdots, n$. Since σ is an epimorphism, there is $f_{ij} \in R_1[[S_1, \leq_1, \omega_1]]$ such that $\sigma(f_{ij}) = \alpha_{ij}$. Therefore, there is $[f_{ij}] \in M_n(R_1[[S_1, \leq_1, \omega_1]])$ such that $[\alpha_{ij}] = [\sigma(f_{ij})] = \tau([f_{ij}])$ for every $[\alpha_{ij}] \in M_n(R_2[[S_2, \leq_2, \omega_2]])$. So, $[\alpha_{ij}] \in \mathrm{Im}(\tau)$. In other words, $M_n(R_2[[S_2, \leq_2, \omega_2]]) \subset \mathrm{Im}(\tau)$. Hence, that τ is surjective. ■

Corollary 5 *Let $M_n(R_1[[S_1, \leq_1, \omega_1]])$ and $M_n(R_2[[S_2, \leq_2, \omega_2]])$ be matrix rings over SGPSR. Let $\tau: M_n(R_1[[S_1, \leq_1, \omega_1]]) \to M_n(R_2[[S_2, \leq_2, \omega_2]])$ is the map that is defined in* (14)*. If δ is an isomorphism, μ is a monomorphism, and σ is an epimorphism, then τ is an isomorphism.*

CONCLUSIONS

A ring homomorphism τ from the matrix ring $M_n(R_1[[S_1, \leq_1, \omega_1]])$ to the matrix ring $M_n(R_2[[S_2, \leq_2, \omega_2]])$ can be constructed by using a strictly ordered monoid homomorphism $\delta: (S_1, \leq_1) \to (S_2, \leq_2)$, and ring homomorphisms $\mu: R_1 \to R_2$ *and* $\sigma: R_1[[S_1, \leq_1, \omega_1]] \to R_2[[S_2, \leq_2, \omega_2]]$. Furthermore, it also proves that $\mathrm{Ker}(\tau)$ is equal to the matrix ring over SGPSR $(\mathrm{Ker}(\mu))[[S_1, \leq_1, \omega_1]]$. Moreover, if δ is an isomorphism and μ is a monomorphism, then τ is a monomorphism. While, if σ is an epimorphism, then τ is an epimorphism. Consequently, τ is an isomorphism if δ is an isomorphism, μ is a monomorphism, and τ is an epimorphism.

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