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CERTAIN OPERATION OF GENERALIZED PETERSEN GRAPHS HAVING LOCATING-CHROMATIC NUMBER FIVE

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Abstract

The locating-chromatic number of a graph is combined two graph concept, coloring vertices and partition dimension of a graph. The locating-chromatic number, denoted by $\chi_L(G)$, is the smallest k such that G has a locating k-coloring. In this paper, we discuss the locating-chromatic number for certain operation of generalized Petersen graphs $s\mathrm{P}(n, 1)$.

1. Introduction

In 2002, Chartrand et al. [7] introduced the locating-chromatic number of a graph, with derived two graph concept, coloring vertices and partition dimension of a graph. Let $G = (V, E)$ be a connected graph and c be a proper k-coloring of G with color $1, 2, ..., k$. Let $\prod = \{C_1, C_2, ..., C_k\}$ be a partition of $V(G)$ which is induced by coloring c. The color code $c_{\Pi}(v)$ of v is the ordered k-tuple $(d(v, C_1), d(v, C_2), ..., d(v, C_k))$, where $d(v, C_i) = \min\{d(v, x) | x \in C_i\}$ for any i. If all distinct vertices of G have distinct color codes, then c is called *k-locating coloring* of G. The locating-chromatic number, denoted by $\chi_L(G)$, is the smallest k such that G has a locating k-coloring. Next, Chartrand et al. [6] determined the locating-chromatic number for some graph classes. On P_n it is a path of order $n \geq 3$, and hence $\chi_L(P_n) = 3$; for a cycle C_n if $n \geq 3$ odd, $\chi_L(C_n) = 3$, and if n even, then $\chi_L(C_n) = 4$; for double star graph $(S_{a,b}), 1 \leq a \leq b$ and $b \geq 2$, obtained $\chi_L(S_{a,b}) = b + 1$.

The following definition of a generalized Petersen graph is taken from Watkins [8]. Let $\{u_1, u_2, ..., u_n\}$ be some vertices on the outer cycle and $\{v_1, v_2, ..., v_n\}$ be some vertices on the inner cycle, for $n \geq 3$. The generalized Petersen graph, denoted by $\mathrm{P}(n, k)$, $n \geq 3$, $1 \leq k \leq \left\lfloor \frac{n-1}{2} \right\rfloor$,

$1 \leq i \leq n$ is a graph that has $2n$ vertices $\{u_i\} \cup \{v_i\}$, and edges $\{u_i u_{i+1}\} \cup \{v_i v_{i+k}\} \cup \{u_i v_i\}$.

Now, we define a new kind of generalized Petersen graph called $sP(n, k)$. Suppose there are s generalized Petersen graphs $P(n, k)$. Some vertices on the outer cycle u_i, $i = 1, 2, ..., n$ for the generalized Petersen graph tth, $t = 1, 2, ..., s$, $s \geq 1$ denoted by u_i^t, while some vertices on the inner cycle v_i, $i = 1, 2, ..., n$ for the generalized Petersen graph tth, $t = 1, 2, ..., s$, $s \geq 1$ denoted by v_i^t. Generalized Petersen graph $sP(n, k)$ obtained from $s \geq 1$ is the graph $P(n, k)$, in which each of vertices on the outer cycle u_i^t, $i \in [1, n]$, $t \in [1, s]$ is connected by a path $(u_i^t u_i^{t+1})$, $t = 1, 2, ..., s - 1$, $s \geq 2$.

The locating-chromatic number for corona product is determined by Baskoro and Purwasih [5], and locating-chromatic number for join graphs is determined by Behtoei and Ambarloei [1]. Additionally, Welyyanti et al. [9, 10] discussed locating-chromatic number for graphs with dominant vertices and locating chromatic number for graph with two homogeneous components. Asmiati obtained the locating-chromatic number of non-homogeneous amalgamation of stars [3]. Next, Asmiati et al. [4] determined some generalized Petersen graphs $P(n, 1)$ having locating-chromatic number 4 for odd $n \geq 3$ or 5; for even $n \geq 4$, certain operation of generalized Petersen graphs $sP(4, 2)$ determined by Irawan et al. [2]. Besides that, in this paper, we will discuss the locating-chromatic number of generalized Petersen graphs $sP(n, 1)$.

The following theorems are basics to determine the lower bound of the locating-chromatic of a graph. The set of neighbours of a vertex y in G is denoted by $N(y)$.

Theorem 1.1 [7]. *Let c be a locating coloring in a connected graph G. If x and y are distinct vertices of G such that* $d(x, w) = d(y, w)$ *for all* $w \in V(G) - \{x, y\}$, *then* $c(x) \neq c(y)$. *In particular, if x and y are non-adjacent vertices such that* $N(x) \neq N(y)$, *then* $c(x) \neq c(y)$.

Theorem 1.2 [7]. *The locating-chromatic number of a cycle* C_n *is* 3 *for odd n and* 4 *for otherwise*.

Theorem 1.3 [4]. *The locating-chromatic number for generalized Petersen graphs* $\mathrm{P}(n, 1)$ *is* 4 *for odd* $n \geq 3$ *or* 5 *for even* $n \geq 4$.

2. Main Results

In this section, we will discuss the locating-chromatic number of new kind generalized Petersen graphs $s\mathrm{P}(n, 1)$.

Theorem 2.1. $\chi_L(s\mathrm{P}(3, 1)) = 5$, *for* $s \geq 2$.

Proof. First, we determine the lower bound of $\chi_L(s\mathrm{P}(3, 1))$ for $s \geq 2$. Because a new kind generalized Petersen graph $s\mathrm{P}(3, 1)$, $s \geq 2$ contains some generalized Petersen graph $\mathrm{P}(n, 1)$, then by Theorem 1.3, $\chi_L(s\mathrm{P}(3, 1)) \geq 4$. Suppose that c is a 4-locating coloring on $s\mathrm{P}(3, 1)$. Consider $c(u_i^1) = i$, $i = 1, 2, 3$ and $c(v_j^1) = j$, $j = 1, 2, 3$ such that $c(u_i^1) \neq c(v_j^1)$ for $c(u_i^1)$ adjacent to $c(v_j^1)$. Observe that if we assign color 4 for any vertices in u_i^2 or v_i^2, then we have two vertices whose the same color codes. Therefore, c is not locating 4-coloring on $s\mathrm{P}(3, 1)$. As the result, $\chi_L(s\mathrm{P}(3, 1)) \geq 5$ for $s \geq 2$.

Next, we determine the upper bound of $\chi_L(s\mathrm{P}(3, 1)) \leq 5$ for $s \geq 2$. Assign the 5-coloring *c* on $s\mathrm{P}(3, 1)$ as follows:

- $c(u_i^t) = \begin{cases} 1 & \text{for } i = 1 \text{ and odd } s; \\ 2 & \text{for } i = 2 \text{ and odd } s; \\ 3 & \text{for } i = 3 \text{ and odd } s; \\ 3 & \text{for } i = 1 \text{ and even } s; \\ 1 & \text{for } i = 2 \text{ and even } s; \\ 4 & \text{for } i = 3 \text{ and even } s. \end{cases}$

- $c(v_i^1) = \begin{cases} 2 & \text{for } i = 1; \\ 3 & \text{for } i = 2; \\ 5 & \text{for } i = 3. \end{cases}$

- $c(v_i^t) = \begin{cases} 3 & \text{for } i = 1 \text{ and odd } s \geq 3; \\ 1 & \text{for } i = 2 \text{ and odd } s \geq 3; \\ 2 & \text{for } i = 3 \text{ and odd } s \geq 3; \\ 4 & \text{for } i = 1 \text{ and even } s; \\ 2 & \text{for } i = 2 \text{ and even } s; \\ 3 & \text{for } i = 3 \text{ and even } s. \end{cases}$

The coloring c will create the partition $\prod$ on $V(s\text{P}(3, 1))$. We show that the color codes of all vertices in $s\text{P}(3, 1)$ are different. For $s = 1$, we have $c_{\Pi}(u_1^1) = (0, 1, 1, 2, 2)$; $c_{\Pi}(u_2^1) = (1, 0, 1, 2, 2)$; $c_{\Pi}(u_3^1) = (1, 1, 0, 1, 1)$; $c_{\Pi}(v_1^1) = (1, 0, 1, 3, 1)$; $c_{\Pi}(v_2^1) = (2, 1, 0, 3, 1)$; $c_{\Pi}(v_3^1) = (2, 1, 1, 2, 0)$. For $s \geq 3$ odd, we have $c_{\Pi}(u_1^t) = (0, 1, 1, 2, i + s)$; $c_{\Pi}(u_2^t) = (1, 0, 1, 2, i + s)$; $c_{\Pi}(u_3^t) = (1, 1, 0, 1, s)$; $c_{\Pi}(v_1^t) = (1, 1, 0, 3, s + 2)$; $c_{\Pi}(v_2^t) = (0, 1, 1, 3, i + s)$; $c_{\Pi}(v_3^t) = (1, 0, 1, 2, s + 1)$. For $s \geq 2$ even, we have $c_{\Pi}(u_1^t) = (1, 1, 0, 1, s + 1)$; $c_{\Pi}(u_2^t) = (0, 1, 1, 1, s)$; $c_{\Pi}(u_3^t) = (1, 2, 1, 0, s)$; $c_{\Pi}(v_1^t) = (2, 1, 1, 0, s + 2)$; $c_{\Pi}(v_2^t) = (1, 0, 1, 1, s + 2)$; $c_{\Pi}(v_3^t) = (1, 1, 0, 1, s + 1)$. Since the color codes of all vertices in $s\text{P}(3, 1)$ are different, it follows that $\chi_L(s\text{P}(3, 1)) \leq 5$ for $s \geq 2$.

Theorem 2.2. $\chi_L(s\mathrm{P}(\mathrm{n}, 1)) = 5,$ *for* $s \geq 2$ *and odd* $n \geq 5.$

Proof. The new kind generalized Petersen graphs $s\mathrm{P}(n, 1),$ for $s \geq 2$ and odd $n \geq 5,$ contain some even cycles. Then, by Theorem 1.2, $\chi_L(s\mathrm{P}(\mathrm{n}, 1)) \geq 4.$ Suppose that c is a locating coloring of $s\mathrm{P}(n, 1),$ for $s \geq 2$ and odd $n \geq 5.$ Let $C_1 = \{u_1^t \,|\, \text{for odd } s\} \cup \{u_n^t \,|\, \text{for even } s\} \cup \{v_1^t \,|\, \text{for even } s\} \cup \{v_n^t \,|\, \text{for odd } s, s \geq 3\};$ $C_2 = \{u_{2j}^t \,|\, \text{for odd } i \text{ and odd } s, j > 0\} \cup \{v_{2j-1}^t \,|\, \text{for odd } i \text{ and odd } s, j > 0\} \cup \{u_{2j-1}^t \,|\, \text{for odd } i \text{ and even } s, j > 0\} \cup \{v_{2j}^t \,|\, \text{for odd } i \text{ and even } s, j > 0\};$ $C_3 = \{u_{2j+1}^t \,|\, \text{for odd } i \text{ and odd } s, j > 0\} \cup \{v_{2j}^t \,|\, \text{for odd } i \text{ and odd } s, j > 0\} \cup \{u_{2j}^t \,|\, \text{for odd } i \text{ and even } s, j > 0\} \cup \{v_{2j+1}^t \,|\, \text{for odd } i \text{ and even } s, j > 0\};$ $C_4 = \{v_n^t \,|\, \text{for odd } s\} \cup \{v_1^t \,|\, \text{for even } s\}$ for $\{i > 0; j > 0\}.$ Then there are some vertices with same color codes, $c_\Pi(u_{n-1}^t) = c_\Pi(v_1^t)$ for even s and $c_\Pi(u_2^t) = c_\Pi(v_1^t)$ for odd; $s \geq 2,$ a contradiction. Therefore, $\chi_L(s\mathrm{P}(n, 1)) \geq 5,$ for $s \geq 2$ and odd $n \geq 5.$

We determine the upper bound of $\chi_L(s\mathrm{P}(n, 1)) \leq 5,$ for $n \geq 5$ odd. The coloring c will create the partition Π on $V(s\mathrm{P}(n, 1))$:

$$C_1 = \{u_1^t \,|\, \text{for odd } s\} \cup \{u_n^t \,|\, \text{for even } s\};$$

$$\begin{aligned} C_2 = \{u_{2j}^t \,|\, &\text{for odd } i \text{ and odd } s, j > 0\} \\ &\cup \{v_{2j-1}^t \,|\, \text{for odd } i \text{ and odd } s, j > 0\} \\ &\cup \{u_{2j-1}^t \,|\, \text{for odd } i \text{ and even } s, j > 0\} \\ &\cup \{v_{2j}^t \,|\, \text{for odd } i \text{ and even } s, j > 0\}; \end{aligned}$$

$$C_3 = \{u_{2j+1}^t \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{u_{2j}^t \mid \text{ for odd } i \text{ and even } s,\ j > 0\}$$

$$\cup \{v_{2j+1}^t \mid \text{ for odd } i \text{ and even } s,\ j > 0\};$$

$$C_4 = \{v_n^t \mid \text{ for odd } s\} \cup \{v_1^t \mid \text{ for even } s\};$$

$$C_5 = \{v_n^1\}.$$

Therefore, the color codes of all the vertices of G are:

(a)

$C_1 = \{u_1^t \mid \text{ for odd } s\} \cup \{u_n^t \mid \text{ for even } s\};$

$c_\Pi(u_1^1) = (0, 1, 2, 2, 1);\ c_\Pi(u_n^t) = (0, 1, 1, 2, s-1)$ for even $s \geq 2;$

$c_\Pi(u_1^t) = (0, 1, 2, 2, s)$ for odd $s \geq 3.$

(b)

$$C_2 = \{u_{2j}^t \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{v_{2j-1}^t \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{u_{2j-1}^t \mid \text{ for odd } i \text{ and even } s,\ j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{ for odd } i \text{ and even } s,\ j > 0\}.$$

Let $u_i^t, 1 \leq i \leq n-1; i = 2j; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$ for odd s; $u_i^t, 1 \leq i \leq n-2;$ $i = 2j-1; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$ for even s and $v_i^t, 1 \leq i \leq n-2; i = 2j-1; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$ for odd s; $v_i^t,\ 2 \leq i \leq n-2;\ i = 2j; 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$ for even $s \geq 2$.

For $i < \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_{\Pi}(u_i^t) = (i-1,\ 0,\ 1,\ i+1,\ s+i-1) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = (i,\ 0,\ 1,\ i,\ s+i) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = (i,\ 0,\ 1,\ i,\ s+i-1) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = (i+1,\ 0,\ 1,\ i-1,\ s+i) \text{ for even } s.$$

For $i = \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j+1}^t) = (i-1,\ 0,\ 1,\ i,\ 2j+s-1) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j}^t) = (i,\ 0,\ 1,\ i-1,\ 2j+s+1) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j}^t) = (i-1,\ 0,\ 1,\ i,\ 2j+s-1) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j+1}^t) = (i,\ 0,\ 1,\ i-1,\ 2j+s-1) \text{ for even } s.$$

For $i > \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j+1}^t) = (2j,\ 0,\ 1,\ 2j,\ 2j+s-2) \text{ for odd } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j}^t) = (2j+2,\ 0,\ 1,\ 2j,\ 2j+s) \text{ for odd } s;$$

$$c_{\Pi}(u_i^t) = c_{\Pi}(u_{n-2j}^t) = (2j,\ 0,\ 1,\ 2j+2,\ 2j+s-1) \text{ for even } s;$$

$$c_{\Pi}(v_i^t) = c_{\Pi}(v_{n-2j+1}^t) = (2j,\ 0,\ 1,\ 2j,\ 2j+s-1) \text{ for even } s.$$

(c)

$$C_3 = \{u_{2j+1}^t \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{u^t_{2j} \mid \text{for odd } i \text{ and even } s,\ j > 0\}$$

$$\cup \{v^t_{2j+1} \mid \text{for odd } i \text{ and even } s,\ j > 0\}.$$

Let $u_i^t, 1 \le i \le n-2;\ i = 2j+1;\ \ 1 \le j \le \left\lfloor \frac{n}{2} \right\rfloor - 1$ for $s = 1;\ \ u_i^t, 1 \le i \le n;\ i = 2j+1; 1 \le j \le \left\lfloor \frac{n}{2} \right\rfloor$ for odd $s \ge 3;\ u_i^t, 1 \le i \le n-1;\ i = 2j; 1 \le j \le \left\lceil \frac{n}{2} \right\rceil$ for even s and $v_i^t, 1 \le i \le n-1;\ i = 2j; 1 \le j \le \left\lfloor \frac{n}{2} \right\rfloor$ for odd s; v_i^t, $1 \le i \le n;\ \ i = 2j+1; 1 \le j \le \left\lceil \frac{n}{2} \right\rceil$ for even $s \ge 1$.

For $i < \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_\Pi(u_i^t) = (i-1, 1, 0, i+1, i+s-1) \text{ for odd } s;$$

$$c_\Pi(v_i^t) = (i, 1, 0, i, i+s) \text{ for odd } s;$$

$$c_\Pi(u_i^t) = (i, 1, 0, i, i+s) \text{ for even } s;$$

$$c_\Pi(v_i^t) = (i+1, 1, 0, i-1, i+s) \text{ for even } s.$$

For $i = \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_\Pi(u_i^t) = c_\Pi(u_{n-2j+1}^t) = (i-1, 1, 0, i, 2j+s-1) \text{ for odd } s;$$

$$c_\Pi(v_i^t) = c_\Pi(v_{n-2j}^t) = (i, 1, 0, i-1, 2j+s) \text{ for odd } s;$$

$$c_\Pi(u_i^t) = c_\Pi(u_{n-2j}^t) = (i-1, 1, 0, i, 2j+s-1) \text{ for even } s;$$

$$c_\Pi(v_i^t) = c_\Pi(v_{n-2j+1}^t) = (i, 1, 0, i-1, 2j+s+1) \text{ for even } s.$$

For $i > \left\lceil \frac{n}{2} \right\rceil$, we have:

$c_\Pi(u_i^t) = c_\Pi(u_{n-2j}^t) = (2j+1, 1, 0, 2j, 2j+s-1)$ for odd s;

$c_\Pi(v_i^t) = c_\Pi(v_{n-2j+1}^t) = (2j+1, 1, 0, 2j-1, 2j+s-1)$ for odd s;

$c_\Pi(u_i^t) = c_\Pi(u_{n-2j+1}^t) = (2j-1, 1, 0, 2j+1, 2j+s-2)$ for even s;

$c_\Pi(v_i^t) = c_\Pi(v_{n-2j+2}^t) = (2j-1, 1, 0, 2j-1, 2j+s-2)$ for even s.

(d)

$$C_4 = \{v_n^t \mid \text{for odd } s\} \cup \{v_1^t \mid \text{for even } s\};$$

$$c_\Pi(v_n^t) = (2, 1, 1, 0, s) \text{ for odd } s;$$

$$c_\Pi(v_1^t) = (1, 2, 1, 0, s+1) \text{ for even } s.$$

(e)

$$C_5 = \{v_n^1\},$$

$$c_\Pi(v_n^1) = (1, 1, 2, 1, 0).$$

Since all the vertices have different color codes, c is a locating coloring of new kind generalized Petersen graphs $s\mathrm{P}(n, 1)$, so $\chi_L(s\mathrm{P}(n, 1)) \leq 5$, for odd $n \geq 5$.

Theorem 2.3. $\chi_L(s\mathrm{P}(n, 1)) = 5$ *for* $s \geq 2$ *and even* $n \geq 4$.

Proof. First, we determine the lower bound of $\chi_L(s\mathrm{P}(n, 1))$ for $s \geq 2$ and even $n \geq 4$. The new kind generalized Petersen graph $s\mathrm{P}(n, 1)$, for $s \geq 2$ and even $n \geq 4$, contains some generalized Petersen graph $\mathrm{P}(n, 1)$, then by Theorem 1.3, $\chi_L(s\mathrm{P}(n, 1)) \geq 5$.

Next, we determine the upper bound of $\chi_L(s\mathrm{P}(n, 1)) \leq 5$ for $s \geq 2$ and $n \geq 4$ even. The coloring c will create the partition Π on $V(s\mathrm{P}(n, 1))$:

$$C_1 = \{u_1^t \mid \text{for odd } s\} \cup \{u_n^t \mid \text{for even } s\};$$

$$C_2 = \{u_{2j}^t \mid \text{for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{v_{2j-1}^t \mid \text{for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{u_{2j-1}^t \mid \text{for odd } i \text{ and even } s,\ j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{for odd } i \text{ and even } s,\ j > 0\};$$

$$C_3 = \{u_{2j+1}^t \mid \text{for odd } i \text{ odd } s,\ j > 0\}$$

$$\cup \{v_{2j}^t \mid \text{for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{u_{2j}^t \mid \text{for odd } i \text{ and even } s,\ j > 0\}$$

$$\cup \{v_{2j+1}^t \mid \text{for odd } i \text{ and even } s,\ j > 0\};$$

$$C_4 = \{u_n^t \mid \text{for odd } s\} \cup \{u_{n-1}^t \mid \text{for even } s\};$$

$$C_5 = \{v_n^1\}.$$

Therefore, the color codes of all the vertices of G are:

(a)

$$C_1 = \{u_1^t \mid \text{for odd } s\} \cup \{u_n^t \mid \text{for even } s\};$$

$$c_\Pi(u_1^1) = (0, 1, 2, 1, 2);\ u_n^t = (0, 1, 2, 1, s) \text{ for even } s \geq 2;$$

$$c_\Pi(u_1^t) = (0, 1, 2, 1, s+1) \text{ for odd } s \geq 3.$$

(b)

$$C_2 = \{u^t_{2j} \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\bigcup \{v^t_{2j-1} \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\bigcup \{u^t_{2j-1} \mid \text{ for odd } i \text{ and even } s,\ j > 0\}$$

$$\bigcup \{v^t_{2j} \mid \text{ for odd } i \text{ and even } s,\ j > 0\}.$$

Let u^t_i, $1 \le i \le n-2$; $i = 2j$; $1 \le j \le \frac{n}{2} - 2$ for odd s; u^t_i, $1 \le i \le n-3$; $i = 2j-1$; $1 \le j \le \frac{n}{2}$ for even s and v^t_i, $1 \le i \le n-1$; $i = 2j-1$; $1 \le j \le \frac{n}{2}$ for odd s; v^t_i, $1 \le i \le n-1$; $i = 2j$; $1 \le j \le \frac{n}{2}$ for even $s \ge 2$.

For $i \le \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_\Pi(u^t_i) = (i-1, 0, 1, i, i+s) \text{ for odd } s;$$

$$c_\Pi(v^t_i) = (i, 0, 1, i, i+s+1) \text{ for odd } s;$$

$$c_\Pi(u^t_i) = (i, 0, 1, i+1, i+s) \text{ for even } s;$$

$$c_\Pi(v^t_i) = (i+1, 0, 1, i+2, i+s+1) \text{ for even } s.$$

For $i > \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_\Pi(u^t_i) = c_\Pi(u^t_{n-2j}) = (2j+1, 0, 1, 2j, 2j+s) \text{ for odd } s;$$

$$c_\Pi(v^t_i) = c_\Pi(v^t_{n-2j-1}) = (2j+1, 0, 1, 2j, 2j+s) \text{ for odd } s;$$

$$c_\Pi(u^t_i) = c_\Pi(u^t_{n-2j-1}) = (2j+1, 0, 1, 2j, 2j+s+1) \text{ for even } s;$$

$$c_\Pi(v^t_i) = c_\Pi(v^t_{n-2j}) = (2j-1, 0, 1, 2j, 2j+s-1) \text{ for even } s.$$

(c)

$$C_3 = \{u^t_{2j+1} \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{v^t_{2j} \mid \text{ for odd } i \text{ and odd } s,\ j > 0\}$$

$$\cup \{u^t_{2j} \mid \text{ for odd } i \text{ and even } s,\ j > 0\}$$

$$\cup \{v^t_{2j+1} \mid \text{ for odd } i \text{ and even } s,\ j > 0\}.$$

Let $u^t_i,\ 1 \le i \le n-1;\ i = 2j+1;\ 1 \le j \le \frac{n}{2} - 1$ for odd s; $u^t_i,\ 1 \le i \le n-2;\ i = 2j;\ 1 \le j \le \frac{n}{2} - 1$ for even s and $v^t_i,\ 1 \le i \le n-2;\ i = 2j;\ 1 \le j \le \frac{n}{2} - 1$ for odd s; $v^t_i,\ 1 \le i \le n-1;\ i = 2j-1;\ 1 \le j \le \frac{n}{2}$ for even $s \ge 2$.

For $i \le \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_\Pi(u^t_i) = (i-1, 1, 0, i, i+s) \text{ for odd } s;$$

$$c_\Pi(v^1_i) = (i, 1, 0, i+1, i);$$

$$c_\Pi(v^t_i) = (i, 1, 0, i+1, i+2s-2) \text{ for odd } s \ge 3;$$

$$c_\Pi(u^t_i) = (i, 1, 0, i+1, i+s) \text{ for even } s;$$

$$c_\Pi(v^t_i) = (i+1, 1, 0, i+1, i+s) \text{ for even } s.$$

For $i > \left\lceil \frac{n}{2} \right\rceil$, we have:

$$c_\Pi(u^t_i) = c_\Pi(u^t_{n-2j+1}) = (2j+1, 1, 0, 2j-1, 2j+s-1) \text{ for odd } s;$$

$$c_\Pi(v^1_i) = c_\Pi(v^t_{n-2j}) = (2j+2, 1, 0, 2j+1, 2j);$$

$c_\Pi(v_i^t) = c_\Pi(v_{n-2j}^t) = (2j+2, 1, 0, 2j+1, 2j+s+1)$ for odd $s \geq 3$;

$c_\Pi(u_i^t) = c_\Pi(u_{n-2j+1}^t) = (2j, 1, 0, 2j-1, 2j+s)$ for even s;

$c_\Pi(v_i^t) = c_\Pi(v_{n-2j}^t) = (2j, 1, 0, 2j-1, 2j+s)$ for even s.

(d)

$$C_4 = \{u_n^t \mid \text{ for odd } s\} \cup \{u_{n-1}^t \mid \text{ for even } s\};$$

$$c_\Pi(u_n^t) = (1, 2, 1, 0, s) \text{ for odd } s;$$

$$c_\Pi(u_{n-1}^t) = (1, 2, 1, 0, s+1) \text{ for even } s.$$

(e)

$$C_5 = \{v_n^1\},$$

$$c_\Pi(v_n^1) = (2, 1, 2, 1, 0).$$

Since all the vertices have different color codes, c is a locating coloring of new kind generalized Petersen graphs $(s\mathrm{P}(n, 1))$, so $\chi_L(s\mathrm{P}(n, 1)) \leq 5$, for even $n \geq 4$.

3. Conclusion

Based on the results, locating-chromatic number of new kind generalized Petersen graphs $s\mathrm{P}(n, 1)$ is 5 for $s \geq 2$ and $n \geq 3$.

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