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# The relativistic mass accretion rate for accretion disk in the equatorial of rapidly rotating neutron stars

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**Abstract.** The accretion process occurs when neutron stars are in a binary system. The hypothesis says that accretion reduces the magnetic field in a neutron star. The correlation between accretion and decrease in the magnetic field is indicated by the magnitude of the magnetic field dynamics and the mass accretion rate. The purpose of this research is to formulate the mass rate of accretion in rapidly rotating neutron stars. The accretion disc is assumed to be at the equator. The method used is theoretical-mathematical analysis. The quantities to be summarized are the Killing vector, 4-vector velocity, the angular velocity, and the Lorentz gamma factor derived from the metric. The metric used is a rapidly rotating neutron star. The rate of mass accretion is calculated from these quantities and the equation for mass sustainability. The equation of mass accretion rate is derived from these variables and the mass conservation equation. The equation of mass accretion rate is affected by the radial velocity of accretion flows and distance between matter and center of a neutron star. The result obtained is covariant with the equation of mass accretion rate for Kerr black holes. The results obtained can provide a dynamic equation for the magnetic field to produce an equation for the decrease in the magnetic field in a rapidly rotating neutron star.

## 1. Introduction

Neutron star is one of massive objects other than white dwarf and black hole. This massive object causes curvature of spacetime so that the surrounding objects will be attracted, surrounding, and possibly enter the massive object. The process is called accretion. If the accretion material emits the thermal energy and increases the entropy, then it is called as an advection-dominated accretion [1,2]. Neutron stars also carry out an accretion process in the binary system.

The result of the accretion process is a decrease in the amount of magnetic field in the neutron stars. Neutron stars have magnetic fields that can reach  $\sim 10^{18}$  G which is the strongest in this universe [3,4]. The magnitude of magnetic field can be reduced, as an example from  $\sim 10^{12}$  G to  $\sim 10^8$  G in  $5 \times 10^6$  years [5]. In general, the accreting neutron stars can be detected from two sources, HMXBs (*High Mass X-ray Binaries*) with  $B \sim 10^{12}$  G and LMXBs (*Low Mass X-ray Binaries*) with  $B \lesssim 10^{10}$  G. Many neutron



stars are detected in LMXBs [6]. Some studies have shown that the decrease of magnetic field is found in the binary system or in neutron stars in accreting process of other objects [7]. Therefore, the decrease of magnetic field in the neutron star is hypothesized due to neutron star accreting the surrounding materials [5,8–13].

The equation of non-relativistic magnetic field is due to the accretion has been generated which is to link the equation of magnetic field dynamics with the accretion equation [5]. Relativistic studies have been conducted by formulating the equation of magnetic field dynamics of neutron stars which rotate both slowly or rapidly [14–18]. Due to one of the neutron stars studies that many astrophysicists do both observational and theoretical realms is a decrease of magnetic field [19]. Meanwhile, the studies of advection-dominated accretion have been reviewed but still in the Kerr black hole [1,2] and extreme Kerr black holes [20]. Therefore, the accretion flows equation is required for a slowly and rapidly rotating neutron stars which will be connected to the equation of magnetic field dynamics that have been generated in the previous studies. The purpose of this research is to formulate the mass rate of accretion in rapidly rotating neutron stars. The accretion disc is assumed to be at the equator.

## 2. Research methods

The research that will be conducted is theoretical-mathematical analysis analytically. This research will go through in three stages, calculating the 4-velocity vector of matter in the accretion disk ( $u^i$ ), calculating the radial velocity of accretion flow ( $V$ ), elaborating the mass conservation and energy in the accretion disk. The most basic thing in formulating the accretion equation is metric. Metrics are used to define the *killing vectors* dan calculate the LNRF basis vector (*Local Non-Rotating Frame*). *Killing vectors* are used to define the angular velocity of the *dragging of inertial frame*, gravitational potential, gyration radius, and radial velocity of accretion flow. Whereas the LNRF basis vector is needed to calculate the 4-velocity of matter in the accretion disk.

Before calculating the radial velocity of accretion flow, the angular velocity is calculated first. The angular velocity in relation to the stationary observer and the angular velocity of the *dragging of inertial frame* is used to calculate the angular velocity in relation to ZAMO. Elaborating the Lorentz gamma factor using the gyration radius and the components of 4-velocity of matter in the accretion disk is needed to obtain the radial velocity of accretion flow. The final stage is elaborating the mass conservation in the accretion disk. The accretion flow is distributed throughout the space of the accretion disk so that the mass conservation equation must be integrated with the volume. The research method is shown in figure 1.

## 3. Results and discussion

### 3.1. The rapidly rotating neutron star metrics in the equatorial plane

The rapidly rotating neutron star metrics in a spherical coordinate system is

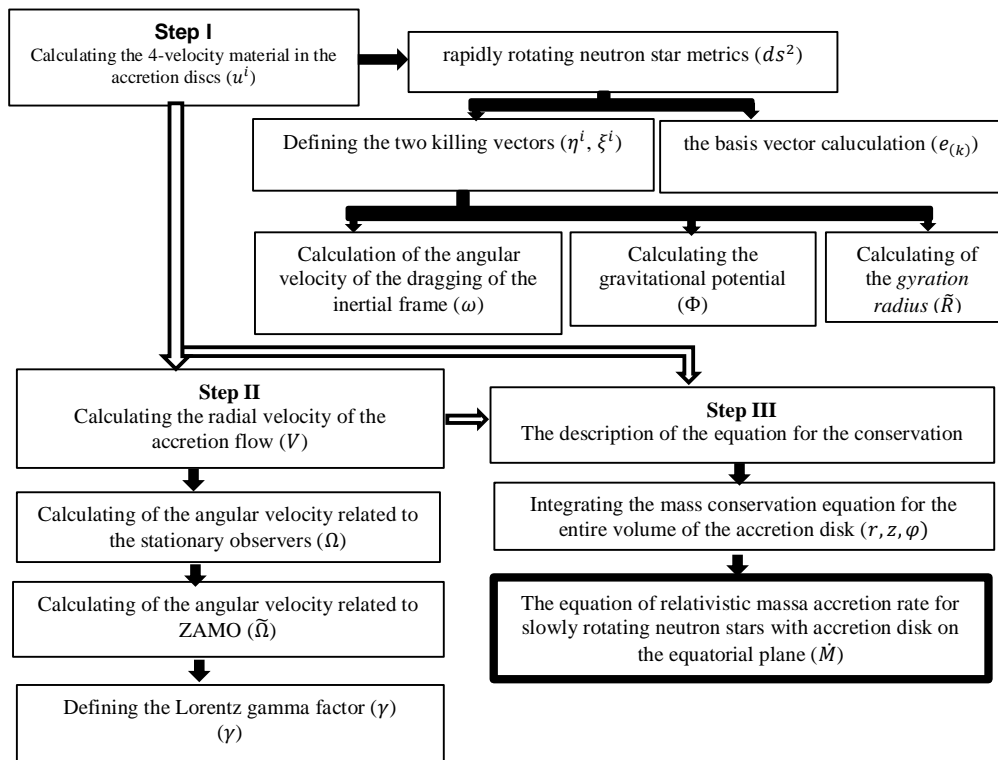
$$ds^2 = -e^{2\Phi} dt^2 + e^{2\lambda} r^2 \sin^2 \theta (d\varphi - \omega dt)^2 + e^{2\alpha} (dr^2 + r^2 d\theta^2), \quad (1)$$

where  $\Phi, \lambda, \omega, \alpha$  are the functions in the parameter of  $r$  and  $\theta$  [21]. The function of  $\omega(r)$  is the angular velocity of the inertial reference frame [21]. If the accretion disk is assumed in the equatorial plane ( $z = 0; \theta = \pi/2$ ) [20], then the metric equation (1) becomes

$$ds^2 = -e^{2\Phi} dt^2 + e^{2\lambda} r^2 (d\varphi - \omega dt)^2 + e^{2\alpha} (dr^2 + dz^2), \quad (2)$$

Equation (2) is the metric equation of a rapidly rotating neutron star in the equatorial plane. The components of the covariance metric are

$$g_{\mu\nu} = \begin{bmatrix} -(e^{2\Phi} - e^{2\lambda} r^2 \omega^2) & 0 & 0 & -e^{2\lambda} r^2 \omega \\ 0 & e^{2\alpha} & 0 & 0 \\ 0 & 0 & e^{2\alpha} & 0 \\ -e^{2\lambda} r^2 \omega & 0 & 0 & e^{2\lambda} r^2 \end{bmatrix}. \quad (3)$$



**Figure 1.** The method used is to formulate equations of mass accretion rate on rapidly rotating neutron stars.

### 3.2. Killing vector

Equation (2) contains two Killing vectors. They are

$$\eta^i = \frac{\partial}{\partial t} = \delta^i_{(t)}, \quad (4)$$

and

$$\xi^i = \frac{\partial}{\partial \varphi} = \delta^i_{(\varphi)}. \quad (5)$$

with  $\delta^i_{(k)}$  is the Kronecker delta [2]. Therefore, the killing vector of a rapidly rotating neutron star metric is

$$\eta\eta = g_{tt} = -(e^{2\Phi} - e^{2\lambda}r^2\omega^2), \quad (6)$$

$$\eta\xi = g_{t\varphi} = -e^{2\lambda}r^2\omega, \quad (7)$$

$$\xi\xi = g_{\varphi\varphi} = e^{2\lambda}r^2. \quad (8)$$

Killing vector is used to define three scalar functions which are

- The angular velocity of dragging of inertial frame ( $\omega$ ) is defined as [2]

$$\omega = -\frac{(\eta\xi)}{(\xi\xi)} = -\frac{g_{t\varphi}}{g_{\varphi\varphi}} = -\frac{(-e^{2\lambda}r^2\omega)}{e^{2\lambda}r^2} = \omega(r). \quad (9)$$

Equation (9) shows the definition of  $\omega = -\frac{(\eta\xi)}{(\xi\xi)}$  corresponding to the rapidly rotating neutron star.

Gravitational potential  $\phi$  is defined as [22]

$$\phi = -\frac{1}{2} \ln[-(\eta\eta) - 2\omega(\eta\xi) - \omega^2(\xi\xi)],$$

$$e^{2\phi} = e^{-2\Phi(r)}. \quad (10)$$

- Gyration radius ( $\tilde{R}$ ) is defined as [2]

$$\tilde{R}^2 = (\xi\xi)e^{2\phi} = e^{2\lambda r^2} e^{-2\Phi(r)}. \quad (11)$$

### 3.3. 4-Velocity vector

The 4-velocity vector of matter of rapidly rotating neutron star accretion disk is

$$u^i = \gamma(e_{(t)}^i + v^{(r)}e_{(r)}^i + v^{(z)}e_{(z)}^i + v^{(\varphi)}e_{(\varphi)}^i), \quad (12)$$

where  $\gamma$  is the Lorentz gamma factor,  $v^{(k)}$  is the velocity measured in LNRF (*Local Non-Rotating Frame*) ( $v^{(k)} = dx^{(k)}/dt$ ), and  $e_{(k)}^i$  is the contravariance component for the rapidly rotating neutron star of the basis vector LNRF  $e_{(k)}$  which defined as [2]

$$e_{(k)} = e_{(k)}^i \frac{\partial}{\partial x^i}. \quad (13)$$

Based on the previous assumption that the accretion disk is in the equatorial ( $z = 0$ ), so the accretion disk is in hydrostatic equilibrium so that the component  $v^{(k)} = 0$  [22]. Equation (12) becomes

$$u^i = \gamma(e_{(t)}^i + v^{(r)}e_{(r)}^i + v^{(\varphi)}e_{(\varphi)}^i). \quad (14)$$

In equation (14) requires a basis vector. The Othonormal basis for a rapidly rotating neutron star in the ZAMO framework is [18]

$$e_{\hat{0}}^v = e^{-\Phi}(1,0,0,\omega), \quad (15)$$

$$e_{\hat{r}}^v = e^{-\alpha}(0,1,0,0), \quad (16)$$

$$e_{\hat{\theta}}^v = e^{-\alpha}r^{-1}(0,0,1,0), \quad (17)$$

$$e_{\hat{\varphi}}^v = \frac{e^{2\Phi} - e^{2\lambda r^2} \sin^2 \theta \omega^2}{e^{\Phi} e^{\lambda r} \sin \theta} e^{-\Phi}(0,0,0,1) \quad (18)$$

For the accretion disk in the equatorial plane, the equation (18) becomes

$$e_{\hat{\varphi}}^v = \frac{e^{2\Phi} - e^{2\lambda r^2} \omega^2}{e^{2\Phi + \lambda r}}(0,0,0,1). \quad (19)$$

The basis vector for the rapidly rotating neutron stars is

$$e_{(t)} = e^{-\Phi} \left( \frac{\partial}{\partial x^0} + \omega \frac{\partial}{\partial x^3} \right), \quad (20)$$

$$e_{(r)} = e^{-\alpha} \frac{\partial}{\partial x^1}, \quad (21)$$

$$e_{(\varphi)} = \frac{e^{2\Phi} - e^{2\lambda r^2} \omega^2}{e^{2\Phi + \lambda r}} \frac{\partial}{\partial x^3}. \quad (22)$$

The 4-velocity can also be written as [2]

$$u^{i\delta} = u^t \partial/\partial x^t + u^r \partial/\partial x^r + u^\varphi \partial/\partial x^\varphi, \quad (23)$$

If it is written in the LNRF basis vector, the equation (23) becomes

$$u^{i\delta} = \gamma(e_{(t)} + v^{(r)}e_{(r)} + v^{(\varphi)}e_{(\varphi)}) \quad (24)$$

Equation (20-22) are substituted into the equation (24) which obtained as

$$u^{i\delta} = \gamma \left( e^{-\Phi} \left( \frac{\partial}{\partial x^0} + \omega \frac{\partial}{\partial x^3} \right) + v^{(r)} e^{-\alpha} \frac{\partial}{\partial x^1} + v^{(\varphi)} \left( \frac{e^{2\Phi} - e^{2\lambda r^2} \omega^2}{e^{2\Phi + \lambda r}} \right) \frac{\partial}{\partial x^3} \right). \quad (25)$$

From the equation (23) and (25) obtained each contravariance component of 4-velocity vector as follows

$$u^t = \gamma e^{-\Phi}, \quad (26)$$

$$u^r = \gamma v^{(r)} e^{-\alpha}, \quad (27)$$

$$u^\varphi = \gamma \left( e^{-\Phi} \omega + v^{(\varphi)} \left( \frac{e^{2\Phi} - e^{2\lambda r^2} \omega^2}{e^{2\Phi + \lambda r}} \right) \right). \quad (28)$$

The equation forms (26-27) have a covariant form with the Kerr Black Holes which corresponds to those obtained previously [22].

### 3.4. Angular Velocity and Lorentz Gamma Factor

The angular velocity related to the stationary observer  $\Omega$  is

$$\Omega = \frac{u^\varphi}{u^t} = \frac{\gamma \left( e^{-\Phi} \omega + v^{(\varphi)} \left( \frac{e^{2\Phi} - e^{2\lambda r^2} \omega^2}{e^{2\Phi + \lambda r}} \right) \right)}{\gamma e^{-\Phi}} = \omega + v^{(\varphi)} e^{\Phi} \left( \frac{e^{2\Phi} - e^{2\lambda r^2} \omega^2}{e^{2\Phi + \lambda r}} \right). \quad (29)$$

The angular velocity that related to the local inertia or ZAMO  $\tilde{\Omega}$  is

$$\tilde{\Omega} = \Omega - \omega = v^{(\varphi)} \left( \frac{e^{2\Phi} - e^{2\lambda} r^2 \omega^2}{e^{\Phi + \lambda r}} \right). \quad (30)$$

The Lorentz gamma factor is defined as follows

$$\gamma = \frac{1}{\sqrt{1 - (v^{(\varphi)})^2 - (v^{(r)})^2}}. \quad (31)$$

To define  $v^{(\varphi)}$  it is needed to do elaborating the equation (30) as follows

$$v^{(\varphi)} = \left( \frac{e^{\Phi + \lambda r}}{e^{2\Phi} - e^{2\lambda} r^2 \omega^2} \right) \tilde{\Omega}, \quad (32)$$

The magnitude of  $v^{(r)}$  is obtained from the equation (27) as follows

$$\gamma v^{(r)} = u^r e^\alpha. \quad (33)$$

The velocity of the radial component  $V$  is defined as follows

$$\begin{aligned} \gamma v^{(r)} &= u^r e^\alpha = V / \sqrt{1 - V^2}, \\ v^{(r)} &= \frac{V}{\gamma \sqrt{1 - V^2}}. \end{aligned} \quad (34)$$

Equation (32) and (34) are substituted into the equation (31) that obtained Lorentz gamma factor as

$$\gamma^2 = \frac{1}{[1 - V^2]} \frac{1}{\left[ 1 - \left( \frac{e^{\Phi + \lambda r}}{e^{2\Phi} - e^{2\lambda} r^2 \omega^2} \right) \tilde{\Omega} \right]^2}. \quad (35)$$

The value of  $V$

$$V = \frac{(v^{(r)})^2}{[1 - (\tilde{\Omega})^2]} = \frac{v^{(r)}}{\sqrt{1 - (v^{(\varphi)})^2}}, \quad (36)$$

is the radial velocity of fluid measured by the observer at constant  $r$  and rotates with the fluid.

### 3.5. The equation of mass accretion rate in a rapidly rotating neutron star

The general equation of relativistic mass conservation is

$$\nabla^i (\rho u_i) = 0, \quad (37)$$

The general equation of relativistic mass conservation for accretion disk that assumed in the equatorial plane is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r u_r) = 0. \quad (38)$$

The material accreted by the neutron star is distributed throughout the accretion space so that the equation (38) is integrated with the volume accretion disk  $(r, \varphi, z)$

$$\begin{aligned} \frac{\partial}{\partial t} \int \rho dV &= - \iiint \frac{1}{r} \frac{\partial}{\partial r} (\rho r u_r) r dr d\varphi dz \\ \frac{\partial M}{\partial t} &= - \int_0^{2\pi} d\varphi \frac{\partial}{\partial r} \int (\rho r u_r) dz \\ \dot{M} &= -2\pi(2h)r \rho \gamma e^{-\alpha} V / \sqrt{1 - V^2}. \end{aligned} \quad (39)$$

The mass accretion rate  $\dot{M}$  depends on the function  $\alpha$ . The result of this elaboration already has a covariant form with the equation of slowly rotating neutron star. In addition, it also has a covariant form with the slowly rotating Kerr black hole with an accretion disk in the equatorial plane [1,2,20]. Killing vector, 4-velocity, angular velocity, and Lorentz gamma factor of rapidly rotating neutron star have the same form as the slowly rotating neutron star and the slowly Kerr black hole [2,20]. However, for rapidly rotating neutron star, the velocity of accretion mass is not only depending on the distance of the accretionary material from the center of star, but also on the position of the equatorial plane.

## 4. Conclusion

Killing vector, 4-velocity, angular velocity, and Lorentz gamma factor of rapidly rotating neutron star have the same form as the slowly rotating neutron star and the slowly Kerr black hole. Likewise, the equation of mass accretion rate in rapidly rotating neutron star with an accretion disk in the equatorial plane ( $\dot{M}$ ) depends on the function  $\alpha$ . The result of this elaboration has a covariant form with the mass

accretion rate equation for slowly rotating neutron stars and slowly rotating Kerr black holes with accretion disk in the equatorial plane. The fundamental difference is in the function  $\alpha$  which depends not only on  $r$  but also on  $\theta$ .

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