PAPER • **OPEN ACCESS**

The Locating Chromatic Number of some Modified Path with Cycle having Locating Number Four

To cite this article: M Damayanti *et al* 2021 *J. Phys.: Conf. Ser.* **1751** 012008

The Locating Chromatic Number of some Modified Path with Cycle having Locating Number Four

M Damayanti[1], Asmiati[2*], Fitriani[2], M Ansori[2], A Faradilla[1]

[1] Graduate School of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Lampung, Jl. Sumantri Brojonegoro no 1, Bandar Lampung, Indonesia
[2] Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Lampung, Jl. Sumantri Brojonegoro no 1, Bandar Lampung, Indonesia

Corresponding author[2*]: asmiati.1976@fmipa.unila.ac.id

Abstract. The locating-chromatic number was introduced by Chartrand in 2002. The locating-chromatic number of a graph is a combined concept between the coloring and partition dimension of a graph. The locating chromatic number of a graph G is defined as the cardinality of minimum color classes. In this paper, we discuss about the locating chromatic number of three types modified path with cycle having locating chromatic number four.

Keyword: *locating chromatic number, path, cycle.*

1. Introduction

The locating-chromatic number of a graph is a combined concept between the coloring and partition dimension of graph. Chartrand et al. has been introduced the concept of the partition dimension of a graph in 1998 [2] and the concept of locating chromatic number of a graph in 2002 [1].

Let $G = (V, E)$ be a connected graph. Let c be a proper k −coloring of G with colors $1,2, \dots, k$. Let $\pi = \{C_1, C_2, \dots, C_k\}$ be a partition of $V(G)$, where C_i is the set of vertices receiving color i. The color code $C_\pi(v)$ of v is the ordered *k-tuple* $\left(d(v, C_1), d(v, C_2), \dots, d(v, C_k)\right)$ where $d(v, C_i) = min\{d(v, x) | x \in C_i\}$ for any i. If all distinct vertices of G have distinct color codes, then c is called a *locating chromatic k-coloring* of G. The locating chromatic number, denoted by $\chi_L(G)$ is the smallest k such that G has a locating coloring with k colors.

The locating-chromatic number has been determined for some classes of graphs, namely cycles [1], multipartite graphs [1], and some classes of trees. Chartrand et al. [1] determined the locating chromatic number of path and double stars, then in [3] also gave a characterization of all graphs of order n with locating-chromatic number n -1. After this, Asmiati et al. [5] determined the locating-chromatic number an amalgamation of stars and non-homogeneous caterpillars and firecracker graphs in [4]. Behtoe and Omoomi [6] found the locating-chromatic number on the Kneser graph. Next, Baskoro and Purwasih [8] determined the locating-chromatic number for the corona product of graphs, then in [7] also gave a characterization of all graphs with locating-chromatic number 3. Recently, Ghanem et al. [9] found the locating chromatic number of powers of the path and powers of cycles.

Journal of Physics: Conference Series **1751** (2021) 012008 doi:10.1088/1742-6596/1751/1/012008

Based on the previous results, the locating chromatic number of modified path graphs have not been studied. Motivated by this, in this paper we determine the locating chromatic number of some modified path with cycle having locating number four.
Let us begin to state the following lemma and theorem which are useful to obtain our main result.

Lemma 1.1. Chartrand et *al.*[1] *Let c be a locating coloring in a connected graph G. If u and u are distinct vertices of G such that $d(u,w) = d(v,w)$ for all $t \in V(G) - \{u,v\}$ then $c(u) \neq c(v)$. In particular, if u and v are non-adjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

Theorem 1.1. Chartrand et *al.*[1] *For $n \geq 3$, the locating chromatic number of a path graph (P_n) is 3.*

2. Results and discussion

In this section, we will discuss the locating chromatic some modified path with locating chromatic number four.

Type 1 is a modified path obtained from a path P_n with v_i vertices $i = 1, \ldots, n$ by adding vertex outside of P_n which will form C_3, denoted by $P_n(C_3)$. The following is given the locating chromatic number of $P_n(C_3)$.

Theorem 2.1. The locating chromatic number of $P_n(C_3)$ is 4.

Proof: Let $P_n(C_3)$, $n \geq 3$, with the vertex set $V(P_n(C_3)) = \{u_i, v_i; 1 \leq i \leq n\}$ and the edge set $E(P_n(C_3)) = \{u_i v_i, u_i v_{i+1}; i \in [1, n-1]\} \cup \{v_n v_{n+1}; i \in [1, n-1]\}$.

First, we determine the lower bound for the modified graph of path for $n \geq 3$. According to Theorem 1.1, it is clear that $\chi_L P_n(C_3) \geq 3$. For a contradiction, suppose c is a locating coloring on $P_n(C_3)$ using 3 colors. Let $c(v_i) = \{1,2,3\} = c(u_i)$. Since u_i adjacent to v_i and v_{i+1}, if $c(u_i) = \{c(v_j)\}$, then $c_\pi(u_i) = c_\pi(v_j), i \neq j$, a contradiction. As a result, needed at least 4 colors to color the modified graph of the path. So, $\chi_L P_n(C_3) \geq 4$.

Next, we determine the upper bound for the modified graph of the path for $n \geq 3$. Let c be a locating coloring using 4 colors as follows:

$c(u_i) = 1, \text{for } i \geq 1$

$$c(v_i) = \begin{cases} 2, & \text{for i} = 2n, n \geq 1 \\ 3, & \text{for i} = 2n+1, n \geq 1 \\ 4, & \text{for i} = 1 \end{cases}$$

The color codes of $(P_n(C_3))$ are:

$$c_\pi(u_i) = \begin{cases} i, & \text{for } 4^{\text{th}} \text{ component, i} \geq 1 \\ 0, & \text{for } 1^{\text{st}} \text{ component}, i \geq 1 \\ 2, & \text{for } 3^{\text{rd}} \text{ component}, i = 1 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\pi(v_i) = \begin{cases} i-1, & \text{for } 4^{\text{th}} \text{ component, i} \geq 1 \\ 0, & \text{for } 2^{\text{nd}} \text{ component, even } i, i \geq 2 \\ & \text{for } 3^{\text{rd}} \text{ component, odd } i \geq 3 \\ 2, & \text{for } 3^{\text{rd}} \text{ component}, i = 1 \\ 1, & \text{otherwise} \end{cases}$$

Since all vertices in $P_n(C_3)$ for $n \geq 3$ have distinct color codes, then c is a locating coloring using 4 colors. As a result $\chi_L P_n(C_3) \leq 4$. Thus, $P_n(C_3) = 4$. ■

Journal of Physics: Conference Series **1751** (2021) 012008 doi:10.1088/1742-6596/1751/1/012008

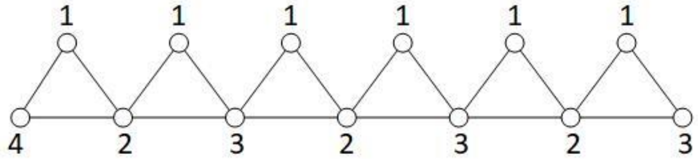

Figure. 1 The minimum locating coloring of $P_7(C_3)$.

Type 2 is obtained from $P_n(C_3)$ which insert one vertex x_i in v_iv_{i+1}, denoted by $P_n^*(C_n)$. The following theorem gives the locating chromatic number of $P_n^*(C_3)$.

Theorem 2.2. The locating chromatic number of $P_n^*(C_n)$ is 4.

Proof: Let $P_n^*(C_3)$, $n > 3$, with the vertex set $V(P_n^*(C_3)) = \{u_i, v_i; 1 \leq i \leq n\}$ and the edge set $E(P_n^*(C_3)) = \{v_{2i-1}u_i; i \in [1, n-1]\} \cup \{u_iv_{2i+1}; i \in [1, n-1]\} \cup \{v_ix_i; i \in [1, n-1]\} \cup \{x_iv_{i+1}; i \in [1, n-1]\}$.

First, we determine the lower bound of $P_n^*(C_3)$. By Theorem 2.1, it is clear that $\chi_L(P_n^*(C_3)) \geq 4$. Next, we determine the upper bound. Let c be a locating coloring using 4 colors as follows:

$c(u_i) = 1, \quad \text{for } i \geq 1$

$$c(v_i) = \begin{cases} 3, & \text{for } i \geq 2 \\ 4, & \text{for } i = 1 \end{cases}$$

$c(x_i) = 2, \quad \text{for } i \geq 1$

The color codes of $(P_n^*(C_n))$ are:

$$c_\pi(u_i) = \begin{cases} i, & \text{for } 4^{\text{th}} \text{ component}, \mathrm{i} \geq 1 \\ 0, & \text{for } 1^{\text{st}} \text{ component}, i \geq 1 \\ 2, & \text{for } 2^{\text{nd}} \text{ component}, i \geq 1 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\pi(v_i) = \begin{cases} 2(i-1), & \text{for } 4^{\text{th}} \text{ component}, \mathrm{i} \geq 1 \\ 0, & \text{for } 3^{\text{rd}} \text{ component}, i \geq 2 \\ 2, & \text{for } 3^{\text{rd}} \text{ component}, i = 1 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\pi(x_i) = \begin{cases} 2i-1, & \text{for } 4^{\text{th}} \text{ component}, \mathrm{i} \geq 1 \\ 0, & \text{for } 2^{\text{nd}} \text{ component}, i \geq 1 \\ 2, & \text{for } 1^{\text{st}} \text{component}, i \geq 1 \\ 1, & \text{otherwise} \end{cases}$$

Since all vertices in $P_n^*(C_3)$ for $n \geq 3$ have distinct color codes, then c is locating coloring using 4 colors. As a result, $\chi_L(P_n^*(C_3)) \leq 4$. Thus, $\chi_L(P_n^*(C_3)) = 4$. ∎

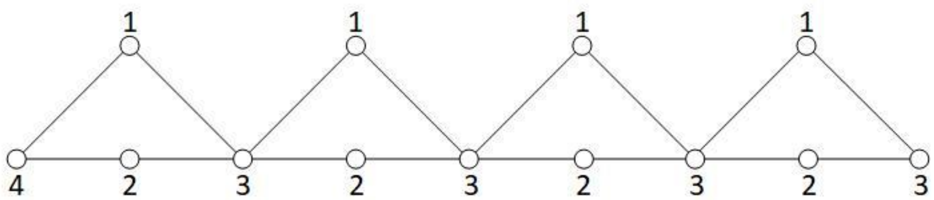

Figure 2. The minimum locating coloring of $P_5^*(C_3)$.

Type 3 is obtained from type 2 adding a vertex y_i outside of $P_n^*(C_3)$ such that $x_{i-1}y_i$ forms C_3, denoted by $sP_n^*(C_3)$. The following theorem gives the locating chromatic number of $sP_n^*(C_3)$.

Theorem 2.3. The locating chromatic number of $sP_n^*(C_3)$ is 4.

Proof: Let $(sP_n^*(C_3))$, $n > 3$, with the vertex set $V(sP_n^*(C_3)) = \{u_i, v_i, x_i, y_i; 1 \leq i \leq n\}$ and the edge set $E(sP_n^*(C_3)) = \{v_iu_i; i \in [1, n-1]\} \cup \{x_iy_i; i \in [1, n-1]\} \cup \{u_iv_{i+1}; i \in [1, n-1]\} \cup \{y_ix_{i+1}; i \in [1, n-1]\} \cup \{v_ix_i; i \in [1, n-1]\} \cup \{x_iv_{i+1}; i \in [1, n-1]\}$.

First, we determine the lower bound of $sP_n^*(C_3)$. By Theorem 2.2, We obtain $\chi_L(sP_n^*(C_3)) \geq 4$. Next, we determine the upper bound of $sP_n^*(C_3)$. Let c be a locating coloring using 4 colors as follows:

$c(u_i) = 1, \quad$ for $i \geq 1$

$$c(v_i) = \begin{cases} 3, & \text{for } i \geq 2 \\ 4, & \text{for } i = 1 \end{cases}$$

$c(x_i) = 2, \quad$ for $i \geq 1$

$c(y_i) = 1, \quad$ for $i \geq 1$

The color codes of $(sP_n^*(C_3))$ are:

$$c_\pi(u_i) = \begin{cases} 2i-1, & \text{for } 4^{\text{th}} \text{ component}, \mathrm{i} \geq 2 \\ 0, & \text{for } 1^{\text{st}} \text{ component}, i \geq 1 \\ 2, & \text{for } 2^{\text{nd}} \text{ component}, i \geq 1 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\pi(v_i) = \begin{cases} 2i-2, & \text{for } 4^{\text{th}} \text{ component}, i \geq 1 \\ 0, & \text{for } 3^{\text{rd}} \text{ component}, i \geq 2 \\ 2, & \text{for } 3^{\text{rd}} \text{component}, i = 1 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\pi(x_i) = \begin{cases} 2i-1, & \text{for } 4^{\text{th}} \text{ component}, \mathrm{i} \geq 1 \\ 0, & \text{for } 2^{\text{nd}} \text{ component}, i \geq 1 \\ 1, & \text{otherwise} \end{cases}$$

$$c_\pi(y_i) = \begin{cases} 2i, & \text{for } 4^{\text{th}} \text{ component}, \mathrm{i} \geq 1 \\ 0, & \text{for } 1^{\text{st}} \text{ component}, i \geq 1 \\ 2, & \text{for } 3^{\text{rd}} \text{component}, i \geq 1 \\ 1, & \text{otherwise} \end{cases}$$

Since all vertices in $P_n^*(C_3)$ for $n \geq 3$ have distinct color codes, then c is locating coloring. So $\chi_L(sP_n^*(C_3)) \leq 4$. Thus, $\chi_L(sP_n^*(C_3)) = 4$. ■

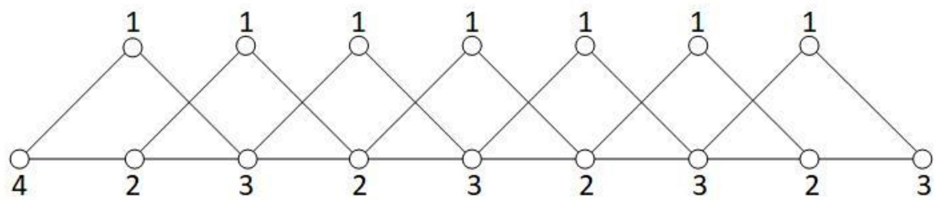

Figure 3. The minimum locating coloring of $sP_5^*(C_3)$.

3. Conclusions

In this research, we have successfully found modified path graph with cycle. Type 1 we get by adding vertex outside of P_n which will form C_3, denoted by $P_n(C_3)$. Type 2 is obtained from type 1 which insert one vertex x_i in v_iv_{i+1}, denoted by $P_n^*(C_n)$. Type 3 is obtained from type 2 adding a vertex y_i outside of $P_n^*(C_3)$ such that $x_{i-1}y_i$ forms C_3, denoted by $sP_n^*(C_3)$. We Prove that the locating chromatic number of $P_n(C_3)$, $P_n^*(C_n)$ and $sP_n^*(C_3)$ are 4.

References

[1] G Chartrand, D Erwin, M A Henning, P J Slater and P Zhang 2002 The locating chromatic number of a graph *Bull. Inst. Combin*. Appl. **36** 89–101

[2] G Chartrand, P Zhang and E Salehi 1998 On the partition dimension of graph *Congr. Numer*. **130** 157–168

[3] G Chartrand, D Erwin, M A Henning, P G Slaler and P Zhang 2003 Graph of Order *n* with Locating Chromatic Number *n*-1 *DiscreteMath.* **269** 65-79

[4] Asmiati 2016 On the locating-chromatic number of non-homogeneous caterpillars and firecracker graphs *Far East Journal of Mathematical Sciences (FJMS)* **100(8)** 1305-1316

[5] Asmiati, H Assiyatun, E T Baskoro 2011 Locating chromatic number of amalgamation of stars. *ITB J* Sci.A43 1-8.

[6] A Behtoei and B Omoomi 2011 On the locating chromatic number of kneser graphs *Discrete Appl.Math.* **159(18)** 2214–2221

[7] Baskoro E T, Asmiati 2013 Characterizing all trees with locating-chromatic number 3 *Electronic Journal of Graph Theory and Applications I(2)* pp 109-117

[8] E T Baskoro, A Purwasih Ira 2012 The locating chromatic number for corona product of graphs *Soatheast-Asian J. of sciences* **1(1)** 126-136

[9] M Ghanem, H Al-ezeh and A Dabour 2019 Locating chromatic number of powers of path and cycles *Symmetry 11* **389** pp 2-6