

# CERTAIN OPERATION OF GENERALIZED PETERSEN GRAPHS HAVING LOCATING- CHROMATIC NUMBER FIVE

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## CERTAIN OPERATION OF GENERALIZED PETERSEN GRAPHS HAVING LOCATING-CHROMATIC NUMBER FIVE

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## Abstract

The locating-chromatic number of a graph is combined two graph concept, coloring vertices and partition dimension of a graph. The locating-chromatic number, denoted by  $\chi_L(G)$ , is the smallest  $k$  such that  $G$  has a locating  $k$ -coloring. In this paper, we discuss the locating-chromatic number for certain operation of generalized Petersen graphs  $SP(n, 1)$ .

## 1. Introduction

In 2002, Chartrand et al. [7] introduced the locating-chromatic number of a graph, with derived two graph concept, coloring vertices and partition dimension of a graph. Let  $G = (V, E)$  be a connected graph and  $c$  be a proper  $k$ -coloring of  $G$  with color  $1, 2, \dots, k$ . Let  $\Pi = \{C_1, C_2, \dots, C_k\}$  be a partition of  $V(G)$  which is induced by coloring  $c$ . The color code  $c_\Pi(v)$  of  $v$  is the ordered  $k$ -tuple  $(d(v, C_1), d(v, C_2), \dots, d(v, C_k))$ , where  $d(v, C_i) = \min\{d(v, x) | x \in C_i\}$  for any  $i$ . If all distinct vertices of  $G$  have distinct color codes, then  $c$  is called  $k$ -locating coloring of  $G$ . The locating-chromatic number, denoted by  $\chi_L(G)$ , is the smallest  $k$  such that  $G$  has a locating  $k$ -coloring. Next, Chartrand et al. [6] determined the locating-chromatic number for some graph classes. On  $P_n$  it is a path of order  $n \geq 3$ , and hence  $\chi_L(P_n) = 3$ ; for a cycle  $C_n$  if  $n \geq 3$  odd,  $\chi_L(C_n) = 3$ , and if  $n$  even, then  $\chi_L(C_n) = 4$ ; for double star graph  $(S_{a,b})$ ,  $1 \leq a \leq b$  and  $b \geq 2$ , obtained  $\chi_L(S_{a,b}) = b + 1$ .

The following definition of a generalized Petersen graph is taken from Watkins [8]. Let  $\{u_1, u_2, \dots, u_n\}$  be some vertices on the outer cycle and  $\{v_1, v_2, \dots, v_n\}$  be some vertices on the inner cycle, for  $n \geq 3$ . The generalized Petersen graph, denoted by  $P(n, k)$ ,  $n \geq 3$ ,  $1 \leq k \leq \lfloor \frac{n-1}{2} \rfloor$ ,

$1 \leq i \leq n$  is a graph that has  $2n$  vertices  $\{u_i\} \cup \{v_i\}$ , and edges  $\{u_i u_{i+1}\} \cup \{v_i v_{i+k}\} \cup \{u_i v_i\}$ .

Now, we define a new kind of generalized Petersen graph called  $sP(n, k)$ . Suppose there are  $s$  generalized Petersen graphs  $P(n, k)$ . Some vertices on the outer cycle  $u_i, i = 1, 2, \dots, n$  for the generalized Petersen graph  $t$ th,  $t = 1, 2, \dots, s, s \geq 1$  denoted by  $u_i^t$ , while some vertices on the inner cycle  $v_i, i = 1, 2, \dots, n$  for the generalized Petersen graph  $t$ th,  $t = 1, 2, \dots, s, s \geq 1$  denoted by  $v_i^t$ . Generalized Petersen graph  $sP(n, k)$  obtained from  $s \geq 1$  is the graph  $P(n, k)$ , in which each of vertices on the outer cycle  $u_i^t, i \in [1, n], t \in [1, s]$  is connected by a path  $(u_i^t u_i^{t+1}), t = 1, 2, \dots, s - 1, s \geq 2$ .

The locating-chromatic number for corona product is determined by Baskoro and Purwasih [5], and locating-chromatic number for join graphs is determined by Behtoei and Ambarloei [1]. Additionally, Welyyanti et al. [9, 10] discussed locating-chromatic number for graphs with dominant vertices and locating chromatic number for graph with two homogeneous components. Asmiati obtained the locating-chromatic number of non-homogeneous amalgamation of stars [3]. Next, Asmiati et al. [4] determined some generalized Petersen graphs  $P(n, 1)$  having locating-chromatic number 4 for odd  $n \geq 3$  or 5; for even  $n \geq 4$ , certain operation of generalized Petersen graphs  $sP(4, 2)$  determined by Irawan et al. [2]. Besides that, in this paper, we will discuss the locating-chromatic number of generalized Petersen graphs  $sP(n, 1)$ .

The following theorems are basics to determine the lower bound of the locating-chromatic of a graph. The set of neighbours of a vertex  $y$  in  $G$  is denoted by  $N(y)$ .

**Theorem 1.1** [7]. Let  $c$  be a locating coloring in a connected graph  $G$ . If  $x$  and  $y$  are distinct vertices of  $G$  such that  $d(x, w) = d(y, w)$  for all  $w \in V(G) - \{x, y\}$ , then  $c(x) \neq c(y)$ . In particular, if  $x$  and  $y$  are non-adjacent vertices such that  $N(x) \neq N(y)$ , then  $c(x) \neq c(y)$ .

**Theorem 1.2** [7]. The locating-chromatic number of a cycle  $C_n$  is 3 for odd  $n$  and 4 for otherwise.

**Theorem 1.3** [4]. The locating-chromatic number for generalized Petersen graphs  $P(n, 1)$  is 4 for odd  $n \geq 3$  or 5 for even  $n \geq 4$ .

## 2. Main Results

In this section, we will discuss the locating-chromatic number of new kind generalized Petersen graphs  $sP(n, 1)$ .

**Theorem 2.1.**  $\chi_L(sP(3, 1)) = 5$ , for  $s \geq 2$ .

**Proof.** First, we determine the lower bound of  $\chi_L(sP(3, 1))$  for  $s \geq 2$ . Because a new kind generalized Petersen graph  $sP(3, 1)$ ,  $s \geq 2$  contains some generalized Petersen graph  $P(n, 1)$ , then by Theorem 1.3,  $\chi_L(sP(3, 1)) \geq 4$ . Suppose that  $c$  is a 4-locating coloring on  $sP(3, 1)$ . Consider  $c(u_i^1) = i$ ,  $i = 1, 2, 3$  and  $c(v_j^1) = j$ ,  $j = 1, 2, 3$  such that  $c(u_i^1) \neq c(v_j^1)$  for  $c(u_i^1)$  adjacent to  $c(v_j^1)$ . Observe that if we assign color 4 for any vertices in  $u_i^2$  or  $v_i^2$ , then we have two vertices whose the same color codes. Therefore,  $c$  is not locating 4-coloring on  $sP(3, 1)$ . As the result,  $\chi_L(sP(3, 1)) \geq 5$  for  $s \geq 2$ .

Next, we determine the upper bound of  $\chi_L(sP(3, 1)) \leq 5$  for  $s \geq 2$ . Assign the 5-coloring  $c$  on  $sP(3, 1)$  as follows:

$$\begin{aligned}
 \bullet c(u_i^t) &= \begin{cases} 1 & \text{for } i = 1 \text{ and odd } s; \\ 2 & \text{for } i = 2 \text{ and odd } s; \\ 3 & \text{for } i = 3 \text{ and odd } s; \\ 3 & \text{for } i = 1 \text{ and even } s; \\ 1 & \text{for } i = 2 \text{ and even } s; \\ 4 & \text{for } i = 3 \text{ and even } s. \end{cases} \\
 \bullet c(v_i^1) &= \begin{cases} 2 & \text{for } i = 1; \\ 3 & \text{for } i = 2; \\ 5 & \text{for } i = 3. \end{cases} \\
 \bullet c(v_i^t) &= \begin{cases} 3 & \text{for } i = 1 \text{ and odd } s \geq 3; \\ 1 & \text{for } i = 2 \text{ and odd } s \geq 3; \\ 2 & \text{for } i = 3 \text{ and odd } s \geq 3; \\ 4 & \text{for } i = 1 \text{ and even } s; \\ 2 & \text{for } i = 2 \text{ and even } s; \\ 3 & \text{for } i = 3 \text{ and even } s. \end{cases}
 \end{aligned}$$

The coloring  $c$  will create the partition  $\Pi$  on  $V(sP(3, 1))$ . We show that the color codes of all vertices in  $sP(3, 1)$  are different. For  $s = 1$ , we have  $c_{\Pi}(u_1^1) = (0, 1, 1, 2, 2)$ ;  $c_{\Pi}(u_2^1) = (1, 0, 1, 2, 2)$ ;  $c_{\Pi}(u_3^1) = (1, 1, 0, 1, 1)$ ;  $c_{\Pi}(v_1^1) = (1, 0, 1, 3, 1)$ ;  $c_{\Pi}(v_2^1) = (2, 1, 0, 3, 1)$ ;  $c_{\Pi}(v_3^1) = (2, 1, 1, 2, 0)$ . For  $s \geq 3$  odd, we have  $c_{\Pi}(u_1^t) = (0, 1, 1, 2, i + s)$ ;  $c_{\Pi}(u_2^t) = (1, 0, 1, 2, i + s)$ ;  $c_{\Pi}(u_3^t) = (1, 1, 0, 1, s)$ ;  $c_{\Pi}(v_1^t) = (1, 1, 0, 3, s + 2)$ ;  $c_{\Pi}(v_2^t) = (0, 1, 1, 3, i + s)$ ;  $c_{\Pi}(v_3^t) = (1, 0, 1, 2, s + 1)$ . For  $s \geq 2$  even, we have  $c_{\Pi}(u_1^t) = (1, 1, 0, 1, s + 1)$ ;  $c_{\Pi}(u_2^t) = (0, 1, 1, 1, s)$ ;  $c_{\Pi}(u_3^t) = (1, 2, 1, 0, s)$ ;  $c_{\Pi}(v_1^t) = (2, 1, 1, 0, s + 2)$ ;  $c_{\Pi}(v_2^t) = (1, 0, 1, 1, s + 2)$ ;  $c_{\Pi}(v_3^t) = (1, 1, 0, 1, s + 1)$ . Since the color codes of all vertices in  $sP(3, 1)$  are different, it follows that  $\chi_L(sP(3, 1)) \leq 5$  for  $s \geq 2$ .

**Theorem 2.2.**  $\chi_L(sP(n, 1)) = 5$ , for  $s \geq 2$  and odd  $n \geq 5$ .

**Proof.** The new kind generalized Petersen graphs  $sP(n, 1)$ , for  $s \geq 2$  and odd  $n \geq 5$ , contain some even cycles. Then, by Theorem 1.2,  $\chi_L(sP(n, 1)) \geq 4$ . Suppose that  $c$  is a locating coloring of  $sP(n, 1)$ , for  $s \geq 2$  and odd  $n \geq 5$ . Let  $C_1 = \{u_1^t \mid \text{for odd } s\} \cup \{u_n^t \mid \text{for even } s\} \cup \{v_1^t \mid \text{for even } s\} \cup \{v_n^t \mid \text{for odd } s, s \geq 3\}$ ;  $C_2 = \{u_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \cup \{v_{2j-1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \cup \{u_{2j-1}^t \mid \text{for odd } i \text{ and even } s, j > 0\} \cup \{v_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\}$ ;  $C_3 = \{u_{2j+1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \cup \{v_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \cup \{u_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\} \cup \{v_{2j+1}^t \mid \text{for odd } i \text{ and even } s, j > 0\}$ ;  $C_4 = \{v_n^t \mid \text{for odd } s\} \cup \{v_1^t \mid \text{for even } s\}$  for  $\{i > 0; j > 0\}$ . Then there are some vertices with same color codes,  $c_\Pi(u_{n-1}^t) = c_\Pi(v_1^t)$  for even  $s$  and  $c_\Pi(u_2^t) = c_\Pi(v_1^t)$  for odd;  $s \geq 2$ , a contradiction. Therefore,  $\chi_L(sP(n, 1)) \geq 5$ , for  $s \geq 2$  and odd  $n \geq 5$ .

We determine the upper bound of  $\chi_L(sP(n, 1)) \leq 5$ , for  $n \geq 5$  odd. The coloring  $c$  will create the partition  $\Pi$  on  $V(sP(n, 1))$ :

$$C_1 = \{u_1^t \mid \text{for odd } s\} \cup \{u_n^t \mid \text{for even } s\};$$

$$C_2 = \{u_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{v_{2j-1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{u_{2j-1}^t \mid \text{for odd } i \text{ and even } s, j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\};$$

$$\begin{aligned}
 C_3 &= \{u_{2j+1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \\
 &\cup \{v_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \\
 &\cup \{u_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\} \\
 &\cup \{v_{2j+1}^t \mid \text{for odd } i \text{ and even } s, j > 0\}; \\
 C_4 &= \{v_n^t \mid \text{for odd } s\} \cup \{v_1^t \mid \text{for even } s\}; \\
 C_5 &= \{v_n^1\}.
 \end{aligned}$$

Therefore, the color codes of all the vertices of  $G$  are:

(a)

$$\begin{aligned}
 C_1 &= \{u_1^t \mid \text{for odd } s\} \cup \{u_n^t \mid \text{for even } s\}; \\
 c_{\Pi}(u_1^1) &= (0, 1, 2, 2, 1); \quad c_{\Pi}(u_n^t) = (0, 1, 1, 2, s-1) \text{ for even } s \geq 2; \\
 c_{\Pi}(u_1^t) &= (0, 1, 2, 2, s) \text{ for odd } s \geq 3.
 \end{aligned}$$

(b)

$$\begin{aligned}
 C_2 &= \{u_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \\
 &\cup \{v_{2j-1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \\
 &\cup \{u_{2j-1}^t \mid \text{for odd } i \text{ and even } s, j > 0\} \\
 &\cup \{v_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\}.
 \end{aligned}$$

$$\begin{aligned}
 &\text{Let } u_i^t, 1 \leq i \leq n-1; i = 2j; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ for odd } s; u_i^t, 1 \leq i \leq n-2; \\
 &i = 2j-1; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ for even } s \text{ and } v_i^t, 1 \leq i \leq n-2; i = 2j-1; 1 \leq j \\
 &\leq \left\lfloor \frac{n}{2} \right\rfloor \text{ for odd } s; v_i^t, 2 \leq i \leq n-2; i = 2j; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ for even } s \geq 2.
 \end{aligned}$$



For  $i < \left\lceil \frac{n}{2} \right\rceil$ , we have:

$$c_{\Pi}(u_i^t) = (i - 1, 0, 1, i + 1, s + i - 1) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = (i, 0, 1, i, s + i) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = (i, 0, 1, i, s + i - 1) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = (i + 1, 0, 1, i - 1, s + i) \text{ for even } s.$$

For  $i = \left\lceil \frac{n}{2} \right\rceil$ , we have:

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j+1}^t) = (i - 1, 0, 1, i, 2j + s - 1) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j}^t) = (i, 0, 1, i - 1, 2j + s + 1) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j}^t) = (i - 1, 0, 1, i, 2j + s - 1) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j+1}^t) = (i, 0, 1, i - 1, 2j + s - 1) \text{ for even } s.$$

For  $i > \left\lceil \frac{n}{2} \right\rceil$ , we have:

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j+1}^t) = (2j, 0, 1, 2j, 2j + s - 2) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j}^t) = (2j + 2, 0, 1, 2j, 2j + s) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j}^t) = (2j, 0, 1, 2j + 2, 2j + s - 1) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j+1}^t) = (2j, 0, 1, 2j, 2j + s - 1) \text{ for even } s.$$

(c)

$$C_3 = \{u_{2j+1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{u_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\}$$

$$\cup \{v_{2j+1}^t \mid \text{for odd } i \text{ and even } s, j > 0\}.$$

Let  $u_i^t, 1 \leq i \leq n-2; i = 2j+1; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor - 1$  for  $s = 1; u_i^t, 1 \leq i \leq n; i = 2j+1; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$  for odd  $s \geq 3; u_i^t, 1 \leq i \leq n-1; i = 2j; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$  for even  $s$  and  $v_i^t, 1 \leq i \leq n-1; i = 2j; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$  for odd  $s; v_i^t, 1 \leq i \leq n; i = 2j+1; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$  for even  $s \geq 1$ .

For  $i < \left\lfloor \frac{n}{2} \right\rfloor$ , we have:

$$c_{\Pi}(u_i^t) = (i-1, 1, 0, i+1, i+s-1) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = (i, 1, 0, i, i+s) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = (i, 1, 0, i, i+s) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = (i+1, 1, 0, i-1, i+s) \text{ for even } s.$$

For  $i = \left\lfloor \frac{n}{2} \right\rfloor$ , we have:

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j+1}^t) = (i-1, 1, 0, i, 2j+s-1) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j}^t) = (i, 1, 0, i-1, 2j+s) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j}^t) = (i-1, 1, 0, i, 2j+s-1) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j+1}^t) = (i, 1, 0, i-1, 2j+s+1) \text{ for even } s.$$

For  $i > \left\lceil \frac{n}{2} \right\rceil$ , we have:

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j}^t) = (2j+1, 1, 0, 2j, 2j+s-1) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j+1}^t) = (2j+1, 1, 0, 2j-1, 2j+s-1) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j+1}^t) = (2j-1, 1, 0, 2j+1, 2j+s-2) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j+2}^t) = (2j-1, 1, 0, 2j-1, 2j+s-2) \text{ for even } s.$$

(d)

$$C_4 = \{v_n^t \mid \text{for odd } s\} \cup \{v_1^t \mid \text{for even } s\};$$

$$c_{\Pi}(v_n^t) = (2, 1, 1, 0, s) \text{ for odd } s;$$

$$c_{\Pi}(v_1^t) = (1, 2, 1, 0, s+1) \text{ for even } s.$$

(e)

$$C_5 = \{v_n^1\},$$

$$c_{\Pi}(v_n^1) = (1, 1, 2, 1, 0).$$

Since all the vertices have different color codes,  $c$  is a locating coloring of new kind generalized Petersen graphs  $sP(n, 1)$ , so  $\chi_L(sP(n, 1)) \leq 5$ , for odd  $n \geq 5$ .

**Theorem 2.3.**  $\chi_L(sP(n, 1)) = 5$  for  $s \geq 2$  and even  $n \geq 4$ .

**Proof.** First, we determine the lower bound of  $\chi_L(sP(n, 1))$  for  $s \geq 2$  and even  $n \geq 4$ . The new kind generalized Petersen graph  $sP(n, 1)$ , for  $s \geq 2$  and even  $n \geq 4$ , contains some generalized Petersen graph  $P(n, 1)$ , then by Theorem 1.3,  $\chi_L(sP(n, 1)) \geq 5$ .

Next, we determine the upper bound of  $\chi_L(sP(n, 1)) \leq 5$  for  $s \geq 2$  and  $n \geq 4$  even. The coloring  $c$  will create the partition  $\Pi$  on  $V(sP(n, 1))$ :

$$C_1 = \{u_1^t \mid \text{for odd } s\} \cup \{u_n^t \mid \text{for even } s\};$$

$$C_2 = \{u_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{v_{2j-1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{u_{2j-1}^t \mid \text{for odd } i \text{ and even } s, j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\};$$

$$C_3 = \{u_{2j+1}^t \mid \text{for odd } i \text{ odd } s, j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{u_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\}$$

$$\cup \{v_{2j+1}^t \mid \text{for odd } i \text{ and even } s, j > 0\};$$

$$C_4 = \{u_n^t \mid \text{for odd } s\} \cup \{u_{n-1}^t \mid \text{for even } s\};$$

$$C_5 = \{v_n^1\}.$$

Therefore, the color codes of all the vertices of  $G$  are:

(a)

$$C_1 = \{u_1^t \mid \text{for odd } s\} \cup \{u_n^t \mid \text{for even } s\};$$

$$c_{\Pi}(u_1^1) = (0, 1, 2, 1, 2); u_n^t = (0, 1, 2, 1, s) \text{ for even } s \geq 2;$$

$$c_{\Pi}(u_1^t) = (0, 1, 2, 1, s+1) \text{ for odd } s \geq 3.$$

(b)

$$\begin{aligned}
C_2 &= \{u_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \\
&\cup \{v_{2j-1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\} \\
&\cup \{u_{2j-1}^t \mid \text{for odd } i \text{ and even } s, j > 0\} \\
&\cup \{v_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\}.
\end{aligned}$$

Let  $u_i^t$ ,  $1 \leq i \leq n-2$ ;  $i = 2j$ ;  $1 \leq j \leq \frac{n}{2} - 2$  for odd  $s$ ;  $u_i^t$ ,  $1 \leq i \leq n-3$ ;  $i = 2j-1$ ;  $1 \leq j \leq \frac{n}{2}$  for even  $s$  and  $v_i^t$ ,  $1 \leq i \leq n-1$ ;  $i = 2j-1$ ;  $1 \leq j \leq \frac{n}{2}$  for odd  $s$ ;  $v_i^t$ ,  $1 \leq i \leq n-1$ ;  $i = 2j$ ;  $1 \leq j \leq \frac{n}{2}$  for even  $s \geq 2$ .

For  $i \leq \left\lceil \frac{n}{2} \right\rceil$ , we have:

$$\begin{aligned}
c_{\Pi}(u_i^t) &= (i-1, 0, 1, i, i+s) \text{ for odd } s; \\
c_{\Pi}(v_i^t) &= (i, 0, 1, i, i+s+1) \text{ for odd } s; \\
c_{\Pi}(u_i^t) &= (i, 0, 1, i+1, i+s) \text{ for even } s; \\
c_{\Pi}(v_i^t) &= (i+1, 0, 1, i+2, i+s+1) \text{ for even } s.
\end{aligned}$$

For  $i > \left\lceil \frac{n}{2} \right\rceil$ , we have:

$$\begin{aligned}
c_{\Pi}(u_i^t) &= c_{\Pi}(u_{n-2j}^t) = (2j+1, 0, 1, 2j, 2j+s) \text{ for odd } s; \\
c_{\Pi}(v_i^t) &= c_{\Pi}(v_{n-2j-1}^t) = (2j+1, 0, 1, 2j, 2j+s) \text{ for odd } s; \\
c_{\Pi}(u_i^t) &= c_{\Pi}(u_{n-2j-1}^t) = (2j+1, 0, 1, 2j, 2j+s+1) \text{ for even } s; \\
c_{\Pi}(v_i^t) &= c_{\Pi}(v_{n-2j}^t) = (2j-1, 0, 1, 2j, 2j+s-1) \text{ for even } s.
\end{aligned}$$

(c)

$$C_3 = \{u_{2j+1}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{for odd } i \text{ and odd } s, j > 0\}$$

$$\cup \{u_{2j}^t \mid \text{for odd } i \text{ and even } s, j > 0\}$$

$$\cup \{v_{2j+1}^t \mid \text{for odd } i \text{ and even } s, j > 0\}.$$

Let  $u_i^t$ ,  $1 \leq i \leq n-1$ ;  $i = 2j+1$ ;  $1 \leq j \leq \frac{n}{2}-1$  for odd  $s$ ;  $u_i^t$ ,  $1 \leq i \leq n-2$ ;  $i = 2j$ ;  $1 \leq j \leq \frac{n}{2}-1$  for even  $s$  and  $v_i^t$ ,  $1 \leq i \leq n-2$ ;  $i = 2j$ ;  $1 \leq j \leq \frac{n}{2}-1$  for odd  $s$ ;  $v_i^t$ ,  $1 \leq i \leq n-1$ ;  $i = 2j-1$ ;  $1 \leq j \leq \frac{n}{2}$  for even  $s \geq 2$ .

For  $i \leq \left\lceil \frac{n}{2} \right\rceil$ , we have:

$$c_{\Pi}(u_i^t) = (i-1, 1, 0, i, i+s) \text{ for odd } s;$$

$$c_{\Pi}(v_i^1) = (i, 1, 0, i+1, i);$$

$$c_{\Pi}(v_i^t) = (i, 1, 0, i+1, i+2s-2) \text{ for odd } s \geq 3;$$

$$c_{\Pi}(u_i^t) = (i, 1, 0, i+1, i+s) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = (i+1, 1, 0, i+1, i+s) \text{ for even } s.$$

For  $i > \left\lceil \frac{n}{2} \right\rceil$ , we have:

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j+1}^t) = (2j+1, 1, 0, 2j-1, 2j+s-1) \text{ for odd } s;$$

$$c_{\Pi}(v_i^1) = c_{\Pi}(v_{n-2j}^t) = (2j+2, 1, 0, 2j+1, 2j);$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j}^t) = (2j + 2, 1, 0, 2j + 1, 2j + s + 1) \text{ for odd } s \geq 3;$$

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j+1}^t) = (2j, 1, 0, 2j - 1, 2j + s) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j}^t) = (2j, 1, 0, 2j - 1, 2j + s) \text{ for even } s.$$

(d)

$$C_4 = \{u_n^t \mid \text{for odd } s\} \cup \{u_{n-1}^t \mid \text{for even } s\};$$

$$c_{\Pi}(u_n^t) = (1, 2, 1, 0, s) \text{ for odd } s;$$

$$c_{\Pi}(u_{n-1}^t) = (1, 2, 1, 0, s + 1) \text{ for even } s.$$

(e)

$$C_5 = \{v_n^1\},$$

$$c_{\Pi}(v_n^1) = (2, 1, 2, 1, 0).$$

Since all the vertices have different color codes,  $c$  is a locating coloring of new kind generalized Petersen graphs  $(sP(n, 1))$ , so  $\chi_L(sP(n, 1)) \leq 5$ , for even  $n \geq 4$ .

### 3. Conclusion

Based on the results, locating-chromatic number of new kind generalized Petersen graphs  $sP(n, 1)$  is 5 for  $s \geq 2$  and  $n \geq 3$ .

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