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Mathematical Modeling of Heat Transper in Agricultural Drying Machine Room (Box Dryer)

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Abstract. Differential equations arise in many fields of science and technology, when the deterministic relationships involving a continuously changing quantity (modeled by a mathematical function) and its rate of change (expressed as a derivative) are known or postulated. This can be seen for example in the problem of heat transfer. The problems raised in this study are how to model the heat equation and how to solve the heat equation model. The research was conducted to find a model of the heating equation in a rice dryer and solve the heat equation using the variable separation method. This discussion is carried out in two conditions, namely the steady state (constant time) and unsteady (changing time).

Keyword: specific terms, used in, this paper, no more, than 6 words.

1. Introduction

Heat transfer is an energy transfer process that occurs because of the difference in temperature between objects or materials. Heat transfer occurs from an object with a high temperature to an object with a low temperature and will stop when the two objects have the same temperature. Heat transfer is widely applied in daily life and in the industrial sector, for example boiling water, radiation processing, manufacturing of sharp objects, drying rice, processing and increasing the selling price of rice.

The problem of changing temperature with time to determine the length of time it takes to reach the desired temperature can be expressed in a mathematical model as a partial differential equation. Partial differential equations are encountered in connection with various physical and geometric problems when the functions involved depend on two or more independent variables. It is no exaggeration to say that only the simplest physical systems can be modeled by the ordinary differential equations of fluid mechanics and solid mechanics, heat transfer, electromagnetic theory and many other fields of physics are full of problems which must be modeled by partial differential equations. In fact, the range of applications for partial differential equations is very large, compared to the range of applications for ordinary differential equations. Independent variables can be time and one or more coordinates in space.

When obtaining a mathematical model in the form of a differential equation, the next step is to solve the differential equation by determining the solution. The solution to the differential equation is a function that satisfies the differential equation. That is, if the function and its derivatives are substituted into the differential equation, a true statement is obtained. The problem of changing temperature with time to determine the length of time needed to reach the desired temperature must be expressed in mathematical modeling. After obtaining the mathematical modeling, then the model solution can be searched. The model solution is then developed and interpreted to obtain a solution to the problem.

In this paper the author limits the discussion of the problem to only heat transfer in a block-shaped agricultural dryer where the machine is in a steady state (constant temperature with time) and unsteady (temperature varies with time). The concept of heat transfer in this agricultural drying system is carried out in a block-filled room as a drying room. This phenomenon can be modeled as a heat equation in space or 3 dimensions.

2. Research Objectives

This research aims to

1. Obtain a model of the time it takes to dry rice until the moisture content in the rice is reduced to the maximum.
2. Know the model solution to the problem of the time it takes to dry rice until the moisture content in the rice is maximally reduced if the machine is in a steady state (constant temperature with time) and unsteady (temperature varies with time).

3. Heat Equation Modeling

The heat equation for an element uses Cartesian coordinates (because it is a block), so the modeling is formed into a heat equation using Cartesian coordinates (x, y, z). Modeling the transfer of heat in objects with 3 dimensions must pay attention to the heat that is transferred into and out of the object in the three coordinate directions.

Given a beam section like Figure 1 below:

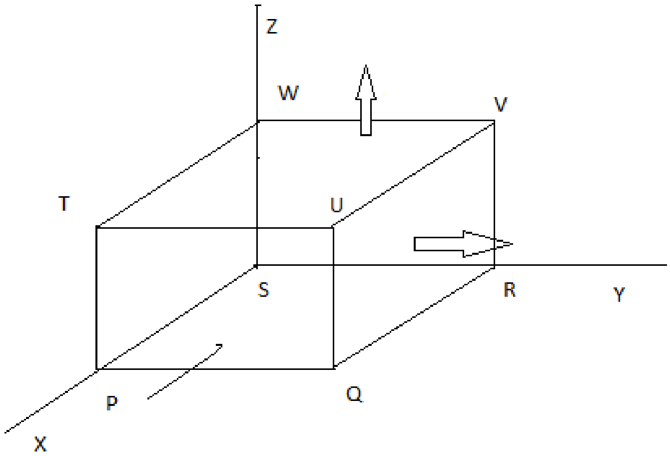

Figure 1. Heat conduction flow in 3-dimensional beam

In fact, heat conduction is found in rod-shaped objects, so that the heat conduction in bars can be viewed as heat conduction in a space bounded by the x, y and z axes. Furthermore, the conduction of heat in bars is known as three-dimensional heat conduction, which is the expression of one- and two-

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dimensional heat conduction. Based on the first and two dimensional heat conduction equations, three dimensional heat conduction equations and their solutions can be determined which meet the initial conditions and certain boundary conditions.

Conduction is defined as the process of flowing heat from an area with a higher temperature to an area with a lower temperature in one medium (solid, it is found, is generally the conduction of heat in a bar or solid block-shaped object. Bars are three-dimensional rigid elements, so as to Determining the heat conduction in bars can be considered as heat conduction in space, which is the area bounded by the x, y, z axes.

4. Results and Discussion

The heat conduction equation is taken as a function T (x, y, z, t) which is the temperature at time t at each point in the coordinates (x, y, z) of an object element PQRSTUVW. The heat needed to raise the temperature of the object by mass m is ΔT with mass m and specific heat c.

$$Q = mc\Delta T$$

The amount of heat entering the element through the SRVW plane is -k ├ ∂T / ∂x┤ |_x and the SRVW field area is ΔyΔz, then the amount of heat entering the element through the plane at padat is

$$Q_1 = -k\frac{\partial T}{\partial x}\bigg|_x \Delta y\Delta z\Delta t$$

and the amount of heat coming out of the SRVW field element is

$$Q_2 = -k\frac{\partial T}{\partial x}\bigg|_{x+\Delta x} \Delta y\Delta z\Delta t$$

The amount of heat in the x-axis direction element

$$Q(x) = \left\{k\frac{\partial T}{\partial x}\bigg|_{x+\Delta x} - k\frac{\partial T}{\partial x}\bigg|_x\right\}\Delta y\Delta z\Delta t \quad (1)$$

Analog of the amount of heat on the elements in the directions y and z is

$$Q(y) = \left\{k\frac{\partial T}{\partial y}\bigg|_{y+\Delta y} - k\frac{\partial T}{\partial y}\bigg|_y\right\}\Delta x\Delta z\Delta t \quad (2)$$

and

$$Q(z) = \left\{k\frac{\partial T}{\partial z}\bigg|_{z+\Delta z} - k\frac{\partial T}{\partial z}\bigg|_z\right\}\Delta x\Delta y\Delta t \quad (3)$$

The total heat obtained by the element is the sum of equations (1), (2) and (3).
This total heat is used to raise the temperature by ΔT. The heat needed to raise the temperature by ΔT with mass m and specific heat is c.

$$Q = mc\Delta T \quad (4)$$

For example, the density of the rigid element is ρ, then the element mass m = ρΔxΔyΔz.
So that equation (4) becomes

$$Q = c\rho\Delta x\Delta y\Delta z\Delta T \quad (5)$$

The total heat (1), (2) and (3) is equal to equation (4).

$$Q(x) + Q(y) + Q(z) = c\rho\Delta x\Delta y\Delta z\Delta T \quad (6)$$

Dividing equation (6) by ΔxΔyΔz and Δt is obtained

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$$\left\{\frac{k\frac{\partial T}{\partial x}\Big|_{x+\Delta x}-k\frac{\partial T}{\partial x}\Big|_{x}}{\Delta x\Delta y\Delta z}\right\}+\left\{\frac{k\frac{\partial T}{\partial y}\Big|_{y+\Delta y}-k\frac{\partial T}{\partial y}\Big|_{y}}{\Delta x\Delta y\Delta z}\right\}+\left\{\frac{k\frac{\partial T}{\partial z}\Big|_{z+\Delta z}-k\frac{\partial T}{\partial z}\Big|_{z}}{\Delta x\Delta y\Delta z}\right\}=c\rho\frac{\Delta T}{\Delta t}$$

With the limit ΔxΔyΔz and Δt going to zero, we get

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)=c\rho\frac{\partial T}{\partial t}$$

or

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=c\rho\frac{\partial T}{\partial t}$$

or

$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\frac{\partial T}{\partial t} \quad (7)$$

With $\alpha=\frac{k}{c\rho}$ heat diffusivity. The equation (ads) is a 3-dimensional heat equation.

If the heat conduction in the element is steady, then the temperature T is independent of time, so we get

$$\frac{\partial T}{\partial t}=0$$

then equation (7)

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0 \quad (8)$$

PQRSTUVW object is bounded by 6 planes which are the sides of the element. The temperature in the x-direction PQRS.TUVW plane bounded 0 <x <a is maintained equal to zero. So that the element of the direction x obtained the boundary condition

$$T(0,y,z)=T(a,y,z)=0$$
$$0<x<a.$$

In the same way, the boundary conditions for the other axis directions are obtained as follows:

$$T(x,0,z)=T(x,b,z)=0$$
$$0<y<b.$$

and

$$T(x,y,0)=T(x,y,c)=0$$
$$0<z<c.$$

Using variable separation by taking T = X (x) Y (y) Z (z), where X is a function of x, Y is a function of y and Z is a function of Z, then equation (8) becomes

$$X''\,YZ+XY''\,Z+XYZ''=0$$

or

$$\frac{X''}{X}=-\frac{Y''}{Y}-\frac{Z''}{Z} \quad (9)$$

Since the right side of equation (9) is a function with the independent variables y and z, while the left side is a function with the independent variable x, then each side must be constant, for example $-\gamma^2$, then

$$\frac{X''}{X}=-\gamma^2$$
$$X''-\gamma^2 X=0$$

and

$$-\frac{Y''}{Y}-\frac{Z''}{Z}=-\gamma^2$$

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$$\frac{Y"}{Y} = -\frac{Z"}{Z} + \gamma^2 \qquad (10)$$

Since the left and right sides of equation (10) are functions of the independent variables y and z, they must be constant, for example $-\tau^2$. So that it is obtained

$\frac{Y"}{Y} = -\tau^2$ or $Y" + \tau^2 Y = 0$

and $\frac{Z"}{Z} = \gamma^2 + \tau^2$ or $Z" - Z(\gamma^2 + \tau^2) = 0$

obtained the problem of order 2 ODE boundary conditions

$$X" - \gamma^2 X = 0, \qquad 0 < x < a \qquad (11)$$

$$Y" + \tau^2 Y = 0, \qquad 0 < y < b \qquad (12)$$

$$Z" - Z(\gamma^2 + \tau^2) = 0, \; 0 < z < c \qquad (13)$$

by provided the limit

$X(0) = X(a) = 0, Y(0) = Y(b) = 0, Z(0) = Z(a) = 0, \gamma^2$ and τ^2 contant. (14)

For equation (11) the solution

$$X(x) = Acos\,\gamma x + Bsin\,\gamma x$$

by provided a limit is obtained

$X(a) = Bsin\,\gamma a = 0$, then $sin\,\gamma a = 0$

$$\gamma a = n\pi, \quad n = 1, 2, 3, \ldots$$

$$\gamma = \frac{n\pi}{a}, \quad n = 1, 2, 3, \ldots$$

$$\gamma_m = \frac{m\pi}{a}, \quad m = 1, 2, 3, \ldots$$

obtained

$$X(x) = Bsin\,\frac{m\pi}{a}x, \quad m = 1, 2, 3, \ldots$$

For equation (12) the solution

$$Y(y) = Acos\,\tau y + Bsin\,\tau y$$

by provided a limit is obtained

$Y(b) = Bsin\,\tau b = 0$, then $sin\,\tau b = 0$

$$\tau b = n\pi, \quad n = 1, 2, 3, \ldots$$

$$\tau = \frac{n\pi}{b}, \quad n = 1, 2, 3, \ldots$$

$$\tau_n = \frac{n\pi}{b}, \quad n = 1, 2, 3, \ldots$$

obstained

$$Y(y) = sin\,\frac{n\pi}{b}y, \quad n = 1, 2, 3, \ldots$$

For equation (13) the solution

Example $\gamma^2 + \tau^2 = \sigma^2$

$$\sigma = \pm\sqrt{\gamma^2 + \tau^2}$$

$$\sigma = \pm\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\sigma = \pm\pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{m}{b}\right)^2}$$

Therefor obstained

$$Z(z) = Ae^{\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{m}{b}\right)^2}} + Be^{-\pi\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{m}{b}\right)^2}}$$

Based on the boundary conditions obtained A = -B

$$Z(z) = Ae^{\sigma_{mn}z} + Be^{-\sigma_{mn}z}$$

$$Z(z) = Ae^{\sigma_{mn}z} - Ae^{-\sigma_{mn}z}$$

$$Z(z) = A(e^{\sigma_{mn}z} - e^{-\sigma_{mn}z})$$

By up to A=1

$$Z(z) = sinh\, \sigma_{mn}z, \quad m, n = 1, 2, 3, \ldots$$

Solutions in general

$$T_{mn}(x, y, z) = B_{mn} sin\, \frac{m\pi}{a} x. sin\, \frac{n\pi}{b} y. sinh\, \sigma_{mn}z$$

$$T(x, y, z) = \sum_{m}^{\infty}\sum_{n}^{\infty} B_{mn}\, sin\, \frac{m\pi}{a} x. sin\, \frac{n\pi}{b} y. sinh\, \sigma_{mn}z$$

This equation is a general solution to the 3 dimensional thermal equation.

5. Conclusion

1. Mathematical model of agricultural dryer (box Dryer) with block form $\alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \frac{\partial T}{\partial t}$, with $\alpha = \frac{k}{c\rho}$ heat diffusivity.
2. General solution of dryer model $T(x, y, z) = \sum_{m}^{\infty}\sum_{n}^{\infty} B_{mn}\, sin\, \frac{m\pi}{a} x. sin\, \frac{n\pi}{b} y. sinh\, \sigma_{mn}z$

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