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Application of Pascoletti-Serafini scalarization modification method to solve multi-objective optimization problems for stock portfolio

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Abstract. One example of a multi-objective optimization problem is stock portfolio management. There are at least two objective functions to be achieved simultaneously, namely to maximize returns and minimize risk. The desire to maximize return and minimize risk are conflicting objectives. In this study, the problem of multi-objective optimization in the selection of Islamic stock portfolios will use the Pascoletti-Serafini scalarization modification method. Furthermore, the solution to the multi objective optimization problem is known as the Pareto optimal solution or efficient solution. In the Pascoletti-Serafini scalarization modification method, a set of Pareto optimal solutions can be constructed so that not only one solution is offered to decision makers, but a set of Pareto optimal solutions. From this research, the results obtained in the form of a set of efficient solutions that can be used as investor preferences in choosing the optimal stock portfolio.

1. Introduction

In many economic problems, decision maker no longer only consider one objective, such as only want to maximize profit, or just want to minimize production cost without considering other aspects. However, decision maker want to optimize several objectives functions at the same time. Furthermore, this problem is called as multi-objective optimization problem. In multi objective optimization problems, one objective function with another objective will conflict with each other so that it is difficult to find a single optimal solution. It is mean difficult to find a single solution that is able to optimize two or more objective functions at the same time.

One example of multi-objective optimization problem is stock portfolio management. There are at least two objective functions to be achieved simultaneously, namely to maximize return and minimize risk. The desire to maximize return and minimize risk are confliting objectives.

To get an optimal stock portfolio requires a good investment management. According to Reilly and Brown [1], an investment is the current commitment of dollars for a period of time in order to derive future payments that will compensate the investor for (1) the time the funds are committed, (2) the expected rate of inflation during this time period, and (3) the uncertainly of the future payments.

According to Duan [2], portfolio optimization plays a critical role in determining portfolio strategies for investors. What investors hope to achieve from portfolio optimization is to maximize portfolio returns and minimize portfolio risk. Since return is compensated based on risk, investors have to balance the risk-return trade-off for their investments. Therefore, there is no a single optimized



portfolio that can satisfy all investors. An optimal portfolio is determined by an investor's risk-return preference.

Many researches on optimization of stock portfolios have been done. Amiri, Ekhtiari, and Yazdani [3], Saputro and Qudratullah [4] use Nadir Compromise Programming to solve multi-objective optimization problems in stock portfolio. Pouya, Solimanpur, and Rezaee [5] use the method of invasive weed optimization. Fitria [6] uses Model Predictive Control (MPC) to solve the portfolio optimization problem. Oh, et.al. [7], and Skolpadungket, Dahal, and Hampupornchai [8] use Genetic Algorithm in solving stock portfolio optimization problems.

Nadir Compromise Programming (NCP) and Multi-Objective Genetica Algorithm (MOGA) are several methods for solving multi-objective optimization problems. There is another method that can solve the multi-objective optimization problem for both linear and nonlinear functions, namely the Pascoletti-Serafini Scalarization method [9]. The Pascoletti-Serafini scalarization method change the multi-objective optimization problem which is a vector optimization problem into a scalar optimization problem. Khorram, Khaledian, and Khaledyan [10] modified the Pascoletti-Serafini scalarization method by restriction certain parameters to generate solutions.

In this research, the problem of multi-objective optimization in the selection of islamic stock portfolios will use the Pascoletti-Serafini scalarization modification method. Furthermore, multi-objective optimization problem solutions are known as Pareto Optimal solution or efficient solution or non-dominated solution. In the Pascoletti-Serafini scalarization modification method, a set of Pareto optimal solution can be constructed so that not only one solution is offered to decision makers, but a set of Pareto optimal solutions.

2. Research method

In this study, the stock data used to determine the optimal portfolio is Islamic stock data which is incorporated in the Jakarta Islamic Index (JII). The Jakarta Islamic Index (JII), which is an index of Islamic stocks in Indonesia, consists of the 30 most liquid stocks listed on the Indonesia Stock Exchange (IDX). The data used in this study are daily stock data from August 27th, 2018 to August 25th, 2020.

2.1. Calculation of return and risk value

The calculation of the discrete return value is formulated as follows:

$$R_t = \frac{S_t}{S_{t-1}} - 1 \quad (1)$$

where R_t is the stock return at time t , and S_t is the stock price at time t .

Next, to calculate the arithmetic average value of the expected return using the following formula:

$$E(R_i) = \frac{\sum_{j=1}^N R_{ij}}{N} \quad (2)$$

where $E(R_i)$ represents the expected return of stock i , R_{ij} is the return of stock i at time to = j , and N is the number of periods of observation.

The risk in investing can be determined by looking at the risk coefficient value of a stock. To calculate the risk coefficient (stock beta) of a stock, you can use the following formula:

$$\beta_i = \frac{Cov(R_i R_m)}{\sigma_m^2} \quad (3)$$

where β_i states the risk coefficient of stock i , σ_m^2 is the variance value of market shares, in this case the Jakarta Islamic Index (JII), and $Cov(R_i R_m)$ states the covariance between stock i and market shares m .

The value of the stock variance i , σ_i^2 from a number of N stock data is formulated in the following equation:

$$\sigma_i^2 = \sum_{j=i}^N \frac{(R_{ij} - E(R_i))^2}{N} \tag{4}$$

Covariance shows the linear attachment of two variables. The covariance of two variables (stock i , and market share m) is formulated as follows:

$$Cov(R_i R_m) = \sum_{j=1}^N \frac{(R_{ij} - E(R_i))(R_{mj} - E(R_m))}{N} \tag{5}$$

2.2. *Pascoletti-Serafini scalarization modification*

Given a multi-objective optimization problem (MOP) which is mathematically expressed as:

$$MOP : \min_{x \in X} f(x) = (f_1(x), f_2(x), \dots, f_p(x)), \tag{MOP (1)}$$

with $X \subset \mathbb{R}^n$ is a non-empty phonetic assembly and f is a vector value function composed of p ($p \geq 2$) real value functions. The image of X in f is denoted as $Y := f(X) \subseteq \mathbb{R}^p$ and points to the image space.

Furthermore, the solution for $\hat{x} \in X$ is called the Pareto efficient/optimal solution of MOP (1) if there is no $x \in X$ such that $f(x) \leq f(\hat{x})$. If $\hat{x} \in X$ is Pareto optimal then $f(\hat{x})$ is called the point that is not dominated [10].

The Pascoletti-Serafini scalarization modification which is hereinafter called the problem $\overline{SP}(a, r)$ is given as follows:

$$\begin{aligned} &\min t \\ &\text{with constraints:} \\ &tr - f(x) \geq 0, \quad \overline{SP}(a, r) \\ &x \in X, \quad t \in \mathbb{R}. \end{aligned}$$

In this study, to limit the selection of the $r \in \mathbb{R}^p$ parameter, it is assumed that MOP (1) has an ideal point. The point $f^* = (f_1^*, \dots, f_p^*)^T$, where $f_i^* = \min_{x \in X} f_i(x)$ untuk $i = 1, \dots, p$, is called the ideal point of MOP (1). Furthermore, after obtaining the ideal point from MOP (1) the objective function is redefined as $f(x) \leftarrow f(x) - f^*$. In other words, the ideal point is shifted to the starting point. In the case of two objective functions are shown in Figure 2.1. With this redefinition it is assumed that the ideal point is equal to zero and the objective function is non-negative [10].

3. **Result**

3.1. *Calculating of expected return and risk coefficient values*

Of the 30 stocks listed in the Jakarta Islamic Index (JII) for the period August 27th, 2018 to August 25th, 2020, the return value and the expected return of each stock are calculated in the following Table 1.

Table 1. The expected return value of each stocks in JII.

Stock name	Value of expected return ($E(R)$)	Stock name	Value of expected return ($E(R)$)	Stock name	Value of expected return ($E(R)$)
ADRO	0,013054266	EXCL	-0,003942662	MNCN	-0,791666667
AKRA	0,001205266	ICBP	0,010502358	PGAS	-0,008345024
ANTM	0,006998945	INCO	0,009009481	PTBA	-0,023513961
ASII	-0,007633199	INDF	0,012538815	PTPP	-0,031071859
BRPT	0,066208497	INTP	-0,012004707	SCMA	-0,032366520
BSDE	-0,010783542	ITMG	-0,041682694	TLKM	-0,034259053
BTPS	0,045716860	JPFA	-0,016709630	TPIA	-0,035736733
CPIN	0,014477447	JSMR	0,005301349	UNTR	-0,037517386
CTRA	0,012336491	KLBF	0,010940747	UNVR	-0,038762560
ERAA	0,009666473	LPPF	-0,047399529	WIKA	0,011659107

Furthermore, stocks with a positive expected return value are selected and then the share risk coefficient value is calculated in the following Table 2.

Table 2. Stocks with positive expected returns.

Stock name	Value of expected return ($E(R)$)	Stock name	Value of expected return ($E(R)$)
ADRO	0.013054266	ERAA	0.009666473
AKRA	0.001205266	ICBF	0.010502358
ANTM	0.006998945	INCO	0.009009481
BRPT	0.066208497	INDF	0.012538815
BTPS	0.045716860	JSMR	0.005301349
CPIN	0.014477447	KLBF	0.010940747
CTRA	0.012336491	WIKA	0.011659107

Of the 14 stocks with a positive expected return, the risk coefficient (β) is calculated. Furthermore, after obtaining the value of the expected return and the risk coefficient, the ratio of each stock is calculated by comparing the value of the expected return and the risk coefficient. The coefficient and ratio values of each share are presented in the following Table 3.

Table 3. Risk coefficient and ratio values of Stock with positive expected return.

Stock name	Risk coefficient (β)	Ratio	Stock name	Risk coefficient (β)	Ratio
ADRO	-34,78840746	-0,000375248	ERAA	1,195516024	0,008085607
AKRA	0,188001228	0,006410949	ICBF	11,43864871	0,000918147
ANTM	1,109264912	0,006309535	INCO	1,545543312	0,005829329
BRPT	6,950584620	0,009525601	INDF	12,97479585	0,000966398
BTPS	11,29147788	0,004048793	JSMR	1,027451655	0,005159707
CPIN	10,17521513	0,001422815	KLBF	3,040220735	0,003598669
CTRA	2,008097185	0,006143374	WIKA	1,838901341	0,006340257

In the formation of a multi-objective optimization problem, it is limited by choosing six decision variables, namely the six stocks with the highest ratio. The six shares were obtained as following in Table 4.

Table 4. List of stocks in the formation of a stock portfolio.

Stock name	Value of expected return ($E(R)$)	Risk coefficient (β)
AKRA	0,001205266	0,188001228
ANTM	0,006998945	1,109264912
BRPT	0,066208497	6,95058462
CTRA	0,012336491	2,008097185
ERAA	0,009666473	1,195516024
WIKA	0,011659107	1,838901341

3.2. Mathematical model formulation of multi-objective optimization problems in the stock portfolio

In the multi objective optimization problem for stock portfolios, there are two aspects to be considered, namely risk and expected return. The first step taken to model the optimization problem is to determine the decision variable. The decision variables (variables) for the multi objective optimization problem for this stock portfolio are as follows:

$$x_i = \text{the proportion of funds to be invested in stocks to-}i ; i = 1,2,3,4,5,6$$

where 1 = AKRA ; 2 = ANTM ; 3 = BRPT ; 4 = CTRA ; 5 = ERAA ; 6 = WIKA.

The next step is to determine the objective function. There are two objective functions that are considered, namely as follows:

1. The objective function to maximize the expected return (profit)

$$\text{Max. } f_1 = 0,001205266x_1 + 0,006998945x_2 + 0,066208497x_3 + 0,012336491x_4 \\ + 0,009666473x_5 + 0,011659107x_6$$

The above objective function can also be expressed as a minimization problem as follows:

$$\text{Min. } f_1' = -(0,001205266x_1 + 0,006998945x_2 + 0,066208497x_3 + 0,012336491x_4 \\ + 0,009666473x_5 + 0,011659107x_6)$$

2. The objective function is to minimize risk

$$\text{Min. } f_2 = 0,188001228x_1 + 1,109264912x_2 + 6,95058462x_3 + 2,008097185x_4 \\ + 1,195516024x_5 + 1,838901341x_6$$

In fulfilling the objective function above, there are several obstacles to consider, namely as follows:

- i. The constraint function is the amount of the proportion of funds.

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$$

- ii. The constraint function is the lower and upper limits of the proportion of funds invested for each share:

$$0 \leq x_i \leq 0,8 ; i = 1,2,3,4,5,6.$$

3.3 Solving multi-objective optimization problems on stock portfolios with the Pascoletti-Serafini scalarization modification method

The first step to solve the multi-objective optimization problem with the Pacoletti-Serafini scalarization modification method is to find the ideal point. In other words, trouble $\min_{x \in \mathbb{R}^2} f_i(x)$; $i = 1,2$ completed. Obtained:

$$\begin{aligned} f_1^* &= \min f_1(x) \\ &= \min [-(0,001205266x_1 + 0,006998945x_2 + 0,066208497x_3 + 0,012336491x_4 \\ &\quad + 0,009666473x_5 + 0,011659107x_6)] \\ &= -0.05543410 \end{aligned}$$

where

$$\hat{x}_1 = \arg \min_{x \in X} f_1(x) = (0; 0; 0,8; 0,2; 0; 0).$$

Then,

$$\begin{aligned} f_2^* &= \min f_2(x) \\ &= \min (0,188001228x_1 + 1,109264912x_2 + 6,95058462x_3 + 2,008097185x_4 + \\ &\quad 1,195516024x_5 + 1,838901341x_6) \\ &= 0.3722540 \end{aligned}$$

where

$$\hat{x}_2 = \arg \min_{x \in X} f_2(x) = (0,8; 0,2; 0; 0; 0; 0).$$

Then the ideal point is obtained $f^* = (f_1^*, f_2^*) = (-0.05543410, 0.3722540)$.

Furthermore, after getting the ideal point of the multi objective optimization problem above, the next step is to redefine the objective functions into $f(x) \leftarrow f(x) - f^*$. Obtained:

$$\min f(x) - f^* = \begin{pmatrix} f'_1 - f_1^* \\ f'_2 - f_2^* \end{pmatrix} = \begin{pmatrix} f'_1 + 0.05543410 \\ f'_2 - 0.3722540 \end{pmatrix}$$

with constraints:

(MOP 2)

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 &= 1 \\ 0 \leq x_i &\leq 0,8 ; i = 1,2,3,4,5,6. \end{aligned}$$

Then the set is defined $R = \left\{ \frac{\beta}{\|\beta\|_2} \mid \beta \in \mathbb{R}^p, \sum_{i=1}^p \beta_i = 1, \beta_i \geq 0 \right\}$. To generate vector spread $\beta = (\beta_1, \beta_2)$ selected $\delta = \frac{1}{10}$ so that eleven unit vectors are obtained $\left(\frac{\beta}{\|\beta\|_2} \right)$ who are members of the set R , as shown in Table 5.

Table 5. Vector spread $\beta = (\beta_1, \beta_2)$ for $\delta = \frac{1}{10}$.

$\beta = (\beta_1, \beta_2)^T$	$\ \beta\ _2$	$\left(\frac{\beta}{\ \beta\ _2}\right)$
$(0,1)^T$	1	$r_1 = (0,1)^T$
$(0,1,0,9)^T$	0,906	$r_2 = (0,110,0,994)^T$
$(0,2,0,8)^T$	0,825	$r_3 = (0,242,0,970)^T$
$(0,3,0,7)^T$	0,762	$r_4 = (0,394,0,919)^T$
$(0,4,0,6)^T$	0,721	$r_5 = (0,555,0,832)^T$
$(0,5,0,5)^T$	0,707	$r_6 = (0,707,0,707)^T$
$(0,6,0,4)^T$	0,721	$r_7 = (0,832,0,555)^T$
$(0,7,0,3)^T$	0,762	$r_8 = (0,919,0,394)^T$
$(0,8,0,2)^T$	0,825	$r_9 = (0,970,0,242)^T$
$(0,9,0,1)^T$	0,906	$r_{10} = (0,994,0,110)^T$
$(1,0)^T$	1	$r_{11} = (1,0)^T$

I. Troubleshooting $\overline{SP}(a, r)$ for $r_2 = (0,110,0,994)^T$

By substituting $r = (0,110,0,994)^T$ and $f(x)$ as in MOP (1) problem $\overline{SP}(a, r)$ becomes:

$$\min f(t, x_1, x_2, x_3, x_4, x_5, x_6) = \min t$$

with constraints:

$$\begin{pmatrix} 0.110t \\ 0.994t \end{pmatrix} - \begin{pmatrix} f'_1 + 0.05543410 \\ f_2 - 0.3722540 \end{pmatrix} \geq 0$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$$

$$0 \leq x_i \leq 0,8 ; i = 1,2,3,4,5,6.$$

$$t \in \mathbb{R}$$

where $f'_1 = -(0,001205266x_1 + 0,006998945x_2 + 0,066208497x_3 + 0,012336491x_4 + 0,009666473x_5 + 0,011659107x_6)$

and $f_2 = 0,188001228x_1 + 1,109264912x_2 + 6,95058462x_3 + 2,008097185x_4 + 1,195516024x_5 + 1,838901341x_6.$

By using the LINGO program, optimal values are obtained $(t, x_1, x_2, x_3, x_4, x_5, x_6) = (0.3429, 0, 0, 0.101, 0.8, 0, 0, 099.)$.

II. Problem solving $\overline{SP}(a, r)$ for $r_{11} = (1,0)^T$

By substituting $r = (1,0)^T$ and $f(x)$ as in MOP (1) problem $\overline{SP}(a, r)$ becomes:

$$\min f(t, x_1, x_2, x_3, x_4, x_5, x_6) = \min t$$

with constraints:

$$\begin{pmatrix} t \\ 0 \end{pmatrix} - \begin{pmatrix} f'_1 + 0.05543410 \\ f_2 - 0.3722540 \end{pmatrix} \geq 0$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$$

$$0 \leq x_i \leq 0,8 ; i = 1,2,3,4,5,6.$$

$$t \in \mathbb{R}$$

where $f'_1 = -(0,001205266x_1 + 0,006998945x_2 + 0,066208497x_3 + 0,012336491x_4 + 0,009666473x_5 + 0,011659107x_6)$

and $f_2 = 0,188001228x_1 + 1,109264912x_2 + 6,95058462x_3 + 2,008097185x_4 + 1,195516024x_5 + 1,838901341x_6$.

By using the LINGO program, optimal values are obtained $(t, x_1, x_2, x_3, x_4, x_5, x_6) = (0.0404, 0, 0, 0.0519, 0.8, 0, 0.1481)$.

The same process is continued to obtain eight other solutions for r_3, r_4, \dots, r_{10} so that the results are as shown in the following table 6.

Table 6. Problem solving $\overline{SP}(a, r)$ for r_3, r_4, \dots, r_{10} .

k	r_k	$(t^{(k)}, \bar{x}^{(k)})$ optimal	$f(\bar{x}^{(k)})$
1	$(0,1)^T$	$(0,0,0, 0.8, 0.2, 0,0)$	$(-0.055434096, 5.962087133)$
2	$(0,110,0,994)^T$	$(0.3429,0,0,0.1010, 0.8,0,0.099.)$	$(-0.017710503, 2.490538027)$
3	$(0,242,0,970)^T$	$(0.1618,0,0,0.0745, 0.8,0,0.1255)$	$(-0.016264944, 2.355078420)$
4	$(0,394,0,919)^T$	$(0.1007,0,0,0.0652, 0.8,0,0.1348)$	$(-0.015757634, 2.307539766)$
5	$(0,555,0,832)^T$	$(0.0719,0,0,0.0605, 0.8,0,0.1395)$	$(-0.015501252, 2.283514855)$
6	$(0,707,0,707)^T$	$(0.0567,0,0,0.0577, 0.8,0,0.1423)$	$(-0.015348514, 2.269202141)$
7	$(0,832,0,555)^T$	$(0.0483,0,0,0.0557, 0.8,0,0.1443)$	$(-0.015239415, 2.258978775)$
8	$(0,919,0,394)^T$	$(0.0438,0,0,0.0544, 0.8,0,0.1456)$	$(-0.015168501, 2.252333587)$
9	$(0,970,0,242)^T$	$(0.0416,0,0,0.0533, 0.8,0,0.1467)$	$(-0.015108497, 2.246710735)$
10	$(0,994,0,110)^T$	$(0.0406,0,0,0.0525, 0.8,0,0.1475)$	$(-0.015064857, 2.242621388)$
11	$(1,0)^T$	$(0.0404,0,0,0.0519, 0.8,0,0.1481)$	$(-0.015032128, 2.239554378)$

4. Conclusion

From the optimization results of six Islamic stocks using the Pascoletti-Serafini Scalarization Modification method, several efficient solutions with 11 different r parameters were obtained. The following results were obtained.

- a. The proportion of invested funds is BRPT shares of 0.8 and CTRA shares of 0.2. From the proportion of funds, the expected return value is 0.055434096 and the risk is 5.962087133. This result is obtained in the selection of the parameter $r = (0,1)^T$.
- b. The other 10 efficient solutions have the same proportion value for CTRA shares, which is 0.8. Furthermore, the shares that get the proportion of funds to be invested are BTPR and WIKA with the sum of the proportions of both of them being 0.2

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References

[1] Frank R and Keith B 2012 *Investment Analysis & Portfolio Management* .
 [2] Duan Y C, College B and Hill C 2007 A Multi-Objective Approach to Portfolio Optimization A Multi-objective Approach to Portfolio Optimization *Rose-Hulman Undergrad. Math. J.* **8**, 1.
 [3] Amiri M, Ekhtiari M and Yazdani M 2011 Nadir compromise programming: A model for optimization of multi-objective portfolio problem *Expert Syst. Appl.* **38**, 6 p. 7222–7226.

- [4] Saputro T A and Qudratullah M F 2017 Optimasi Multi Objektif Pada Pemilihan Portofolio Saham Syariah Menggunakan Compromise Programming (CP) dan Nadir Compromise Programming (NCP) *J. Fourier* **6**, 2 p. 91.
- [5] Rezaei, Pouya A, Solimanpur M and Jahangoshai Rezaee M 2016 Solving multi-objective portfolio optimization problem using invasive weed optimization *Swarm Evol. Comput.* **28** p. 42–57.
- [6] Fitria I 2016 Prediksi Harga Saham Investment Management Based on Stock.
- [7] Oh K J, Kim T Y, Min S H and Lee H Y 2006 Portfolio algorithm based on portfolio beta using genetic algorithm *Expert Syst. Appl.* **30**, 3 p. 527–534.
- [8] Skolpadungket P, Dahal K and Harnpornchai N 2007 Portfolio optimization using multi-objective genetic algorithms *2007 IEEE Congr. Evol. Comput. CEC 2007* p. 516–523.
- [9] Pascoletti A and Serafini P 1984 Scalarizing vector optimization problems *J. Optim. Theory Appl.* **42**, 4 p. 499–524.
- [10] Khorram E, Khaledian K and Khaledyan M 2014 A numerical method for constructing the Pareto front of multi-objective optimization problems *J. Comput. Appl. Math.* **261** p. 158–171.