

Subdivision of Certain Barbell Operation of Origami Graphs has Locating- Chromatic Number Five

By Kurnia Muludi

Subdivision of Certain Barbell Operation of Origami Graphs has Locating-Chromatic Number Five

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Abstract

The locating-chromatic number denote by $\chi_L(G)$, is the smallest t such that G has a locating t -coloring. In this research, we determined locating-chromatic number for subdivision of certain barbell operation of origami graphs.

Key words:

locating-chromatic number, subdivision, certain barbell operation, origami graphs.

1. Introduction

The concept of partition dimension was introduced by Chartrand et al. [1] as the development of the concept of metric dimension. The application of metric dimension can be found in robotic navigation [2], chemical data classification [26] and the optimization of threat detecting sensors [4]. The locating-chromatic number was first discovered by Chartrand et al. [5] in 2002, with obtained two graph concepts, coloring vertices and partition dimension of a graph. The locating-chromatic number denote by $\chi_L(G)$, is the smallest t such that G has a locating t -coloring. Ne²⁰ investigated the locating-chromatic number for a path graph P_n , a cycle graph C_n , and k -ble star graph $S_{a,b}$. Furthermore, Chartrand et al. [6] characterized all graphs order n with locating-chromatic number $n - 1$. Baskoro and Asmiati [7] characterized all trees with locating-chromatic number 3.

The locating-chromatic number of the join of graphs was introduced by B³oei and Anbarloei [8]. Purwasih et al. [9], obtained locating-chromatic number for a subdivision of a graph on one edge. For graph with dominant vertices have been studied in [10]. In [11], Asmiati found the locating-chromatic number of non-homogeneous caterpillar and firecrackers graph, [12] certain barbell graphs $B_{m,n}$ and $B_{P(n,1)}$. In 2019, Irawan et al. [13] obtained locating-chromatic number for certain operation of generalized Petersen graphs $sP(4,2)$. Furthermore, in [14] determined the locating-chromatic number for $sP(n,1)$, origami graphs [15] and certain barbell origami graphs [16]. The locating-chromatic

number of a graph is a newly interesting topic to study cause there is no general theorem for determining the locating-chromatic number of any graph. In this research, we specifying about locating-chromatic number for subdivision of certain barbell operation of origami graphs, called $B_{O_n}^s$. This study is a continuation of previous research.

The following definition of the locating-chromatic number of a graph, dominant vertices, origami graph, and certain barbell origami graphs is taken from [5, 17, 18, 16]. We use some theorems that is basics to work out a lower bound of the locating-chromatic number of a graph is taken from [5, 15]. The set of neighbours of a vertex l in G , denoted by $N(l)$.

Theorem 1.1. [5] Let c be a locating coloring in a connected graph. If k and l are distinct vertices of G such that $d(k,w) = d(l,w)$ for all $w \in V(G) - \{k,l\}$, then $c(k) \neq c(l)$. In particular, if k and l are non-adjacent vertices of G such that $N(k) \neq N(l)$, then $c(k) \neq c(l)$.

Theorem 1.2. [15] Let O_n be an origami graph for $n \geq 3$. The locating chromatic number of an origami graphs O_n is 4 for $n=3$ and 5 otherwise.

2. Results and Discussion

In this section, we will discuss the locating-chromatic number for subdivision of certain barbell operation of origami graphs, denoted by $B_{O_n}^s$.

Theorem 2.1. Let $B_{O_n}^s$ be a subdivision of certain barbell operation of origami graphs for $n \geq 3, s \geq 1$. Then the locating-chromatic number of $B_{O_n}^s$ is five, $\chi_L(B_{O_n}^s) = 5$.

Proof. Let $B_{O_n}^s$ be a subdivision of certain barbell operation of origami graphs for $n \geq 3, s \geq 1$, with $V(B_{O_n}^s) = \{u_i, u_{n+i}, v_i, v_{n+i}, w_i, w_{n+i} : i \in \{1, \dots, n\}\} \cup \{x_i$

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$i \in \{1, \dots, s\}$ and $E(B_{O_n}^s) = \{u_i w_i, u_i v_i, v_i w_i, u_i u_{i+1}, w_i u_{i+1}; i \in \{1, \dots, n\}\} \cup \{u_{n+i} w_{n+i}, u_{n+i} v_{n+i}, v_{n+i} w_{n+i}, u_{n+i} u_{n+i+1}, w_{n+i} u_{n+i+1}; i \in \{1, \dots, n-1\}\} \cup \{u_n x_1, x_s u_{n+1}\} \cup \{x_i x_{i+1}; i \in \{1, \dots, s-1\}\}$.

To prove the theorem, we will be divided into two cases :

Cases 1. For $n = 3$

First, we determine lower bound of $\chi_L(B_{O_3}^s)$. Since subdivision of certain barbell operation of origami graphs, containing origami graphs O_3 , then by Theorem 1.2. $\chi_L(B_{O_3}^s) \geq 4$. Next, we will show that 4 colors are not enough. Origami graph $B_{O_3}^s$ there are six complete graph with four vertices, denote by K_4 . Without loss of generality, we assign three colors for any K_4 in $B_{O_3}^s$, and then the six vertices are dominant vertices. As a result, if we use four colors it is not enough because there are more than one K_4 in $B_{O_3}^s$. So $\chi_L(B_{O_3}^s) \geq 5$.

Next, we determined the upper bound of $\chi_L(B_{O_3}^s) \leq 5$. To show that $\chi_L(B_{O_3}^s) \leq 5$, consider the 5-coloring c on $B_{O_3}^s$ as follow,

$$\begin{aligned} C_1 &= \{u_1, w_2, u_6, v_5\}; \\ C_2 &= \{u_4, w_1, w_5\}; \\ C_3 &= \{u_2, v_1, w_3, u_5, v_4, v_6\} \cup \{x_i | \text{for odd } i, i \geq 1\}; \\ C_4 &= \{u_3, v_2, w_4, w_6\} \cup \{x_i | \text{for even } i, i \geq 2\}; \\ C_5 &= \{v_3\}; \end{aligned}$$

The coloring c will create partition Π on $V(B_{O_3}^s)$. We shall show that the color codes of all vertices in $B_{O_3}^s$ are different. We have $c_\Pi(u_1) = (0, 2, 1, 1, 1)$; $c_\Pi(u_2) = (1, 1, 0, 1, 2)$; $c_\Pi(u_3) = (1, 2, 1, 0, 1)$; $c_\Pi(u_4) = (1, 0, 1, 1, s + 3)$; $c_\Pi(u_5) = (1, 1, 1, 0, s + 4)$; $c_\Pi(u_6) = ((0, 1, 1, s + 4)$; $c_\Pi(v_1) = (1, 3, 2, 0, 1)$; $c_\Pi(v_2) = (1, 3, 0, 1, 2)$; $c_\Pi(v_3) = (2, 0, 1, 1, 3)$; $c_\Pi(v_4) = (2, 1, 1, 0, s + 4)$; $c_\Pi(v_5) = (0, 1, 2, 1, s + 5)$; $c_\Pi(v_6) = (1, 2, 1, 0, s + 5)$; $c_\Pi(w_1) = (1, 3, 2, 1, 0)$; $c_\Pi(w_2) = (0, 2, 1, 1, 2)$; $c_\Pi(w_3) = (1, 1, 1, 0, 2)$; $c_\Pi(w_4) = (2, 1, 0, 1, s + 4)$; $c_\Pi(w_5) = (1, 0, 2, 1, s + 5)$; $c_\Pi(w_6) = (1, 1, 0, 1, s + 4)$. For $s = 1$, we have $c_\Pi(x_i) = (i + 1, 1, 1, 0, i + 2)$. For i odd, $i \leq \lfloor \frac{s}{2} \rfloor$, $s \geq 2$, we have $c_\Pi(x_i) = (i + 1, i + 1, 1, 0, i + 2)$. For i even, $i \leq \lfloor \frac{s}{2} \rfloor$, $s \geq 2$, we have $c_\Pi(x_i) = (i + 1, i + 1, 0, 1, i + 2)$. For i odd, $i > \lfloor \frac{s}{2} \rfloor$, $s \geq 2$, we have $c_\Pi(x_i) = (s - i + 2, s - i + 1, 1, 0, i + 2)$. For i even, $i > \lfloor \frac{s}{2} \rfloor$, $s \geq 2$, we have $c_\Pi(x_i) = (s - i + 2, s - i + 1, 0, 1, i + 2)$.

Since the color codes of all vertices $B_{O_3}^s$ are different, thus c is a locating coloring. So $\chi_L(B_{O_3}^s) \leq 5$.

Case 2. For $n \geq 4$

First, we determine lower bound of $\chi_L(B_{O_n}^s)$ for $n \geq 4$. Since subdivision of certain barbell operation of origami graphs, containing origami graphs O_n , then by Theorem 1.2 it is clear that $\chi_L(B_{O_n}^s) \geq 5$.

To show the upper bound for the locating-chromatic number for subdivision of certain barbell operation of origami graphs $\chi_L(B_{O_n}^s) \geq 5$ for $n \geq 4$. Let us different some subcases.

Subcase 2.1. (odd n), for $\lfloor \frac{n}{2} \rfloor$ odd, $n \geq 5$

Let c be a coloring for subdivision of certain barbell operation of origami graph $B_{O_n}^s$, for $\lfloor \frac{n}{2} \rfloor$ odd, $n \geq 5$ we make the partition Π of $V(B_{O_n}^s)$:

$$\begin{aligned} C_1 &= \{w_1 | 1 \leq i \leq n\} \cup \{u_{n+1}\}; \\ C_2 &= \{u_i | \text{for odd } i, 3 \leq i \leq n\} \cup \{v_i | \text{for even } i, 2 \leq i \leq n-1\} \cup \{u_{n+i} | \text{for odd } i, 3 \leq i \leq \lfloor \frac{n}{2} \rfloor - 2\} \cup \{u_{n+i} | \text{for odd } i, \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n\} \cup \{v_{n+i} | \text{for even } i, 2 \leq i \leq n-1\} \cup \{x_i | \text{for } i, i \geq 2\}; \\ C_3 &= \{u_i | \text{for even } i, 2 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1\} \cup \{u_i | \text{for even } i, \lfloor \frac{n}{2} \rfloor + 3 \leq i \leq n-1\} \cup \{v_i | \text{for odd } i, 1 \leq i \leq 27\} \cup \{u_{n+i} | \text{for even } i, 2 \leq i \leq n-1\} \cup \{v_{n+i} | \text{for odd } i, 1 \leq i \leq n\}; \\ C_4 &= \{u_1\} \cup \{w_{n+i} | 1 \leq i \leq n\} \cup \{x_i | \text{for odd } i, i \geq 1\}; \\ C_5 &= \{u_{\lfloor \frac{n}{2} \rfloor + 1}\} \cup \{u_{n+\lfloor \frac{n}{2} \rfloor}\}. \end{aligned}$$

For $\lfloor \frac{n}{2} \rfloor$ odd $n \geq 5$, the color codes of all the vertices of $V(B_{O_n}^s)$ are :

$$c_\Pi(u_i) = \begin{cases} 0, & \text{for 2nd ordinate, even } i, 3 \leq i \leq n, n \geq 5 \\ 2, & \text{for 3rd ordinate, even } i, 2 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1, n \geq 5 \\ i-1, & \text{for 3rd ordinate, even } i, \lfloor \frac{n}{2} \rfloor + 3 \leq i \leq n-1, n \geq 9 \\ i-1, & \text{for 4th ordinate, } i = 1 \\ i-1, & \text{for 5th ordinate, } i = \lfloor \frac{n}{2} \rfloor + 1 \\ i-1, & \text{for 3rd ordinate, } i = \lfloor \frac{n}{2} \rfloor + 1 \\ i-1, & \text{for 4th ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ n-i+1, & \text{for 4th ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ i - \lfloor \frac{n}{2} \rfloor - 1, & \text{for 5th ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ \lfloor \frac{n}{2} \rfloor - i + 1, & \text{for 5th ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ \lfloor \frac{n}{2} \rfloor - i, & \text{for 5th ordinate, } i = 1 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(v_i) = \begin{cases} 0, & \text{for } 2^{nd} \text{ ordinate, } 1 \leq i \leq n-1, n \geq 5 \\ 9, & \text{for } 3^{rd} \text{ ordinate, odd } i, 1 \leq i \leq n, n \geq 5 \\ 2, & \text{for } 2^{nd} \text{ ordinate, } i = 1 \\ 3, & \text{for } 3^{rd} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor + 1 \\ i, & \text{for } 4^{th} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ n - i + 2, & \text{for } 4^{th} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ \lfloor \frac{n}{2} \rfloor, & \text{for } 5^{th} \text{ ordinate, } i = 1 \\ \lfloor \frac{n}{2} \rfloor - i + 2, & \text{for } 5^{th} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ i - \lfloor \frac{n}{2} \rfloor, & \text{for } 5^{th} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_i) = \begin{cases} 0, & \text{for } 1^{st} \text{ ordinate, } 1 \leq i \leq n, n \geq 5 \\ 2, & \text{for } 2^{nd} \text{ ordinate, } i = 1 \\ 19, & \text{for } 3^{rd} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ i, & \text{for } 4^{th} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ n - i + 1, & \text{for } 4^{th} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ \lfloor \frac{n}{2} \rfloor - i + 1, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ i - \lfloor \frac{n}{2} \rfloor, & \text{for } 5^{th} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(u_{n+i}) = \begin{cases} i - 1, & \text{for } 1^{st} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ n - i + 1, & \text{for } 1^{st} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ 0, & \text{for } 1^{st} \text{ ordinate, } i = 1 \\ & \text{for } 2^{nd} \text{ ordinate, odd } i, 3 \leq i \leq \lfloor \frac{n}{2} \rfloor - 2, n \geq 9 \\ & \text{for } 2^{nd} \text{ ordinate, odd } i, \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n, n \geq 5 \\ & \text{for } 3^{rd} \text{ ordinate, even } i, 3 \leq i \leq n-1, n \geq 5 \\ & \text{for } 5^{th} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor + 1 \\ \lfloor \frac{n}{2} \rfloor - 1, & \text{for } 1^{st} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ 2, & \text{for } 2^{nd} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ \lfloor \frac{n}{2} \rfloor - i, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1, n \geq 5 \\ i - \lfloor \frac{n}{2} \rfloor, & \text{for } 5^{th} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(v_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ n - i + 2, & \text{for } 1^{st} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ 0, & \text{for } 2^{nd} \text{ ordinate, even } i, 2 \leq i \leq n-1, n \geq 5 \\ & \text{for } 3^{rd} \text{ ordinate, odd } i, 1 \leq i \leq n, n \geq 5 \\ & \text{for } 2^{nd} \text{ ordinate, } i = 1 \\ 3, & \text{for } 2^{nd} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ \lfloor \frac{n}{2} \rfloor - i + 1, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ i - \lfloor \frac{n}{2} \rfloor + 1, & \text{for } 5^{th} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 5 \\ n - i + 1, & \text{for } 1^{st} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 5 \\ 2, & \text{for } 2^{nd} \text{ ordinate, } i = 1 \text{ and } i = \lfloor \frac{n}{2} \rfloor \\ 0, & \text{for } 4^{th} \text{ ordinate, } 29 \leq i \leq n, n \geq 5 \\ \lfloor \frac{n}{2} \rfloor - i, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1, n \geq 5 \\ i - \lfloor \frac{n}{2} \rfloor + 1, & \text{for } 5^{th} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor \leq i \leq n, n \geq 5 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(x_i) = \begin{cases} s - i + 1, & \text{for } 1^{st} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ i + 1, & \text{for } 1^{st} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ & \text{for } 3^{rd} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor \\ 1, & \text{for } 1^{st} \text{ ordinate, } i = s \\ & \text{for } 2^{nd} \text{ ordinate, odd } i, i \geq 1 \\ & \text{for } 4^{th} \text{ ordinate, even } i, i \geq 2 \\ s - i + 2, & \text{for } 3^{rd} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor \\ i + \lfloor \frac{n}{2} \rfloor - 2, & \text{for } 5^{th} \text{ ordinate, } i < \lfloor \frac{s}{2} \rfloor \\ s - i + \lfloor \frac{n}{2} \rfloor + 1, & \text{for } 5^{th} \text{ ordinate, } i \geq \lfloor \frac{s}{2} \rfloor \\ 0, & \text{otherwise.} \end{cases}$$

Since for odd n all vertices have different color codes, c is a locating coloring for subdivision of certain barbell operation of origami graphs $B_{O_n}^s$, so that $\chi_L(B_{O_n}^s) \leq 5$, for $\lfloor \frac{n}{2} \rfloor$ odd, $n \geq 5$.

Subcase 2.2. (odd n), for $\lfloor \frac{n}{2} \rfloor$ even, $n \geq 7$

Let c be a coloring for subdivision of certain barbell operation of origami graph $B_{O_n}^s$, for $\lfloor \frac{n}{2} \rfloor$ even, $n \geq 7$ we make the partition Π of $V(B_{O_n}^s)$:

$$C_1 = \{w_i | 1 \leq i \leq n\} \cup \{u_{n+1}\};$$

$$C_2 = \{u_i | \text{for odd } i, 3 \leq i \leq n\} \cup \{v_i | \text{for even } i, 2 \leq i \leq n-1\} \cup \{u_{n+i} | \text{for even } i, 2 \leq i \leq \lfloor \frac{n}{2} \rfloor - 2\} \cup \{u_{n+i}\}$$

for even $i, \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n-1 \} \cup \{v_{n+i} | \text{for odd } i, 1 \leq i \leq n\}$;

$C_3 = \{u_i | \text{for even } i, 2 \leq i \leq \lfloor \frac{n}{2} \rfloor - 2\} \cup \{u_i | \text{for even } i, \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n-1\} \cup \{v_i | \text{for odd } i, 1 \leq i \leq n\} \cup \{u_{n+i} | \text{for odd } i, 3 \leq i \leq n\} \cup \{v_{n+i} | \text{for even } i, 2 \leq i \leq n-1\} \cup \{x_i | \text{for even } i, i \geq 2\}$;

$C_4 = \{u_1\} \cup \{w_{n+i} | 1 \leq i \leq n\} \cup \{x_i | \text{for odd } i, i \geq 1\}$;

$C_5 = \{u_{\lfloor \frac{n}{2} \rfloor + 1}\} \cup \{u_{n+\lfloor \frac{n}{2} \rfloor}\}$.

For $\lfloor \frac{n}{2} \rfloor$ even $n \geq 7$, the color codes of all the vertices of $V(B_{\delta_n}^S)$ are :

$$c_{\Pi}(u_i) = \begin{cases} 0, & \text{for } 2^{\text{nd}} \text{ ordinate, odd } i, 3 \leq i \leq n, n \geq 7 \\ & \text{for } 3^{\text{rd}} \text{ ordinate, even } i, 2 \leq i \leq \lfloor \frac{n}{2} \rfloor - 2, n \geq 7 \\ & \text{for } 3^{\text{rd}} \text{ ordinate, even } i, \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n-1, n \geq 7 \\ & \text{for } 4^{\text{th}} \text{ ordinate, } i = 1 \\ & \text{for } 5^{\text{th}} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ 2, & \text{for } 3^{\text{rd}} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ i-1, & \text{for } 4^{\text{th}} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 7 \\ n-i+1, & \text{for } 4^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ i - \lfloor \frac{n}{2} \rfloor, & \text{for } 5^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ \lfloor \frac{n}{2} \rfloor - i, & \text{for } 5^{\text{th}} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1, n \geq 7 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(v_i) = \begin{cases} 0, & \text{for } 2^{\text{nd}} \text{ ordinate, } i \geq 4, n, 2 \leq i \leq n-1, n \geq 7 \\ & \text{for } 3^{\text{rd}} \text{ ordinate, odd } i, 1 \leq i \leq n, n \geq 7 \\ 2, & \text{for } 2^{\text{nd}} \text{ ordinate, } i = 1 \\ 3, & \text{for } 3^{\text{rd}} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ i, & \text{for } 4^{\text{th}} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 7 \\ n-i+2, & \text{for } 4^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ \lfloor \frac{n}{2} \rfloor - i + 1, & \text{for } 5^{\text{th}} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 7 \\ i - \lfloor \frac{n}{2} \rfloor + 1, & \text{for } 5^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_i) = \begin{cases} 0, & \text{for } 1^{\text{st}} \text{ ordinate, } 1 \leq i \leq n, n \geq 7 \\ 2, & \text{for } 2^{\text{nd}} \text{ ordinate, } i = 1 \\ & \text{for } 3^{\text{rd}} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ i, & \text{for } 4^{\text{th}} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 7 \\ n-i+1, & \text{for } 4^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ \lfloor \frac{n}{2} \rfloor - i, & \text{for } 5^{\text{th}} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 7 \\ i - \lfloor \frac{n}{2} \rfloor + 1, & \text{for } 5^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(u_{n+i}) = \begin{cases} i-1, & \text{for } 1^{\text{st}} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 7 \\ n-i+1, & \text{for } 1^{\text{st}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ 0, & \text{for } 1^{\text{st}} \text{ ordinate, } i = 1 \\ & \text{for } 2^{\text{nd}} \text{ ordinate, even } i, 2 \leq i \leq \lfloor \frac{n}{2} \rfloor - 2, n \geq 7 \\ & \text{for } 2^{\text{nd}} \text{ ordinate, even } i, \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n-1, n \geq 7 \\ & \text{for } 3^{\text{rd}} \text{ ordinate, odd } i, 3 \leq i \leq n, n \geq 7 \\ & \text{for } 5^{\text{th}} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ 2, & \text{for } 2^{\text{nd}} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ \lfloor \frac{n}{2} \rfloor - i, & \text{for } 5^{\text{th}} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1, n \geq 7 \\ i - \lfloor \frac{n}{2} \rfloor, & \text{for } 5^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(v_{n+i}) = \begin{cases} i, & \text{for } 1^{\text{st}} \text{ ordinate, } 2 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1, n \geq 7 \\ n-i+2, & \text{for } 1^{\text{st}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ 0, & \text{for } 2^{\text{nd}} \text{ ordinate, odd } i, 1 \leq i \leq n, n \geq 7 \\ & \text{for } 3^{\text{rd}} \text{ ordinate, even } i, 2 \leq i \leq n-1, n \geq 7 \\ 2, & \text{for } 3^{\text{rd}} \text{ ordinate, } i = 1 \\ 3, & \text{for } 2^{\text{nd}} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ \lfloor \frac{n}{2} \rfloor - i + 1, & \text{for } 5^{\text{th}} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 7 \\ i - \lfloor \frac{n}{2} \rfloor + 1, & \text{for } 5^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_{n+i}) = \begin{cases} i, & \text{for } 1^{\text{st}} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor, n \geq 7 \\ n-i+1, & \text{for } 1^{\text{st}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor + 1 \leq i \leq n, n \geq 7 \\ 2, & \text{for } 2^{\text{nd}} \text{ ordinate, } i = \lfloor \frac{n}{2} \rfloor \\ & \text{for } 3^{\text{rd}} \text{ ordinate, } i = 1 \\ 0, & \text{for } 4^{\text{th}} \text{ ordinate, } 1 \leq i \leq n, n \geq 7 \\ \lfloor \frac{n}{2} \rfloor - i, & \text{for } 5^{\text{th}} \text{ ordinate, } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1, n \geq 7 \\ i - \lfloor \frac{n}{2} \rfloor + 1, & \text{for } 5^{\text{th}} \text{ ordinate, } \lfloor \frac{n}{2} \rfloor \leq i \leq n, n \geq 7 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(x_i) = \begin{cases} s - i + 1, & \text{for } 1^{st} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ i + 1, & \text{for } 1^{st} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ 1, & \text{for } 1^{st} \text{ ordinate, } i = s \\ & \text{for } 3^{rd} \text{ ordinate, odd } i, i \geq 1 \\ & \text{for } 4^{th} \text{ ordinate, even } i, i \geq 2 \\ i, & \text{for } 2^{nd} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor \\ s - i + 2, & \text{for } 2^{nd} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor \\ 2, & \text{for } 3^{rd} \text{ ordinate, } s = 1 \\ i + \lfloor \frac{n}{2} \rfloor - 2, & \text{for } 5^{th} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor \\ s - i + \lfloor \frac{n}{2} \rfloor, & \text{for } 5^{th} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor \\ 0, & \text{otherwise.} \end{cases}$$

Since for odd n all vertices have different color codes, c is a locating coloring for subdivision of certain barbell operation of origami graphs $B_{O_n}^s$, so that $\chi_L(B_{O_n}^s) \leq 5$, for $\lfloor \frac{n}{2} \rfloor$ even, $n \geq 7$.

Subcase 2.3. (even n), for $\frac{n}{2}$ odd, $n \geq 6$

Let c be a coloring for subdivision of certain barbell operation of origami graph $B_{O_n}^s$, for $\frac{n}{2}$ odd, $n \geq 6$ we make the partition $\Pi V(B_{O_n}^s)$:

$$\begin{aligned} C_1 &= \{w_i | 1 \leq i \leq \frac{n}{2} - 1\} \cup \{w_i | \frac{n}{2} + 1 \leq i \leq n\} \cup \{u_{n+1}\}; \\ C_2 &= \{u_i | \text{for odd } i, 3 \leq i \leq n - 1\} \cup \{v_i | \text{for even } i, 1 \leq i \leq n\} \cup \{u_{n+i} | \text{for even } i, 2 \leq i \leq n\} \cup \{v_{n+i} | \text{for odd } i, 1 \leq i \leq n - 1\}; \\ C_3 &= \{u_i | \text{for even } i, i \leq n\} \cup \{v_i | \text{for odd } i, 3 \leq i \leq n - 1\} \cup \{u_{n+i} | \text{for odd } i, 3 \leq i \leq n - 1\} \cup \{v_{n+i} | \text{for even } i, 2 \leq i \leq n\} \cup \{x_i | \text{for odd } i, i \geq 1\}; \\ C_4 &= \{u_i\} \cup \{w_{n+i} | 1 \leq i \leq \frac{n}{2} - 1\} \cup \{w_{n+i} | \frac{n}{2} + 1 \leq i \leq n\} \cup \{x_i | \text{for odd } i, i \geq 2\}; \\ C_5 &= \{w_n\} \cup \{w_{\frac{n}{2}}\}. \end{aligned}$$

For $\frac{n}{2}$ odd $n \geq 6$, the color codes of all the vertices of $V(B_{O_n}^s)$ are:

$$c_{\Pi}(u_i) = \begin{cases} 0, & \text{for } 2^{nd} \text{ ordinate, odd } i, 3 \leq i \leq n - 1, n \geq 6 \\ & \text{for } 3^{rd} \text{ ordinate, even } i, 2 \leq i \leq n, n \geq 6 \\ & \text{for } 4^{th} \text{ ordinate, } i = 1 \\ 2, & \text{for } 3^{rd} \text{ ordinate, } i = 1 \\ i - 1, & \text{for } 4^{th} \text{ ordinate, } 2 \leq i \leq \frac{n}{2}, n \geq 6 \\ n - i + 1, & \text{for } 4^{th} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 6 \\ \frac{n}{2} - i + 1, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 6 \\ i - \frac{n}{2}, & \text{for } 5^{th} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 6 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(v_i) = \begin{cases} 3, & \text{for } 2^{nd} \text{ ordinate, } i = 1 \\ 0, & \text{for } 2^{nd} \text{ ordinate, even } i, 2 \leq i \leq n, n \geq 6 \\ & \text{for } 3^{rd} \text{ ordinate, odd } i, 1 \leq i \leq n - 1, n \geq 6 \\ i, & \text{for } 4^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 6 \\ n - i + 2, & \text{for } 4^{th} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 6 \\ \frac{n}{2} - i + 2, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2} - 1, n \geq 6 \\ i - \frac{n}{2} + 1, & \text{for } 5^{th} \text{ ordinate, } \frac{n}{2} \leq i \leq n, n \geq 6 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_i) = \begin{cases} 0, & \text{for } 1^{st} \text{ ordinate, } 1 \leq i \leq \frac{n}{2} - 1, n \geq 6 \\ & \text{for } 1^{st} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 6 \\ & \text{for } 5^{th} \text{ ordinate, } i = \frac{n}{2} \\ 2, & \text{for } 2^{nd} \text{ ordinate, } i = 1 \\ i, & \text{for } 4^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 6 \\ n - i + 1, & \text{for } 4^{th} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 6 \\ \frac{n}{2} - i + 1, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2} - 1, n \geq 6 \\ i - \frac{n}{2} + 1, & \text{for } 5^{th} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 6 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(u_{n+i}) = \begin{cases} i - 1, & \text{for } 1^{st} \text{ ordinate, } 2 \leq i \leq \frac{n}{2}, n \geq 6 \\ n - i + 1, & \text{for } 1^{st} \text{ ordinate, } \frac{n}{2} \leq i \leq n, n \geq 6 \\ 0, & \text{for } 2^{nd} \text{ ordinate, even } i, 2 \leq i \leq n, n \geq 6 \\ & \text{for } 3^{rd} \text{ ordinate, odd } i, 3 \leq i \leq n - 1, n \geq 6 \\ 10, & \text{for } 1^{st} \text{ ordinate, } i = 1 \\ & \text{for } 3^{rd} \text{ ordinate, } i = 1 \\ \frac{n}{2} - i + 1, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 6 \\ i - \frac{n}{2}, & \text{for } 5^{th} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 6 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(v_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ ordinate, } 2 \leq i \leq \frac{n}{2}, n \geq 6 \\ n - i + 2, & \text{for } 1^{st} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 6 \\ 0, & \text{for } 2^{nd} \text{ ordinate, odd } i, 1 \leq i \leq n - 1, n \geq 6 \\ & \text{for } 3^{rd} \text{ ordinate, even } i, 2 \leq i \leq n, n \geq 6 \\ & \text{for } 2^{nd} \text{ ordinate, } i = 1 \\ 3, & \text{for } 2^{nd} \text{ ordinate, } i = 1 \\ \frac{n}{2} - i + 1, & \text{for } 5^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2} - 1, n \geq 6 \\ i - \frac{n}{2} + 1, & \text{for } 5^{th} \text{ ordinate, } \frac{n}{2} \leq i \leq n, n \geq 6 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_{n+i}) = \begin{cases} i, & \text{for 1}^{st} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 6 \\ n-i+1, & \text{for 1}^{st} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 6 \\ 2, & \text{for 2}^{nd} \text{ ordinate, } i=1 \\ 0, & \text{for 4}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}-1, n \geq 6 \\ & \text{for 4}^{th} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 6 \\ \frac{n}{2}, & \text{for 5}^{th} \text{ ordinate, } i = \frac{n}{2} \\ \frac{n}{2}-i+1, & \text{for 5}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}-1, n \geq 6 \\ i-\frac{n}{2}+1, & \text{for 5}^{th} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 6 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(x_i) = \begin{cases} s-i+1, & \text{for 1}^{st} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ i+1, & \text{for 1}^{st} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ 0, & \text{for 3}^{rd} \text{ ordinate, even } i, i \geq 2 \\ & \text{for 4}^{th} \text{ ordinate, odd } i, i \geq 1 \\ i, & \text{for 2}^{nd} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ s-i+2, & \text{for 2}^{nd} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ i+\frac{n}{2}-1, & \text{for 5}^{th} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ s-i+\frac{n}{2}, & \text{for 5}^{th} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ 1, & \text{otherwise.} \end{cases}$$

Since for odd n all vertices have different color codes, c is a locating coloring for subdivision of certain barbell operation of origami graphs $B_{O_n}^s$, so that $\chi_L(B_{O_n}^s) \leq 5$, for $\frac{n}{2}$ odd, $n \geq 6$.

Subcase 2.4. (even n), for $\frac{n}{2}$ even, $n \geq 4$

Let c be a coloring for subdivision of certain barbell operation of origami graph $B_{O_n}^s$, $\frac{n}{2}$ even, $n \geq 4$ we make the partition Π of $V(B_{O_n}^s)$:

$$\begin{aligned} C_1 &= \{w_i | 1 \leq i \leq \frac{n}{2}-1\} \cup \{w_i | \frac{n}{2}+1 \leq i \leq n\} \cup \{u_{n+1}\}; \\ C_2 &= \{u_i | \text{for odd } i, 1 \leq i \leq n-1\} \cup \{v_i | \text{for even } i, 2 \leq i \leq n\} \cup \{u_{n+i} | \text{for odd } i, 3 \leq i \leq n\} \cup \{v_{n+i} | \text{for even } i, 1 \leq i \leq n-1\}; \\ C_3 &= \{u_i | \text{for even } i, 2 \leq i \leq n-2\} \cup \{v_i | \text{for odd } i, 1 \leq i \leq n-1\} \cup \{u_{n+i} | \text{for even } i, 3 \leq i \leq n-1\} \cup \{v_{n+i} | \text{for odd } i, 2 \leq i \leq n\} \cup \{x_i | \text{for odd } i, i \geq 1\}; \\ C_4 &= \{u_n\} \cup \{w_{n+i} | 1 \leq i \leq \frac{n}{2}\} \cup \{w_{n+i} | \frac{n}{2}+2 \leq i \leq n\} \cup \{x_i | \text{for even } i, i \geq 2\}; \\ C_5 &= \{w_n\} \cup \{w_{n+\frac{n}{2}+1}\}. \end{aligned}$$

For $\frac{n}{2}$ even $n \geq 4$, the color codes of all the vertices of $V(B_{O_n}^s)$ are:

$$c_{\Pi}(u_i) = \begin{cases} 0, & \text{for 2}^{nd} \text{ ordinate, odd } i, 1 \leq i \leq n-1, n \geq 4 \\ & \text{for 3}^{rd} \text{ ordinate, even } i, 2 \leq i \leq n, n \geq 4 \\ & \text{for 4}^{th} \text{ ordinate, } i=n \\ i, & \text{for 4}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ n-i, & \text{for 4}^{th} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n-1, n \geq 4 \\ \frac{n}{2}+i+1, & \text{for 5}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ i-\frac{n}{2}, & \text{for 5}^{th} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 4 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(v_i) = \begin{cases} 0, & \text{for 2}^{nd} \text{ ordinate, even } i, 2 \leq i \leq n, n \geq 4 \\ & \text{for 3}^{rd} \text{ ordinate, odd } i, 1 \leq i \leq n-1, n \geq 4 \\ i+1, & \text{for 4}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ n-i+1, & \text{for 4}^{th} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 4 \\ \frac{n}{2}-i+2, & \text{for 5}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}-1, n \geq 4 \\ i-\frac{n}{2}+1, & \text{for 5}^{th} \text{ ordinate, } \frac{n}{2} \leq i \leq n, n \geq 4 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_i) = \begin{cases} 0, & \text{for 1}^{st} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}-1, n \geq 4 \\ & \text{for 1}^{st} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 4 \\ & \text{for 5}^{th} \text{ ordinate, } i = \frac{n}{2} \\ 2, & \text{for 1}^{st} \text{ ordinate, } i = \frac{n}{2} \\ & \text{for 3}^{rd} \text{ ordinate, } i=n \\ i+1, & \text{for 4}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ n-i, & \text{for 4}^{th} \text{ ordinate, } \frac{n}{2} \leq i \leq n-1, n \geq 4 \\ \frac{n}{2}-i+1, & \text{for 5}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ i-\frac{n}{2}+1, & \text{for 5}^{th} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 4 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_{\Pi}(w_{n+i}) = \begin{cases} i-1, & \text{for 1}^{st} \text{ ordinate, } 2 \leq i \leq \frac{n}{2}, n \geq 4 \\ n-i+1, & \text{for 1}^{st} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 4 \\ 0, & \text{for 2}^{nd} \text{ ordinate, even } i, 2 \leq i \leq n, n \geq 4 \\ & \text{for 3}^{rd} \text{ ordinate, odd } i, 3 \leq i \leq n-1, n \geq 4 \\ & \text{for 1}^{st} \text{ ordinate, } i=1 \\ 2, & \text{for 3}^{rd} \text{ ordinate, } i=1 \\ \frac{n}{2}-i+1, & \text{for 5}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ i-\frac{n}{2}, & \text{for 5}^{th} \text{ ordinate, } \frac{n}{2}+1 \leq i \leq n, n \geq 4 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_n(v_{n+i}) = \begin{cases} i, & \text{for 1}^{st} \text{ ordinate, } 2 \leq i \leq \frac{n}{2} + 1, n \geq 4 \\ n - i + 2, & \text{for 1}^{st} \text{ ordinate, } \frac{n}{2} + 2 \leq i \leq n, n \geq 4 \\ 0, & \text{for 2}^{nd} \text{ ordinate, odd } i, 1 \leq i \leq n - 1, n \geq 4 \\ & \text{for 3}^{rd} \text{ ordinate, even } i, 2 \leq i \leq n, n \geq 4 \\ 3, & \text{for 2}^{nd} \text{ ordinate, } i = 1 \\ \frac{n}{2} - i + 3, & \text{for 5}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ i - \frac{n}{2}, & \text{for 5}^{th} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 4 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_n(w_{n+i}) = \begin{cases} i, & \text{for 1}^{st} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ n - i + 1, & \text{for 1}^{st} \text{ ordinate, } \frac{n}{2} + 1 \leq i \leq n, n \geq 4 \\ 0, & \text{for 4}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ & \text{for 4}^{th} \text{ ordinate, } \frac{n}{2} + 2 \leq i \leq n, n \geq 4 \\ & \text{for 5}^{th} \text{ ordinate, } i = \frac{n}{2} + 1 \\ 2, & \text{for 4}^{th} \text{ ordinate, } i = \frac{n}{2} + 1 \\ \frac{n}{2} - i + 2, & \text{for 5}^{th} \text{ ordinate, } 1 \leq i \leq \frac{n}{2}, n \geq 4 \\ i - \frac{n}{2}, & \text{for 5}^{th} \text{ ordinate, } \frac{n}{2} + 2 \leq i \leq n, n \geq 4 \\ 1, & \text{otherwise.} \end{cases}$$

$$c_n(x_i) = \begin{cases} s - i + 1, & \text{for 1}^{st} \text{ ordinate, } i > \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ i + 1, & \text{for 1}^{st} \text{ ordinate, } i \leq \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ & \text{for 2}^{nd} \text{ ordinate, } i < \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ s - i + 2, & \text{for 2}^{nd} \text{ ordinate, } i \geq \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ 0, & \text{for 3}^{rd} \text{ ordinate, odd } i, i \geq 1 \\ & \text{for 4}^{th} \text{ ordinate, even } i, i \geq 2 \\ i + \frac{n}{2}, & \text{for 5}^{th} \text{ ordinate, } i < \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ s - i + \frac{n}{2} + 1, & \text{for 5}^{th} \text{ ordinate, } i \geq \lfloor \frac{s}{2} \rfloor, s \geq 2 \\ 1, & \text{otherwise.} \end{cases}$$

Since for odd n all vertices have different color codes, c is a locating coloring for subdivision of certain barbell operation of origami graphs $B_{O_n}^s$, so that $\chi_L(B_{O_n}^s) \leq 5$, for $\frac{n}{2}$ even, $n \geq 4$. This completes the proof of the theorem. \square

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