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Ring Stability of Underground Toroidal Tanks

Asnawi Lubis[1, a)] and Ahmad Su'udi[1, b)]

[1]*Structure Mechanics Research Group, Department of Mechanical Engineering, University of Lampung Jalan Professor Soemantri Brojonegoro No.1, Bandar Lampung, 35145, Indonesia*
[a)]Corresponding author: asnawi.lubis@eng.unila.ac.id
b)ahmad.suudi@eng.uni.a.ac.id

Abstract. The design of pressure vessels subjected to internal pressure is governed by its strength, while the design of pressure vessels subjected to external pressure is governed by its stability, which is for circular cross-section is called the ring stability. This paper presented the results of finite element study of ring stability of circular toroidal tank without stiffener under external pressure. The tank was placed underground and external pressure load from soil was simulated as pressure at the top of the vessel along 30^0 circumferentially. One might ask the reason for choosing toroidal rather than cylindrical tank. Preliminary finite element studies showed that toroidal shells can withstand higher external pressure than cylindrical shells. In this study, the volume of the tank was fixed for 15,000 litters. The buckling external pressure (p_L) was calculated for radius ratio (*R/r*) of 2, 3, and 4. The corresponding cross-section radiuses were 724.3 mm, 632.7 mm, and 574.9 mm, respectively. The selected element type was SHELL 281 from the ANSYS element library. To obtain the buckling load, the arc-length method was used in the nonlinear analysis. Both material and geometric nonlinearities were activated during the analysis. The conclusion of this study is that short-radius and thin-walled toroidal shell produces higher buckling load.

INTRODUCTION

Toroidal shells are used in many areas of engineering, such as space technology, nuclear power plant, under water field, and pressure vessel. Toroidal shells can be found in the complete form and incomplete form. The application of incomplete toroidal shell can be found in the piping system as elbows or pipe bends. Complete toroidal shells are currently used as LPG fuel tank for passenger cars in some countries in Europe.

The design of toroidal pressure vessel subjected to internal pressure loading is governed by its strength. Stress behavior of toroidal shells has been investigated by many researchers. Reference [1] investigated the influence of major to minor axis ratio of elliptic cross-section toroidal shell on its limit pressure, and found that toroidal shell of circular cross-section withstand the highest limit pressure of all. The influence of cross-section ovality of toroidal shell on limit pressure was further studied by Ref. [2] and confirmed that toroidal shell with circular cross section withstand the maximum internal pressure higher than those of oval cross-section. Nonlinear behavior of toroidal shell having in-plane and out-of-plane oval cross section under internal pressure load was reported by Ref. [3]. In addition, the influence of radius ratio on limit internal pressure was also studied by Ref. [4] that found out in various radius ratios *R/r* in the range of 2 to 10. The volume and thickness was constant and cross-section radius was different for each value of radius ratio. It found that radius ratio of 4 (four) could withstand the highest limit pressure. The influence of circumferential position of radial flush nozzle on limit pressure of toroidal tanks was reported by Ref. [4].

The design of pressure vessels subjected to external pressure is governed by its stability or buckling. The type of buckling for shell structures is snap-through buckling. For axially loaded shell the buckling type is called post buckling/bifurcation snap-through. For shells under external pressure load, such as underwater or underground

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pressure vessel, the type of buckling is called limit point snap-through buckling [5]. Toroidal shell loaded by internal pressure would undergo limit point type of snap-through buckling. Numerous studies of buckling behavior of toroidal shell were reported in scientific and engineering literatures. Stability analysis of an orthotropic complete toroidal shell based on the Sanders-Budiansky buckling shell theory was reported by Ref. [6], which obtained numerical results by using differential quadrature method. References [7] and [8] theoretically solved for stress and displacement of toroidal shell with ring-stiffened ribs subjected to uniform external pressure by using the simple curved beam theory. In the previous cited papers, it was assumed that the external pressure works uniformly on the entire external surface of the shell. This paper presented the results of a finite element study of ring stability of circular toroidal tank loaded by external pressure. For underground application, it was assumed that the external pressure worked on 30^0 on the top surface of circumference of the tank as a simulation of soil load from the top. The plane view of the tank was placed horizontally accordingly.

BEHAVIOUR OF TOROIDAL SHELLS UNDER EXTERNAL PRESSURE

A complete toroidal shell is a special shell of revolution which can be obtained by rotating a circle of radius r along a line circle of radius R. The ratio of major radius R to cross-section radius r is called radius ratio. Small radius ratio is called short radius toroidal shell and high value of radius ratio is called long radius toroidal shell, as shown in Fig. 1.

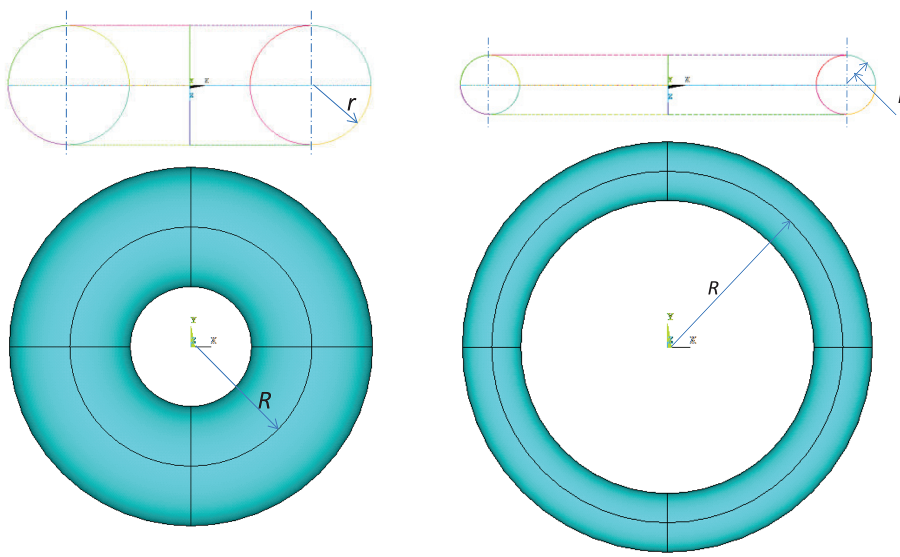

FIGURE 1. Toroidal shell, (a) short radius, (b) long radius

Figure 2 shows the geometric parameters of a complete circular toroidal shell, where R is the major toroidal radius (the distance between the axis of rotation and the centre of the circular cross-section); r is the radius of the circular cross-section; t is the shell thickness; θ is the angular coordinate along the meridian, and φ is the angular coordinate along the hoop circle of revolution for the toroid, measured from extrados.

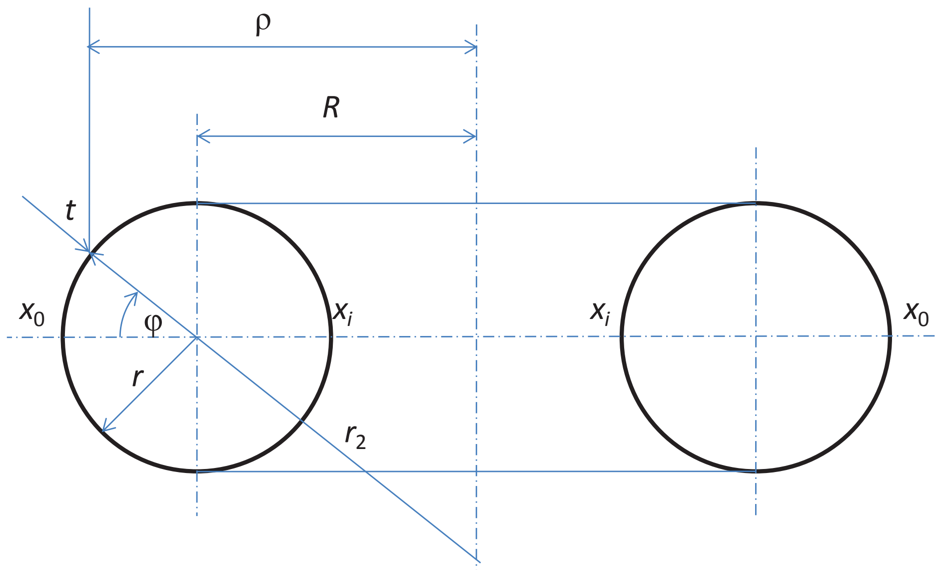

FIGURE 2. Geometry parameters of a toroidal shell

The shape of the toroidal shell is characterized by r_1 and r_2 (the principal radii of curvature of the shell mid surface in the meridional plane and the second principal plane respectively):

$$r_1 = \pm r \tag{1}$$

$$r_2 = \frac{\rho}{\cos\varphi} = \frac{R \pm r\cos\varphi}{\cos\varphi} \tag{2}$$

Toroidal shell has double curvature, i.e., positive Gaussian curvature for $-90^0 < \varphi < 90^0$, negative Gaussian curvature for $\varphi < -90^0$, and $\varphi > 90^0$, and has turning point at $\varphi = \pm 90^0$. The angle φ is circumferential position measured from extrados (x_o). The classical buckling analysis was performed by Ref. [9] and proposed the buckling pressure due to external pressure as follows:

$$p = \frac{2}{\sqrt{3\left(1-\upsilon^2\right)}} E\left(\frac{t}{r}\right)^2 \tag{3}$$

Where, *E* is Young's modulus, *t* is shell thickness, *r* is cross-section radius, *and* υ is Poisson's ratio.

Ring stability is defined as resistance to collapse due to external pressure (and internal vacuum). For underground toroidal tank, resistance to collapse depends upon ring flexibility *D/t* and soil embedment. Soil support increases critical vacuum that depends on soil quality, density of the soil, and ring deflection. However, a water table above the tank decreases the soil support. The classical equation for collapse circular ring subjected to uniform external pressure is calculated by the following equation [10]:

$$\frac{pr^3\left(1-\upsilon^2\right)}{EI} = 3 \tag{4}$$

In the present study, the load was vertical soil pressure at the top of the tank, simulated as internal pressure along 30^0 of circumferential angle. The tank position was horizontal and external pressure load acted along 15^0 toward extrados and 15^0 toward intrados, from crown position. Radial displacement of a node representing nodes at crown

(90^0 from extrados) was plotted in a graph of load (pressure) vs. displacement and would exhibit either of the two primary types of snap-through buckling: post buckled/ bifurcation snap-through or limit point snap-through as shown in Fig. 3.

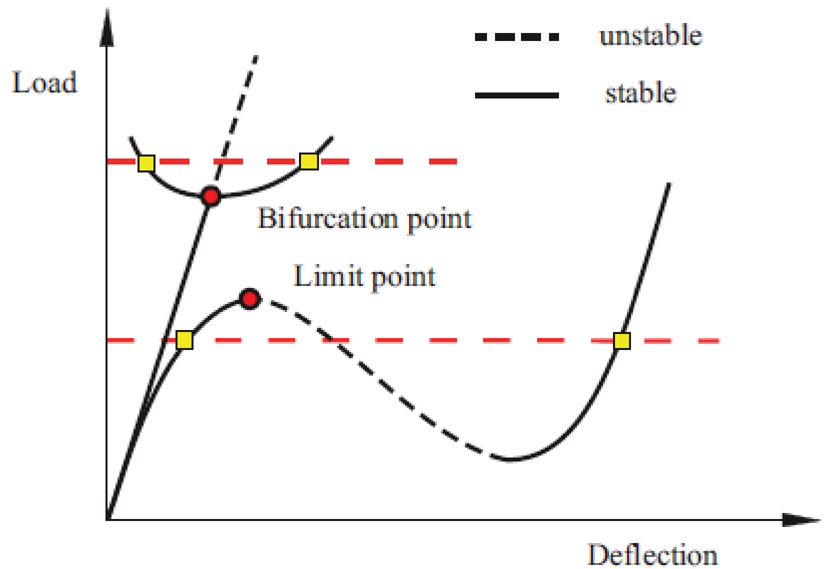

FIGURE 3. Typical nonlinear load-displacement relation for snap-through buckling [6]

FINITE ELEMENT MODELING AND ANALYSIS

The tank was designed for 15,000 litters in capacity. The volume of a toroidal tank can be calculated by using the following formula:

$$V = \pi r^2 2\pi R \quad (5)$$

where, r is radius cross-section and R is major radius of a toroidal tank. If $\rho = R/r$ is radius ratio, equation (5) can be written as,

$$V = 2\rho\pi^2 r^3 \quad (6)$$

If volume is constant, the cross-section radius r can be calculated according to the following formula:

$$r = \left(\frac{V}{2\rho\pi^2}\right)^{1/3} \quad (7)$$

For volume of 15,000 litters, the cross-section radius for corresponding radius ratio of 2, 3, and 4 were presented in Table 1. Geometrical in-plane view of the tanks is shown in Fig. 4.

TABLE 1. Toroidal geometry parameter for 15,000 liter capacity

R/r	2	3	4
r, mm	724.3	632.7	574.9

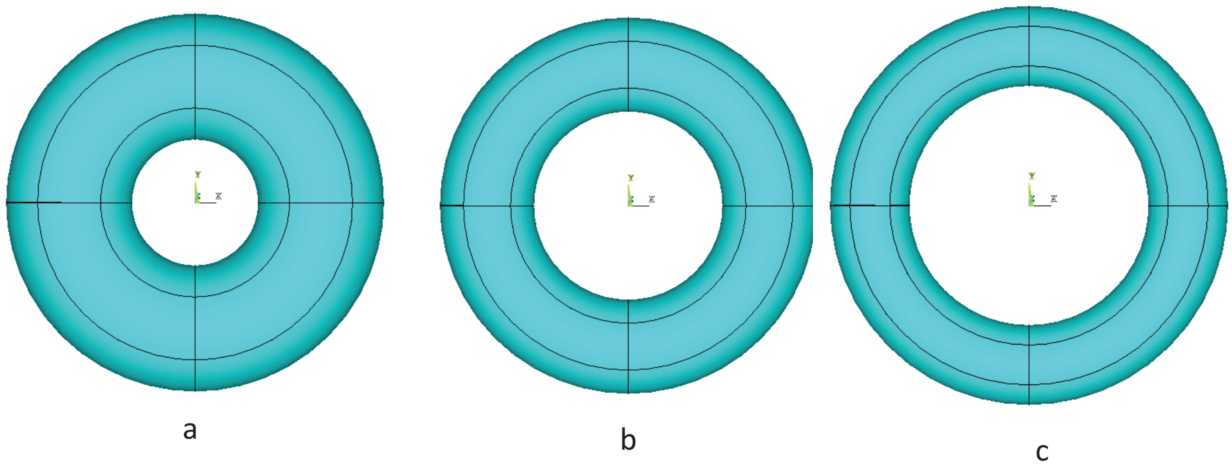

FIGURE 4. Circular cross-section toroidal tanks, (a) *R/r* = 2, (b) *R/r*= 3, (c) *R/r* = 4

ANSYS shell281 element was used in finite element modeling. Element sizes in longitudinal and circumferential direction were 5^0 and 2^0, respectively. The total number of element was 12,960. Typical finite element modeling is shown in Fig. 5.

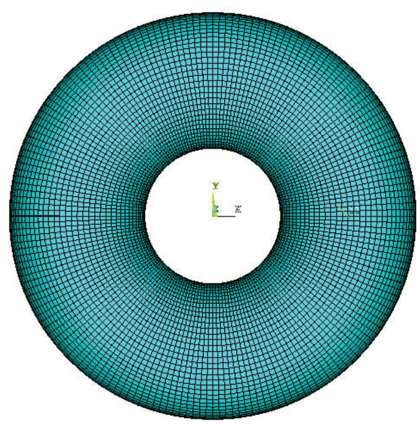

FIGURE 5. Typical finite element modeling

The tank would be placed underground in horizontal. The vertical loading came from soil on the top of the tank. Nodes on bottom part of the tank (crown position and 15^0 toward extrados and intrados) had zero displacement in vertical direction. External pressure from soil at the top part was applied at opposite crown along 30^0 of the circumference. The arc length method to solve nonlinear problem was used to obtain the buckling load. Typical boundary condition and applied loading is shown in Fig. 6.

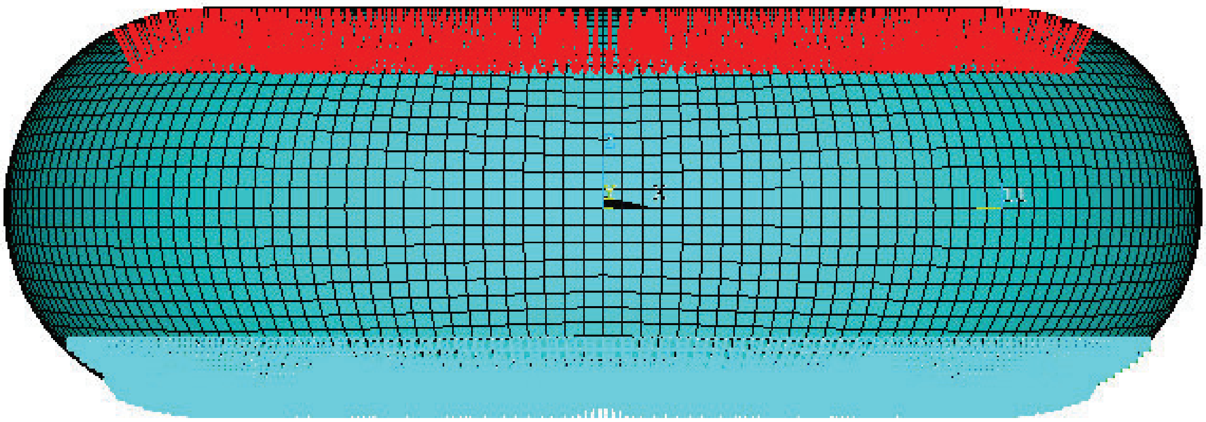

FIGURE 6. Boundary conditions and external pressure loading act on a toroidal tank

RESULTS AND DISCUSSION

Figure 7 shows the relation of external pressure and radial displacement of nodes of crown (90^0 from extrados). It was plot for radius ratio of 2, 3, and 4. The volume and thickness were kept constant for 15,000 litter and 4 mm respectively. The corresponding ring flexibility *D/t* for radius ratio of 2, 3, and 4 were 362, 316, and 287, respectively. It indicated that the toroidal tank exhibited limit point snap-through buckling behavior with the highest buckling load was obtained from radius ratio of 2, and decreased in accordance to higher radius ratio.

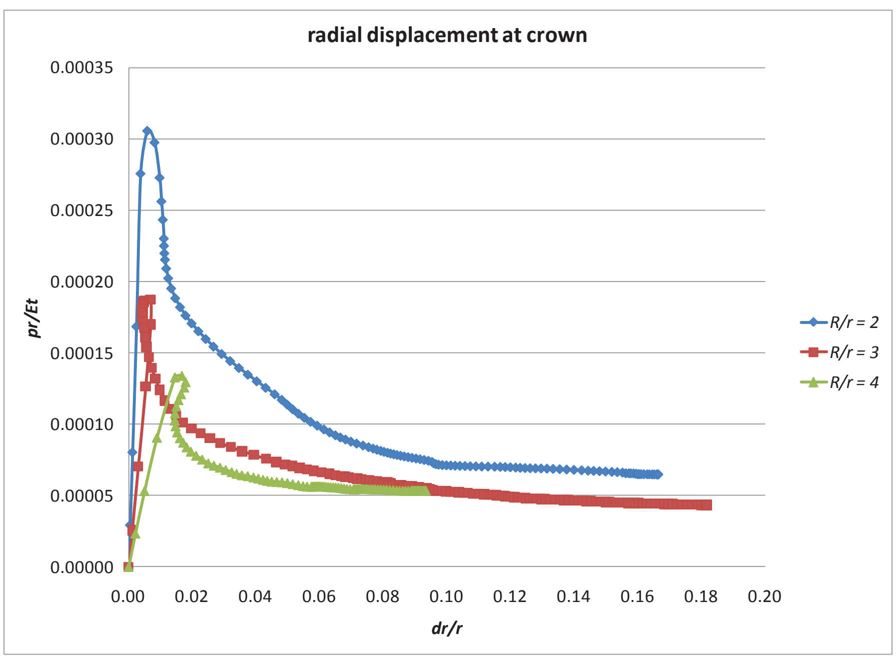

FIGURE 7. Load (external pressure)-displacement relation for toroidal tanks

Recalling the formula of pressure to yield p_Y for a toroidal shell subjected to internal pressure [1], which was:

$$p_Y = \frac{2t\sigma_Y}{r}\frac{\rho - 1}{2\rho - 1} \tag{8}$$

where $\rho = R/r$ is radius ratio. Equation (8) indicates that pressure to yield (also limit internal pressure) will be higher for long radius toroidal. However, due to external pressure as shown in Figure 7, buckling load becomes higher for short-radius toroidal.

Figure 8 shows the load-displacement for radius ratio of 2 and ring deflection of 362, 290, and 241. It can be seen that limit point (buckling load) was the highest for ring flexibility *D/t* of 362 and it decreased in accordance to the increase of the ring flexibility. In Equation (8) for internal pressure, limit pressure is higher in linear to the higher thickness. Surprisingly, the present result for external pressure showed that buckling load was lower in relation to the increased thickness of the shell.

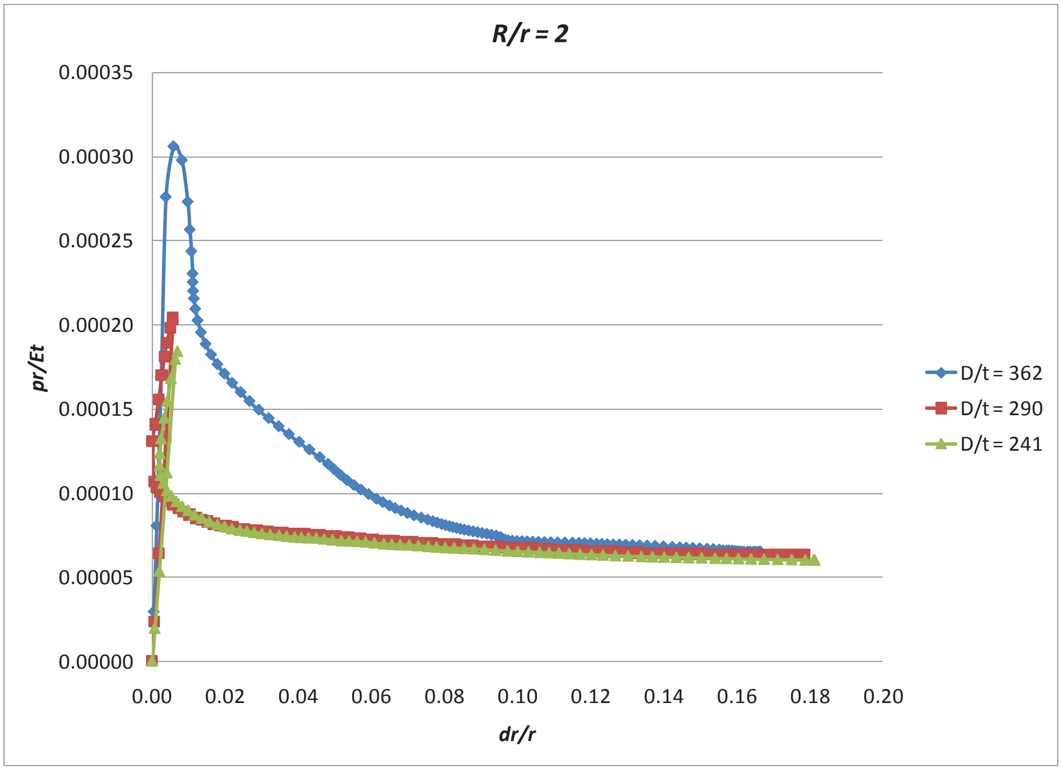

FIGURE 8. Load-displacement relation for toroidal shell under vertical external pressure

CONCLUSION

From this analysis, it was found that the behavior of toroidal tank under external pressure was opposed the behavior under internal pressure:

- Buckling load is higher for short radius toroidal shell and is lower for long radius toroidal shell. However, for internal pressure load, limit pressure is higher for long radius toroidal shell.
- Buckling load is higher for thin shell toroidal shell and is lower as the thickness of the shell increase. Again, it contradicted the behavior under internal pressure, where higher limit pressure would be obtained by higher thickness of the shell.

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