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The Locating-Chromatic Number of Certain Barbell Origami Graphs

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Abstract. The locating-chromatic number of a graph combined two graph concept, coloring vertices and partition dimension of a graph. The locating-chromatic number, denoted by $\chi_L(G)$, is the smallest k such that G has a locating k-coloring. In this paper, we discuss the locating-chromatic number for certain barbell Origami graphs.

Keyword: coloring, locating-chromatic number, barbell origami graphs

1. Introduction

Chartrand et *al* firstly found the locating-chromatic number of a graph in 2002[1], with derived two graph concept, coloring vertices and partition dimension of a graph. Let $G = (V, E)$ be a connected graph and *c* be a proper k-coloring of G with color $1, 2, \dots, k$. Let $\Pi = \{C_1, C_2, \dots, C_k\}$ be a partition of $V(G)$ which is induced by coloring c. The color code $c_\Pi(v)$ of v is the ordered k-tuple $(d(v, C_1), d(v, C_2),\dots, d(v, C_k))$ where $d(v, C_i)$ = min $\{d(v, x) | x \in C_i\}$ for any i. If all distinct vertices of G have distinct color codes, then c is called k-locating coloring of G. The locating-chromatic number, denoted by $\chi_L(G)$, is the smallest k such that G has a locating k-coloring. Nex in 2003, Chartrand et *al.* [2] succeed in constructing tree graphs with locating-chromatic numbers ranging from 3 to *n*, except (*n* – *1*).

Several researcher have studied about the locating-chromatic number of a graph. Asmiati et *al* [3] found locating-chromatic number of amalgamation of star. The locating-chromatic numbers of the join graph determinded by Behtoei and Omoomi [4]. Furthermore, Behtoei and Anbarloei [5] found the locating chromatic number of Cartesian product of graphs. Next, Asmiati [6] determined the locating chromatic number of Non-Homogeneous Almagamation of Stars. Futher Asmiati [7] have found locating-chromatic number for non-homogeneous caterpillars and firecrackers graphs.

Some researchers have determined the locating-chromatic number for certain operation. Specially for generalized Petersen graphs 2019, Irawan et *al* [8] found the locating-chromatic number certain operation generalized Petersen graphs P(4,2), next the locating-chromatic number for new kind generalized Petersen graphs $s\mathrm{P}(n,1)$[9]. Asmiati et *al* [10] determined locating-chromatic number for certain barbell graphs $B_{P(n,1)}$. In this paper, we discuss the locating-chromatic number of certain barbell Origami graphs $\boldsymbol{B_{O_n}}$.

The following definition of a Origami graph is taken from [11]. Let $n \in \mathbb{N}$ with $n \geq 3$. An Origami graph O_n on $3n$ vertices is a graph with $V(O_n) = \{u_i, v_i, w_i | 1 \leq i \leq n\}$ and $(O_n) = \{u_i w_i, u_i v_i, v_i w_i, u_i u_{i+1}, w_i u_{i+1} |\ 1 \leq i \leq n\}$. The barbell graph is constructed by connecting two arbitrary connected graphs G and H by a bridge. Now, we define certain barbell Origami graphs $\boldsymbol{B}_{O_n}$. Let $\boldsymbol{B}_{O_n}$ $n \in \mathbb{N}$ with $n \geq 3$, be the barbell graph where G and H are two copies of Origami graph O_n with $V(\boldsymbol{B}_{O_n}) = \{u_i, u_{n+i}, v_i, v_{n+i}, w_i, w_{n+i} | 1 \leq i \leq n\}$ and $E(\boldsymbol{B}_{O_n}) = \{u_i w_i, u_i v_i, v_i w_i, u_i u_{i+1}, w_i u_{i+1} | 1 \leq i \leq n\} \cup \{u_{n+i} w_{n+i}, u_{n+i} v_{n+i}, v_{n+i} w_{n+i}, u_{n+i} u_{n+i+1}, w_{n+i} u_{n+i+1} | 1 \leq i \leq n\} \cup \{u_n u_{n+1}\}$.

The following theorems is basic to determine the locating chromatic number of a graph. The set of neighbours of a vertex s in G, denoted by $N(s)$.

Theorem 1.1. Chartrand et *al.*[1] *Let c be a locating coloring in a connected graph G. If k and l are distinct vertices of G such that d(k,w)=d(l,w) for all $w \in V(G) - \{k,l\}$, then $c(k) \neq c(l)$. In particular, if k and l are non-adjacent vertices of Gsuch that $N(k) \neq N(l)$, then $c(k) \neq c(l)$.*

Theorem 1.2. Chartrand et al.[1] *The locating chromatic number of a cycle C_n, is 3 for odd n and 4 for even n.*

2. Result and Discusion

In this section we will discuss the locating chromatic number of certain barbell Origami graphs $\boldsymbol{B_{O_n}}$.

Theorem 2.1 The locating-chromatic number of certain barbell Origami graphs $\boldsymbol{B_{O_n}}$is 5, with $\boldsymbol{n \geq 3}$.

Proof. Let $n \in \mathbb{N}$ with $n \geq 3$, with $V(\boldsymbol{B}_{O_n}) = \{u_i, u_{n+i}, v_i, v_{n+i}, w_i, w_{n+i} | 1 \leq i \leq n\}$ and $E(\boldsymbol{B}_{O_n}) = \{u_i w_i, u_i v_i, v_i w_i, \quad u_i u_{i+1}, \quad w_i u_{i+1} | 1 \leq i \leq n\} \cup \{u_{n+i} w_{n+i}, u_{n+i} v_{n+i}, v_{n+i} w_{n+i}, u_{n+i} u_{n+i+1}, w_{n+i} u_{n+i+1} |\ 1 \leq i \leq n\} \cup \{u_n u_{n+1}\}$. First, we determine lower bound of $\chi_L(\boldsymbol{B}_{O_n})$ for $n \geq 3$. Since barbell Origami graphs $\boldsymbol{B}_{O_n}$ for $n \geq 3$ contains two ishomorphic copies of Origami graph O_n, then by Theorem 1.2, we have $\chi_L(\boldsymbol{B}_{O_n}) \geq 4$ for $n \geq 3$. Next, we will show that 4 colors are not enough. For a contradiction, assume that there exsists a 4-locating coloring c on $\boldsymbol{B}_{O_n}$ for $n \geq 3$. Then $\{c(u_i), c(v_i), c(w_i), c(u_{i+1})\} = \{1, 2, 3, 4\}$. Since $\boldsymbol{B}_{O_n}$ for $n \geq 3$ contain n even cycles, then there is $c(u_i) = c(w_i)$ for $i \neq j$ so that $c_\Pi(u_i) = c_\Pi(w_j)$ for $i \neq j$ because u_i and w_j are dominant vertices, a contradiction. Therefore, $\chi_L(\boldsymbol{B}_{O_n}) \geq 5$.

Next, we determine the upper bound of $\chi_L(\boldsymbol{B_{O_n}}) \leq 5$ for $n \geq 3$. To prove the upper bound, next, we consider the following some cases :

Case 1. $\chi_L(\boldsymbol{B}_{O_3}) \leq 5$

To show that $\chi_L(\boldsymbol{B}_{O_3}) \leq 5$, consider the 5-coloring c on $\boldsymbol{B}_{O_3}$ as follow.

$$c(u_i) = \begin{cases} 1 & \text{for } i = 1 \\ 3 & \text{for } i = 2 \\ 4 & \text{for } i = 3 \end{cases}$$

$$c(v_i) = i + 2, i = 1,2,3$$

$$c(w_i) = \begin{cases} 2 & \text{for } i = 1 \\ 1 & \text{for } i = 2 \\ 3 & \text{for } i = 3 \end{cases}$$

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$$c(u_{n+i}) = \begin{cases} 2 & \text{for } i = 1 \\ 3 & \text{for } i = 2 \\ 1 & \text{for } i = 3 \end{cases}$$
$$c(v_{n+i}) = \begin{cases} 3 & \text{for } i = 1,3 \\ 1 & \text{for } i = 2 \end{cases}$$
$$c(w_{n+i}) = \begin{cases} 4 & \text{for } i = 1,3 \\ 2 & \text{for } i = 2 \end{cases}$$

The coloring c will create a partition Π on $V\left(\boldsymbol{B}_{O_3}\right)$. We shall show that the color codes of all vertices in $\boldsymbol{B}_{O_3}$ are different. For $c_\Pi(u_1) = (0,1,1,2,2)$; $c_\Pi(u_2) = (1,1,0,1,2)$; $c_\Pi(u_3) = (1,1,1,0,1)$; $c_\Pi(u_4) = (1,0,1,12)$; $c_\Pi(u_5) = (1,1,0,1,3)$; $c_\Pi(u_6) = (0,1,1,1,3)$; $c_\Pi(v_1) = (1,1,0,2,3)$; $c_\Pi(v_2) = (1,2,1,0,3)$; $c_\Pi(v_3) = (2,2,1,1,0)$; $c_\Pi(v_4) = (2,1,0,2,3)$; $c_\Pi(v_5) = (0,1,1,1,4)$; $c_\Pi(v_6) = (1,2,0,1,4)$; $c_\Pi(w_1) = (1,0,1,2,3)$; $c_\Pi(w_2) = (0,2,1,1,2)$; $c_\Pi(w_3) = (1,2,0,1,1)$; $c_\Pi(w_4) = (2,1,1,0,3)$; $c_\Pi(w_5) = (1,0,1,2,4)$; $c_\Pi(w_6) = (1,1,10,3)$. Since the color codes of all vertices in $\boldsymbol{B}_{O_3}$ are different, thus c is a locating-chromatic coloring. So $\chi_L\left(\boldsymbol{B}_{O_3}\right) \le 5$.

Case 2. $\chi_L\left(\boldsymbol{B}_{O_n}\right) \le 5$ for $n \ge 4$

To show the upper bound for the locating-chromatic number of barbell Origami graph $\boldsymbol{B}_{O_n}$ for $n \ge 4$. Let us different two subcases.

Subcase 2.1(odd *n*)

First, for $\left\lceil \frac{n}{2} \right\rceil$ odd. Let c be a coloring of barbell Origami graph $\boldsymbol{B}_{O_n}$, we make the partition Π of $V(\boldsymbol{B}_{O_n})$:

$$c(u_i) = \begin{cases} 2, & \text{for odd } i, 3 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ & \text{for odd } i, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n \\ 3, & \text{for even } i, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ & \text{for even } i, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n-1 \\ 4, & \text{for } i = 1 \\ 5, & \text{for } i = \left\lceil \frac{n}{2} \right\rceil + 1 \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 1, & \text{for } i = 1 \\ 2, & \text{for odd } i, 3 \le i \le \left\lceil \frac{n}{2} \right\rceil - 2 \\ & \text{for odd } i, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n \\ 3, & \text{for even } i, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ & \text{for even } i, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n-1 \\ 5, & \text{for } i = \left\lceil \frac{n}{2} \right\rceil \end{cases}$$

$$c(v_i) = \begin{cases} 2, & \text{for even } i, 2 \le i \le n-1 \\ 3, & \text{for odd } i, 1 \le i \le n \end{cases}$$
$$c(v_{n+i}) = \begin{cases} 2, & \text{for even } i, 2 \le i \le n-1 \\ 3, & \text{for odd } i, 1 \le i \le n \end{cases}$$
$$c(w_i) = 1, \quad 1 \le i \le n$$
$$c(w_{n+i}) = 4, \quad 1 \le i \le n$$

Therefore the color codes of all the vertices of $V(\boldsymbol{B}_{O_n})$ are :

$$c_\Pi(u_i) = \begin{cases} 1, & \text{for } 1^{st}, 2^{nd} \text{ and } 3^{rd} \text{component}, i = 1 \\ & \text{for } 1^{st} \text{ and } 2^{nd} \text{component}, i \text{ even}, 2 \le i \le n-1, n \ge 5 \\ & \text{for } 1^{st} \text{ and } 3^{rd} \text{component}, i \text{ odd}, 3 \le i \le n, n \ge 5 \\ 2, & \text{for } 3^{rd} \text{ component}, i = \left\lceil \frac{n}{2} \right\rceil + 1 \\ i-1, & \text{for } 4^{th} \text{ component}, i \text{ even}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 5 \\ n-i+1, & \text{for } 4^{th} \text{ component}, i \text{ odd}, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n, n \ge 5 \\ i-2, & \text{for } 4^{th} \text{ component}, i = \left\lceil \frac{n}{2} \right\rceil + 1 \\ \left\lceil \frac{n}{2} \right\rceil - 2, & \text{for } 5^{th} \text{ component}, i = 1 \\ \left\lceil \frac{n}{2} \right\rceil - i + 1, & \text{for } 5^{th} \text{ component}, i \text{ odd}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 5 \\ i - \left\lceil \frac{n}{2} \right\rceil - 1, & \text{for } 5^{th} \text{ component}, i \text{ odd}, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n, n \ge 5 \\ 0, & \text{otherwise} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} 1, & \text{for } 2^{nd}, 3^{rd} \text{ and } 4^{th} \text{component}, i = 1 \\ & \text{for } 2^{nd} \text{ and } 4^{th} \text{component}, 2 \le i \le n-1, n \ge 5 \\ & \text{for } 3^{rd} \text{ and } 4^{th} \text{component}, 3 \le i \le n, n \ge 5 \\ 2, & \text{for } 2^{nd} \text{ component}, i = \left\lceil \frac{n}{2} \right\rceil \\ i-1, & \text{for } 1^{st} \text{ component}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 5 \\ \left\lceil \frac{n}{2} \right\rceil - 1, & \text{for } 5^{th} \text{ component}, i = 1 \\ \left\lceil \frac{n}{2} \right\rceil - i, & \text{for } 5^{th} \text{ component}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 5 \\ i - \left\lceil \frac{n}{2} \right\rceil, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n, n \ge 5 \\ n-i+1, & \text{for } 1^{st} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 5 \\ 0, & \text{otherwise} \end{cases}$$

$$c_\Pi(v_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 5 \\ n-i+2, & \text{for } 4^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 5 \\ 0, & \text{for } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n \\ & \text{for } 2^{nd} \text{ component}, i \text{ even}, 2 \le i \le n-1 \\ \left\lceil \frac{n}{2} \right\rceil - i + 2, & \text{for } 5^{th} \text{ component}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 5 \\ \left\lceil \frac{n}{2} \right\rceil & \text{for } 5^{th} \text{ component}, i = 1 \\ i - \left\lceil \frac{n}{2} \right\rceil, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 5 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(v_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 5 \\ n-i+2, & \text{for } 1^{st} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 5 \\ 0, & \text{for } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n \\ & \text{for } 2^{nd} \text{ component}, i \text{ even}, 2 \le i \le n-1 \\ \left\lceil \frac{n}{2} \right\rceil - i + 1, & \text{for } 5^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 5 \\ i - \left\lceil \frac{n}{2} \right\rceil + 1, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil \le i \le n, n \ge 5 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} 0, & \text{for } 1^{st} \text{ component}, 1 \le i \le n \\ 2, & \text{for } 2^{nd} \text{ component}, i = 1 \\ i, & \text{for } 4^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 5 \\ n-i+2, & \text{for } 4^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 5 \\ \left\lceil \frac{n}{2} \right\rceil - i + 1, & \text{for } 5^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 5 \\ i - \left\lceil \frac{n}{2} \right\rceil, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 5 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} 0, & \text{for } 4^{th} \text{component}, 1 \le i \le n \\ 2, & \text{for } 2^{nd} \text{ component}, i = 1 \\ i, & \text{for } 1^{st} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 5 \\ n-i+1, & \text{for } 1^{st} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 5 \\ \left\lceil \frac{n}{2} \right\rceil - i, & \text{for } 5^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 5 \\ i - \left\lceil \frac{n}{2} \right\rceil + 1, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil \le i \le n, n \ge 5 \\ 1, & \text{otherwise} \end{cases}$$

Next, for $\left\lceil \frac{n}{2} \right\rceil$ even. Let c be a coloring of barbell Origami graph $\boldsymbol{B}_{O_n}$, we make the partition Π of $V(\boldsymbol{B}_{O_n})$:

$$c(u_i) = \begin{cases} 2, & \text{for odd } i, 3 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ & \text{for odd } i, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n \\ 3, & \text{for even } i, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 2 \\ & \text{for even } i, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n-1 \\ 4, & \text{for } i = 1 \\ 5, & \text{for } i = \left\lceil \frac{n}{2} \right\rceil \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 1, & \text{for } i = 1 \\ 2, & \text{for odd } i, 3 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ & \text{for odd } i, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n \\ 3, & \text{for even } i, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ & \text{for even } i, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n-1 \\ 5, & \text{for } i = \left\lceil \frac{n}{2} \right\rceil \end{cases}$$

$$c(v_i) = \begin{cases} 2, & \text{for even } i, 2 \le i \le n-1 \\ 3, & \text{for odd } i, 1 \le i \le n \end{cases}$$

$$c(v_{n+i}) = \begin{cases} 2, & \text{for even } i, 2 \le i \le n-1 \\ 3, & \text{for odd } i, 1 \le i \le n \end{cases}$$

$$c(w_i) = 1, \quad 1 \le i \le n$$

$$c(w_{n+i}) = 4, \quad 1 \le i \le n$$

Therefore the color codes of all the vertices of $V(\boldsymbol{B}_{O_n})$ are :

$$c_\Pi(u_i) = \begin{cases} 1, & \text{for } 1^{st}, 2^{nd} \text{ and } 3^{rd} \text{component}, i = 1 \\ & \text{for } 1^{st} \text{ and } 2^{nd} \text{component}, i \text{ even}, 2 \le i \le n-1, n \ge 7 \\ & \text{for } 1^{st} \text{ and } 3^{rd} \text{component}, i \text{ odd}, 3 \le i \le n, n \ge 7 \\ 2, & \text{for } 3^{rd} \text{ component}, i = \left\lceil \frac{\mathrm{n}}{2} \right\rceil \\ i-1, & \text{for } 4^{th} \text{ component}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 7 \\ n-i+1, & \text{for } 4^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ \left\lceil \frac{n}{2} \right\rceil - i, & \text{for } 5^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 7 \\ i - \left\lceil \frac{n}{2} \right\rceil, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ 0, & \text{otherwise} \end{cases}$$

$$c_\Pi(u_{n+i}) = \begin{cases} 1, & \text{for } 2^{nd}, 3^{rd} \text{ and } 4^{th} \text{ component}, i = 1 \\ & \text{for } 2^{nd} \text{ and } 4^{th} \text{ component}, i \text{ even}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 7 \\ & \text{for } 3^{rd} \text{and } 4^{th} \text{ component}, i \text{ odd}, 3 \le i \le n, n \ge 7 \\ 2, & \text{for } 3^{rd} \text{ component}, i = \left\lceil \frac{\mathrm{n}}{2} \right\rceil \\ i-1, & \text{for } 1^{st} \text{ component}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 7 \\ n-i+1, & \text{for } 1^{st} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ \left\lceil \frac{n}{2} \right\rceil - i, & \text{for } 5^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 7 \\ i - \left\lceil \frac{n}{2} \right\rceil, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ 0, & \text{otherwise} \end{cases}$$

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$$c_\Pi(v_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 7 \\ n-i+2, & \text{for } 4^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n, n \ge 7 \\ 0, & \text{for } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n \\ & \text{for } 2^{nd} \text{ component}, i \text{ even}, 2 \le i \le n-1 \\ \left\lceil \frac{n}{2} \right\rceil - i + 1, & \text{for } 5^{th} \text{ component}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 7 \\ \left\lceil \frac{n}{2} \right\rceil - 1 & \text{for } 5^{th} \text{ component}, i = 1 \\ i - \left\lceil \frac{n}{2} \right\rceil + 1, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(v_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 7 \\ n-i+2, & \text{for } 1^{st} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ 0, & \text{for } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n \\ & \text{for } 2^{nd} \text{ component}, i \text{ even}, 2 \le i \le n-1 \\ \left\lceil \frac{n}{2} \right\rceil - i + 1, & \text{for } 5^{th} \text{ component}, 2 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 7 \\ \left\lceil \frac{n}{2} \right\rceil - 1 & \text{for } 5^{th} \text{ component}, i = 1 \\ i - \left\lceil \frac{n}{2} \right\rceil + 1, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} 0, & \text{for } 1^{st} \text{ component}, 1 \le i \le n \\ 2, & \text{for } 2^{nd} \text{ component}, i = 1 \\ i, & \text{for } 4^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 7 \\ n-i+1, & \text{for } 4^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ \left\lceil \frac{n}{2} \right\rceil - i, & \text{for } 5^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 7 \\ i - \left\lceil \frac{n}{2} \right\rceil + 1, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil \le i \le n, n \ge 7 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} 0, & \text{for } 4^{th} \text{ component}, 1 \le i \le n \\ 2, & \text{for } 2^{nd} \text{ component}, i = 1 \\ i, & \text{for } 1^{st} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, n \ge 7 \\ n-i+1, & \text{for } 1^{st} \text{ component}, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n, n \ge 7 \\ \left\lceil \frac{n}{2} \right\rceil - i, & \text{for } 5^{th} \text{ component}, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, n \ge 7 \\ i - \left\lceil \frac{n}{2} \right\rceil + 1, & \text{for } 5^{th} \text{ component}, \left\lceil \frac{n}{2} \right\rceil \le i \le n, n \ge 7 \\ 1, & \text{otherwise} \end{cases}$$

Subcase 2.2(even *n*).

First, for $\frac{n}{2}$ odd. Let c be a coloring of certain barbell Origami graph $\boldsymbol{B}_{O_n}$, We make the partition Π of $V(\boldsymbol{B}_{O_n})$:

$$c(u_i) = \begin{cases} 2, & \text{for odd } i, 3 \le i \le n-1 \\ 3, & \text{for even } i, 2 \le i \le n \\ 4, & \text{for } i = 1 \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 1, & \text{for } i = 1 \\ 2, & \text{for odd } i, 3 \le i \le n-1 \\ 3, & \text{for even } i, 2 \le i \le n \end{cases}$$

$$c(v_i) = \begin{cases} 2, & \text{for even } i, 2 \le i \le n \\ 3, & \text{for odd } i, 1 \le i \le n-1 \end{cases}$$

$$c(v_{n+i}) = \begin{cases} 2, & \text{for even } i, 2 \le i \le n \\ 3, & \text{for odd } i, 1 \le i \le n-1 \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for } 1 \le i \le \frac{n}{2}-1 \\ & \text{for } \frac{n}{2}+1 \le i \le n \\ 5 & \text{for } i = \frac{n}{2} \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 4, & \text{for } 1 \le i \le \frac{n}{2}-1 \\ & \text{for } \frac{n}{2}+1 \le i \le n \\ 5 & \text{for } i = \frac{n}{2} \end{cases}$$

Therefore the color codes of all the vertices of $V(B_{\boldsymbol{O_n}})$ are :

$$c_\Pi(u_i) = \begin{cases} 1, & \text{for } 1^{st} \text{ and } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n-1, n \ge 6 \\ & \text{for } 1^{st} \text{ and } 2^{nd} \text{component}, i \text{ even}, 2 \le i \le n, n \ge 6 \\ 2, & \text{for } 3^{rd} \text{ component}, i = 1 \\ i-1, & \text{for } 4^{th} \text{ component}, 2 \le i \le \frac{n}{2}, n \ge 6 \\ n-i+1, & \text{for } 4^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 6 \\ \frac{n}{2}, & \text{for } 5^{th} \text{ component}, i = 1 \\ \frac{n}{2}-i+1, & \text{for } 5^{th} \text{ component}, 2 \le i \le \frac{n}{2}, n \ge 6 \\ i-\frac{n}{2}, & \text{for } 5^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 6 \\ 0, & \text{otherwise} \end{cases}$$

$$c_{\Pi}(u_{n+i}) = \begin{cases} 1, & \text{for } 3^{rd} \text{ and } 4^{th} \text{ component}, i \text{ odd}, 1 \le i \le n-1, n \ge 6 \\ & \text{for } 2^{nd} \text{ and } 4^{th} \text{ component}, i \text{ even}, 2 \le i \le n, n \ge 6 \\ 2, & \text{for } 2^{nd} \text{ component}, i = 1 \\ i-1, & \text{for } 1^{st} \text{ component}, 2 \le i \le \frac{n}{2}, n \ge 6 \\ n-i+1, & \text{for } 1^{st} \text{ component}, , \frac{n}{2}+1 \le i \le n, n \ge 6 \\ \frac{n}{2}, & \text{for } 5^{th} \text{ component}, i = 1 \\ \frac{n}{2}-i+1, & \text{for } 5^{th} \text{ component}, 2 \le i \le \frac{n}{2}, n \ge 6 \\ i-\frac{n}{2}, & \text{for } 5^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 6 \\ 0, & \text{otherwise} \end{cases}$$

$$c_{\Pi}(v_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, 1 \le i \le \frac{n}{2}, n \ge 6 \\ n-i+2, & \text{for } 4^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 6 \\ 0, & \text{for } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n-1 \\ & \text{for } 2^{nd} \text{ component}, i \text{ even}, 2 \le i \le n \\ \frac{n}{2}-i+2, & \text{for } 5^{th} \text{ component}, 1 \le i \le \frac{n}{2}-1, n \ge 6 \\ i-\frac{n}{2}+1, & \text{for } 5^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 6 \\ 2 & \text{for } 1^{st} \text{ component}, i = \frac{n}{2} \\ 1, & \text{otherwise} \end{cases}$$

$$c_{\Pi}(v_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component}, 1 \le i \le \frac{n}{2}, n \ge 6 \\ n-i+2, & \text{for } 1^{st} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 6 \\ 0, & \text{for } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n-1 \\ & \text{for } 2^{nd} \text{ component}, i \text{ even}, 2 \le i \le n \\ \frac{n}{2}-i+2, & \text{for } 5^{th} \text{ component}, 1 \le i \le \frac{n}{2}-1, n \ge 6 \\ i-\frac{n}{2}+1, & \text{for } 5^{th} \text{ component}, i \text{ odd}, \frac{n}{2}+1 \le i \le n, n \ge 6 \\ 2 & \text{for } 4^{th} \text{ component}, i = \frac{n}{2} \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} 0, & \text{for } 1^{st} \text{ component}, 1 \le i \le \frac{n}{2} - 1 \\ & \text{for } 1^{st} \text{ component}, \frac{n}{2} + 1 \le i \le n \\ 2, & \text{for } 1^{st} \text{ component}, i = \frac{n}{2} \\ i, & \text{for } 4^{th} \text{ component}, 1 \le i \le \frac{n}{2}, n \ge 6 \\ n - i + 1, & \text{for } 4^{th} \text{ component}, \frac{n}{2} + 1 \le i \le n, n \ge 6 \\ \frac{n}{2} - i + 1, & \text{for } 5^{th} \text{ component}, 1 \le i \le \frac{n}{2} - 1, n \ge 6 \\ i - \frac{n}{2}, & \text{for } 5^{th} \text{ component}, \frac{n}{2} + 1 \le i \le n, n \ge 6 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} 0, & \text{for } 4^{th} \text{ component}, 1 \le i \le \frac{n}{2} - 1 \\ & \text{for } 4^{th} \text{ component}, \frac{n}{2} + 1 \le i \le n \\ 2, & \text{for } 4^{th} \text{ component}, i = \frac{n}{2} \\ i, & \text{for } 1^{st} \text{ component}, 1 \le i \le \frac{n}{2}, n \ge 6 \\ n - i + 1, & \text{for } 1^{st} \text{ component}, \frac{n}{2} + 1 \le i \le n, n \ge 6 \\ \frac{n}{2} - i + 1, & \text{for } 5^{th} \text{ component}, 1 \le i \le \frac{n}{2} - 1, n \ge 6 \\ i - \frac{n}{2}, & \text{for } 5^{th} \text{ component}, \frac{n}{2} + 1 \le i \le n, n \ge 6 \\ 1, & \text{otherwise} \end{cases}$$

Next, for $\frac{n}{2}$ even. Let c be a coloring of barbell Origami graph $\boldsymbol{B}_{O_n}$, We make the partition Π of $V(\boldsymbol{B}_{O_n})$:

$$c(u_i) = \begin{cases} 2, & \text{for odd } i, 3 \le i \le n-1 \\ 3, & \text{for even } i, 2 \le i \le n \\ 4, & \text{for } i = 1 \end{cases}$$

$$c(u_{n+i}) = \begin{cases} 1, & \text{for } i = 1 \\ 2, & \text{for odd } i, 3 \le i \le n-1 \\ 3, & \text{for even } i, 2 \le i \le n \end{cases}$$

$$c(v_i) = \begin{cases} 2, & \text{for even } i, 2 \le i \le n \\ 3, & \text{for odd } i, 1 \le i \le n-1 \end{cases}$$

$$c(v_{n+i}) = \begin{cases} 2, & \text{for even } i, 2 \le i \le n \\ 3, & \text{for odd } i, 1 \le i \le n-1 \end{cases}$$

$$c(w_i) = \begin{cases} 1, & \text{for } 1 \le i \le \frac{n}{2} \\ & \text{for } \frac{n}{2} + 2 \le i \le n \\ 5 & \text{for } i = \frac{n}{2} + 1 \end{cases}$$

$$c(w_{n+i}) = \begin{cases} 4, & \text{for } 1 \le i \le \frac{n}{2} \\ & \text{for } \frac{n}{2} + 1 \le i \le n \\ 5 & \text{for } i = \frac{n}{2} + 1 \end{cases}$$

Therefore the color codes of all the vertices of $V(\boldsymbol{B_{O_n}})$ are :

$$c_{\Pi}(u_i) = \begin{cases} 1, & \text{for } 1^{st} \text{ and } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n-1, n \ge 4 \\ & \text{for } 1^{st} \text{ and } 2^{nd} \text{component}, i \text{ even}, 2 \le i \le n, n \ge 4 \\ 2, & \text{for } 2^{nd} \text{ component}, i = 1 \\ i-1, & \text{for } 4^{th} \text{ component}, 2 \le i \le \frac{n}{2}, n \ge 4 \\ n-i+1, & \text{for } 4^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 4 \\ \frac{n}{2}, & \text{for } 5^{th} \text{ component}, i = 1 \\ \frac{n}{2}-i+2, & \text{for } 5^{th} \text{ component}, 2 \le i \le \frac{n}{2}+1, n \ge 4 \\ i-\frac{n}{2}-1, & \text{for } 5^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 4 \\ 0, & \text{otherwise} \end{cases}$$

$$c_{\Pi}(u_{n+i}) = \begin{cases} 1, & \text{for } 3^{rd} \text{ and } 4^{th} \text{ component}, 1 \le i \le n-1, n \ge 4 \\ & \text{for } 2^{nd} \text{ and } 4^{th} \text{ component}, i \text{ even}, 2 \le i \le n, n \ge 4 \\ 2, & \text{for } 2^{nd} \text{ component}, i = 1 \\ i-1, & \text{for } 1^{st} \text{ component}, 2 \le i \le \frac{n}{2}, n \ge 4 \\ n-i+1, & \text{for } 1^{st} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 4 \\ \frac{n}{2}, & \text{for } 5^{th} \text{ component}, i = 1 \\ \frac{n}{2}-i+2, & \text{for } 5^{th} \text{ component}, 2 \le i \le \frac{n}{2}+1, n \ge 4 \\ i-\frac{n}{2}-1, & \text{for } 5^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 4 \\ 0, & \text{otherwise} \end{cases}$$

$$c_{\Pi}(v_i) = \begin{cases} i, & \text{for } 4^{th} \text{ component}, 1 \le i \le \frac{n}{2}, n \ge 4 \\ n-i+2, & \text{for } 4^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 4 \\ 0, & \text{for } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n-1 \\ & \text{for } 2^{nd} \text{ component}, i \text{ even}, 2 \le i \le n \\ \frac{n}{2}-i+3, & \text{for } 5^{th} \text{ component}, 2 \le i \le \frac{n}{2}, n \ge 4 \\ \frac{n}{2}+1 & \text{for } 5^{th} \text{ component}, i = 1 \\ i-\frac{n}{2}, & \text{for } 5^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 4 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(v_{n+i}) = \begin{cases} i, & \text{for } 1^{st} \text{ component}, i \text{ odd}, 1 \le i \le \frac{n}{2}, n \ge 4 \\ n-i+2, & \text{for } 1^{st} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 4 \\ 0, & \text{for } 3^{rd} \text{ component}, i \text{ odd}, 1 \le i \le n-1 \\ & \text{for } 2^{nd} \text{ component}, i \text{ even}, 2 \le i \le n \\ \frac{n}{2}-i+3, & \text{for } 5^{th} \text{ component}, 2 \le i \le \frac{n}{2}, n \ge 4 \\ \frac{n}{2}+1 & \text{for } 5^{th} \text{ component}, i = 1 \\ i-\frac{n}{2}, & \text{for } 5^{th} \text{ component}, i \text{ odd}, \frac{n}{2}+1 \le i \le n, n \ge 4 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(w_i) = \begin{cases} 0, & \text{for } 1^{st} \text{ component}, 1 \le i \le \frac{n}{2}-1 \\ & \text{for } 1^{st} \text{ component}, \frac{n}{2}+1 \le i \le n \\ & \text{for } 5^{th} \text{ component}, i = \frac{n}{2}+1 \\ 2, & \text{for } 1^{st} \text{ component}, i = \frac{n}{2}+1 \\ i, & \text{for } 4^{th} \text{ component}, 1 \le i \le \frac{n}{2}, n \ge 4 \\ n-i+1, & \text{for } 4^{th} \text{ component}, \frac{n}{2}+2 \le i \le n, n \ge 4 \\ \frac{n}{2}-i+3, & \text{for } 5^{th} \text{ component}, 1 \le i \le \frac{n}{2}-1, n \ge 8 \\ i-\frac{n}{2}, & \text{for } 5^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 8 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\Pi(w_{n+i}) = \begin{cases} 0, & \text{for } 4^{th} \text{ component}, 1 \le i \le \frac{n}{2}-1 \\ & \text{for } 4^{th} \text{ component}, \frac{n}{2}+1 \le i \le n \\ & \text{for } 5^{th} \text{ component}, i = \frac{n}{2}+1 \\ 2, & \text{for } 4^{th} \text{ component}, i = \frac{n}{2}+1 \\ i, & \text{for } 1^{st} \text{ component}, 1 \le i \le \frac{n}{2}, n \ge 4 \\ n-i+1, & \text{for } 1^{st} \text{ component}, \frac{n}{2}+2 \le i \le n, n \ge 4 \\ \frac{n}{2}-i+3, & \text{for } 5^{th} \text{ component}, 1 \le i \le \frac{n}{2}-1, n \ge 8 \\ i-\frac{n}{2}, & \text{for } 5^{th} \text{ component}, \frac{n}{2}+1 \le i \le n, n \ge 8 \\ 1, & \text{otherwise} \end{cases}$$

Since all the vertices have different color codes, c is a locating coloring of certain barbell Origami graphs $\boldsymbol{B}_{O_n}$, so that $\chi_L(\boldsymbol{B}_{O_n}) = 5$, for $n \ge 3$. This concludes the proof. □

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