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The Locating Chromatic Number for Split Graph of Cycle

The Locating Chromatic Number for Split Graph of Cycle

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Abstract.The minimum number of colors in a locating coloring of G is called the locating chromatic number of graph G, denoted by $\chi_L(G)$. Split graph of cycle with a set of vertices $\{v_1, v_2, v_3, \dots, v_n\}$ is graph obtained by adding on vertex v_i as many new k vertices $v_i^1, v_i^2, v_i^3, \dots, v_i^k$, so that each vertices $v_i^1, v_i^2, v_i^3, \dots, v_i^k$ neighbouring with each vertex that is neighbouring to vertex v_i in the cycle graph. Split graph of cycle, denoted by $spl(C_n)$ In this paper will be discussed about the locating chromatic number for split graph of cycle.

***Keyword*:** color code, locating chromatic number, split graph of cycle.

1. Introduction

The locating chromatic number one of material in the graph theory examined by Chartrand *et al* [7]. The locating chromatic number determine by minimizing the number of colors used in the locating coloring location with different color codes at each vertex in the graph.

Let c be a proper coloring of a connected graph G with $c(u) \neq c(v)$ for adjacent vertices u and v in G. Let c_i is a set of vertices receiving color i, for $i \in [1, k]$ then $\pi = \{C_1, C_2, \dots, C_k\}$ be a partition of $V(G)$. The color code $C_\pi(v)$ of a vertex v in G is the ordered *k-tuple* $\big(d(v, C_1), d(v, C_2), \dots, d(v, C_k)\big)$ with $d(v, C_i) = min\{d(v, x) | x \in C_i\}$ for $i \in [1, k]$. If all distinct vertices of G have distinct color codes, then c is called a locating coloring of G. The minimum number of colors in a locating coloring of G is called the locating chromatic number of graph G, denoted by $\chi_L(G)$.

Chartrand *et al* [7] determined the locating chromatic number for classes of graph, namely complete graph, $\chi_L(K_n) = n$; the cycle graph obtained $\chi_L(C_n) = 3$ for odd n odd and 4 for even n. Chartrand *et al* [6] characterized all graph of order n with the locating number $n - 1$. They also gave some conditions of graph G in which $n - 2$ is an upper bound of its locating chromatic number. Asmiati [1] determined the locating chromatic number of amalgamation of stars, Asmiati *et al* [2] determined the locating chromatic number of non-homogeneous amalgamations of stars and also Asmiati [3] determined locating-chromatic number for non-homogeneous caterpillars and firecracker graphs. Welyyanti *et al* [4] found on locating chromatic number for graph with dominant vertices. Sofyan *et al* [5] studied locating chromatic number of homogeneous lobster. Purwasih and Baskoro [8] studied the locating chromatic number of certain halin graph. Recently, Ghanem *et al* [9] studied and found locating chromatic number of power of paths and cycles.

The following theorem is a basic theorem proved by Chartrand *et al* [7]. The neighbourhood of vertex u in connected graph G, denoted by $N(u)$ is the set of vertices adjacent to u.

Theorem 1.1(see[7]). *Let c be a locating coloring in a connected graph G. If u and u are distinct vertices of G such that $d(u,w) = d(v,w)$ for all $t \in V(G) - \{u,v\}$ then $c(u) \neq c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

The split graph is obtained by adding on each vertex v on G one new vertex v', so that v' neighbouring with each vertex that is neighbouring with v in G, denoted by $spl\ (G)$. Split graph of cycle with a set of vertex $\{v_1, v_2, v_3, \dots, v_n\}$ is graph obtained by adding on vertex v_i as many new k vertex $v_i^1, v_i^2, v_i^3, \dots, v_i^k$, so that each vertices $v_i^1, v_i^2, v_i^3, \dots, v_i^k$ neighbouring with each vertex that is neighbouring to vertex v_i in the cycle graph. Split graph of cycle, denoted by $spl(C_n)$.
Next theorem about the locating chromatic number for a cycle graph (C_n).

Theorem 1.2(see [7]). *For $n \geq 3$, the locating chromatic number of a cycle graph (C_n) is 3 for odd n and 4 for even n.*

As long as the research, there is no theorem can determine the locating chromatic number for any graph. Research continues to get the locating chromatic numbers for other graph. Therefore, this paper will discuss about the locating chromatic number for split graph of cycle.

2. Results and discussion

In this section, we will discuss the locating chromatic number for $(spl(C_n))$.

Theorem 2.1. Let $spl(C_n)$ be a split graph of cycle for $n \geq 3$. Then the locating chromatic number of $spl(C_n)$ is:

$$\chi_L\big(spl(C_n)\big) = \begin{cases} 4, & \text{for odd } n \\ 5, & \text{for even } n \end{cases}$$

Proof: Let $\big(spl(C_n)\big)$, $n \geq 3$, be the split graph of cycle with the vertex set $V\big(spl(C_n)\big) = \{v_i, v_i'; 1 \leq i \leq n\}$ and the edge set $E\big(spl(C_n)\big) = \{v_i v_{i+1}; i \in [1, n-1]\} \cup \{v_n v_1\} \cup \{v_i v_{i+1}'; i \in [1, n-1]\} \cup \{v_n v_1'\} \cup \{v_{i+1} v_i'; i \in [1, n-1]\} \cup \{v_1 v_n'\}$. We distinguish two cases.

Case 1 (odd n). First, We determine the lower bound of the locating chromatic number of $spl(C_n)$ for odd n. Since $spl(C_n)$ contains C_n, then by Theorem 1.2, we have $\chi_L\big(spl(C_n)\big) \geq 3$. For a contradiction, assume we have locating coloring using 3 colors. Let $\{c(v_i)\} = \{1, 2, 3\} = \{c(v_i^1)\}$. Observe that vertex v_i^1 adjacent to vertex v_{i-1} and v_{i+1}, as well as vertex v_i adjacent to vertex v_{i-1} and v_{i+1}. Let vertex $v_j \in spl(C_n)$, with $j \neq \{i, i-1, i+1\}$. If $c(v_i^1) = c(v_j)$, then $c_\pi(v_i^1) = c_\pi(v_j)$. Consequence, if $c(v_i^1) = c(v_i)$, then $c_\pi(v_i^1) = c_\pi(v_i)$, a contradiction. So, we have $\chi_L\big(spl(C_n)\big) \geq 4$.

To construct the upper bound of $spl(C_n)$. Let c be a vertex coloring using 4 colors like this

$$c(v_i) = \begin{cases} 1, & \text{for } i = 1 \\ 2, & \text{for even } i \\ 3, & \text{for odd } i \end{cases}$$

$$c(v_i') = 4, \quad \text{for } 1 < i < n$$

For odd n the color codes of $V\big(spl(C_n)\big)$ are:

$$c_\pi(v_i) = \begin{cases} i-1, & \text{for } 1^{st} \text{ component}, i \le \frac{n+1}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component}, i > \frac{n+1}{2} \\ 0, & \text{for } 2^{nd} \text{ component, even } i, 2 \le i \le n-1 \\ & \text{for } 3^{rd} \text{ component, odd } i, 3 \le i \le n \\ 1, & \text{otherwise} \end{cases}$$

$$c_\pi(v_i') = \begin{cases} i-1, & \text{for } 1^{st} \text{ component}, 2 \le i \le \frac{n+1}{2} \\ n-i+1, & \text{for } 1^{st} \text{ component}, i > \frac{n+1}{2} \\ 0, & \text{for } 4^{th} \text{ component}, 1 \le i \le n \\ 2, & \text{for } 1^{st} \text{ component } i = 1 \\ & \text{for } 2^{nd} \text{ component, even } i, 2 \le i \le n-1 \\ & \text{for } 3^{rd} \text{ component, odd } i, 3 \le i \le n \\ 1, & \text{otherwise} \end{cases}$$

Since all vertices in $spl(C_n)$ have distinct color codes, then c is a locating coloring. So, $\chi_L(spl(C_n)) \le 4$ for odd n.

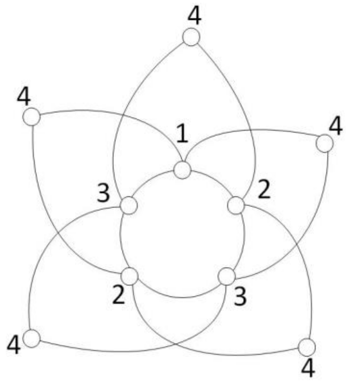

Figure 1. The minimum locating coloring of $spl(C_5)$.

Case 2 (even n). First, We determine the lower bound of the locating chromatic number of $spl(C_n)$ for even n. Since $spl(C_n)$ contains C_n for even n, then by Theorem 1.2, we have $\chi_L(spl(C_n)) \ge 4$. For a contradiction, assume we have locating coloring using 4 colors. Let $\{c(v_i)\} = \{1, 2, 3,4\} = \{c(v_i^1)\}$. Observe that vertex v_i^1 adjacent to vertex v_{i-1} and v_{i+1}, as well as vertex v_i adjacent to vertex v_{i-1} and v_{i+1}. Let vertex $v_j \in spl(C_n)$, with $j \ne \{i, i-1, i+1\}$. If $c(v_i^1) = c(v_j)$, then $c_\pi(v_i^1) = c_\pi(v_j)$. Consequence, if $c(v_i^1) = c(v_i)$, then $c_\pi(v_i^1) = c_\pi(v_i)$, a contradiction. So, we have $\chi_L(spl(C_n)) \ge 5$

Let c be a vertex coloring and assign using 5 colors:

$$c(v_i) = \begin{cases} 1, & \text{for } i = 1 \\ 2, & \text{for } i = 2 \\ 3, & \text{for odd } i \\ 4, & \text{for even } i \end{cases}$$

$$c(v_i') = \begin{cases} 3, & \text{for } i = 1 \\ 4, & \text{for } i = n \\ 5, & \text{for } 2 \le i \le n-1 \end{cases}$$

the color codes are:

$$c_\pi(v_i) = \begin{cases} i-1, & \text{for } 1^{st} \text{ component}, i \le \frac{n}{2}+1 \\ n-i+1, & \text{for } 1^{st} \text{ component}, i > \frac{n}{2}+1 \\ i-2, & \text{for } 2^{nd} \text{ component}, 2 \le i \le \frac{n}{2}+1 \\ n-i+2, & \text{for } 2^{nd} \text{ component } i > \frac{n}{2}+1 \\ 0, & \text{for } 3^{rd} \text{ component, odd } i, 3 \le i \le n-1 \\ & \text{for } 4^{th} \text{ component, even } i, 4 \le i \le n \\ 2, & \text{for } 3^{rd} \text{ component } i = 1 \\ & \text{for } 4^{th} \text{ component } i = 2 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\pi(v_i') = \begin{cases} i-1, & \text{for } 1^{st} \text{ component}, 2 \le i \le \frac{n}{2}+1 \\ n-i+1, & \text{for } 1^{st} \text{ component}, i > \frac{n}{2}+1 \\ i, & \text{for } 2^{nd} \text{ component}, i = 1 \textit{ and } 2 \\ i-2, & \text{for } 2^{nd} \text{ component}, 3 \le i \le \frac{n}{2}+1 \\ n-i+2, & \text{for } 2^{nd} \text{ component } i > \frac{n}{2}+1 \\ 0, & \text{for } 3^{rd} \text{ component } i = 1 \\ & \text{for } 4^{th} \text{ component } i = n \\ & \text{for } 5^{th} \text{ component}, 2 \le i \le n-1 \\ 2, & \text{for } 1^{st} \text{ component } i = 1 \\ & \text{for } 3^{rd} \text{ component, odd } i, 3 \le i \le n-1 \\ & \text{for } 4^{th} \text{ component, even } i, 2 \le i \le n-2 \\ & \text{for } 3^{rd} \text{ component, odd } i, i = 1 \textit{ and } 2 \\ 1, & \text{otherwise} \end{cases}$$

Since for even n all vertices of $spl(C_n)$ have distinct color codes then c is a locating coloring. As a result, we have $\chi_L(spl(C_n)) \le 5$. This concludes the proof. ∎

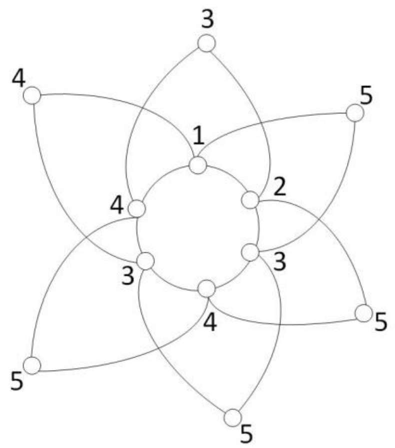

Figure 2. The minimum locating coloring of $spl(C_6)$.

3. Conclusions

Based on the result, to determine the locating chromatic number for split graph of cycle, by deviding two cases. The first case when odd n and second case when even n. So that, obtained the locating chromatic number for split graph of cycle is $\chi_L\big(spl(C_n)\big) = 4$ for odd n and 5 for even n.

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