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The Development of Combined Gravity-Multinomial Logit Estimated from Traffic Count Under Equilibrium Condition

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Abstract: The model examined was the **Gravity (GR)** model combined with the **Multi-Nomial-Logit (MNL)** model. **Non-Linear-Least- Squares (NLLS)** estimation methods were used to calibrate the parameter of the combined model. Three of the stages of this process (four steps model), trip distribution, modal split and traffic assignment, combine to estimate expected O-D demands, and as such, are of relevance to this research. Iterative solution algorithms, that are modifications of the Newton Raphson and Elimination Gauss-Jourdan techniques, are proposed to solve each of the model formulations.

The procedure described here assumes that the observed network is at equilibrium assignment condition. In other words, the observed link flows and travel times represent the (observed) equilibrium conditions. The objective of the solution approach is to find an O-D matrix such that when this matrix is assigned to the network, the resulting O-D travel times will be equal to the observed O-D travel times.

***Key Words**: Combined Gravity-Multinomial Logit Model, Traffic Count, Equilibrium Assignment*

1. INTRODUCTION

The notion of '**O-D matrix**' has been adopted by transport planners to represent the most important features of this travel pattern. When an O-D matrix is assigned onto the network, a flow pattern will be produced. From examining this, one can identify the problems and some kind of solution may be devised. Therefore, an O-D matrix plays a very important role in various transport studies. Unfortunately, '**conventional methods**' for estimating O-D matrix rely much on extensive surveys and interviews such as roadside interview and home interview carried out in the two large transport planning studies for the city of Jakarta. (Tamin, 2008)

The high cost of manpower and other expenses associated with conventional survey techniques for O-D estimation have motivated the development of models that can inexpensively estimate these flows from easily available traffic link volumes. Approaches for estimate of O-D matrices from traffic counts have been motivated primarily by the practical realities of limited data availability, and relative easy of obtaining link traffic counts compared to more elaborate survey procedures. Thus it is clear that there is a need for the estimation of the O-D matrices, to improve traffic management techniques.

The traffic counts on some particular links, the embodiment and the reflection of the O-D matrix, provide direct information about the sum of all O-D pairs which use those links. Some reasons why traffic counts are so attractive used as a data base are: firstly, they are routinely collected by many authorities and transport planning bodies due to their multiple uses in many transport planning tasks i.e. traffic safety study, monitoring flow level, intersection improvement, etc.

The problem then becomes one of estimating the unknown O-D demands on the basis of the known link use proportions and the observed link traffic flows. It may be characterized as an integrated two-mode traffic equilibrium method. This method combines a zonal aggregate-demand model with an equilibrium-type road assignment and a transit assignment method. The model examined was the **Gravity (GR)** model combined with the **Multi-Nomial-Logit (MNL)** model. **Non-Linear-Least- Squares (NLLS)** estimation methods were used to calibrate the parameter of the combined model. Three of the stages of this process (four step model), trip distribution, modal split and traffic assignment, combine to estimate expected O-D demands, and as such, are of relevance to this research. The separation distance is referred to as the deterrence or impedance function and is often measured by travel time. Three kinds of friction factor functions can be selected: power, exponential, and combined.

Iterative solution algorithms, that are modifications of the Newton Raphson and Gauss Jourdan techniques, are proposed to solve each of the model formulations. The methods proposed in this research are based on the least-squares (LS) estimation technique. In the formulation of the problem in a transportation network, link-flow proportions play a key role. The model that is proposed assumes the sum of the squared deviations between the observed and estimated link flows as the error function that is to be minimized, which are used in the road assignment to calibrate the bus-automobile travel-time relationship. The authors use a new simplified BPR formula to recalibrate the volume-delay curves.

Research development about combination modeling has the important role in transport modeling for use in effective and efficient transport system planning. Route Choice is a major element which has to be considered carefully by travelers as an attempt to minimize their travel time. The main objective of the route choice model is to predict the correct throughput of traffic on each road (flow distribution). (Suyuti, 2005). The previous research still in a burden condition of “All or Nothing” which was assumption that driver who select a route try to minimize its expense, not depend on traffic flow level, so all driver will select the same route. This method is not realistic for some congested road network in urban area because it never consider to the traffic jam effect and various perception in considering of route selection. (Tamin, 2002)

Referring to the previous research, the main objective of the research development is develop of a combined trip distribution-mode choice model estimated from traffic count under equilibrium condition.

As mentioned above, the first problem in estimating O-D matrices is to make sure that the estimated matrix, when assigned to the network, reproduces the observed conditions. The procedure described here assumes that the observed network is at equilibrium. In other words, the observed link flows and travel times represent the (observed) equilibrium conditions. The objective of the solution approach is to find an O-D matrix such that when this matrix is assigned to the network, the resulting O-D travel times will be equal to the observed O-D travel times.

2. GRAVITY-MULTINOMIAL LOGIT MODEL DEVELOPMENT

The total volume of flow ($\hat{V}_l$) in a particular link $\underline{l}$ is the summation of the contributions of all trips interchanges between zones within the study area to that link. Thus the flow on each link is a result of:

- trip interchanges from zone $\underline{i}$ to zone $\underline{d}$ or combination of several types of movement travelling between zones within a study area (=$\underline{T_{id}}$); and
- the proportion of trips travelling from zone $\underline{i}$ to zone $\underline{d}$ whose trips use link $\underline{l}$ which is defined by p_{id}^l ($\mathbf{0} \leq p_{id}^l \leq \mathbf{1}$). (Tamin, 2008)

Mathematically, it can be expressed as follows:

$$V_l^k = \sum_i \sum_d T_{id}^k p_{id}^{lk} \tag{1}$$

The value of p_{id}^l is determined by using trip assignment technique. The previous research (**Tamin *et al*, 2001**) used **all-or-nothing assignment** to obtain the value of p_{id}^l. By using this method, the value of p_{id}^l is either **0** or **1**. In this research, the uses of **equilibrium assignment** method which consider the congestion effect of route choice selection was introduced. Hence, the value of p_{id}^l obtained is between 0 and 1 ($\mathbf{0} \leq p_{id}^l \leq \mathbf{1}$).

Given all the p_{id}^l and all the observed traffic ounts ($\hat{V}_l$), then there will be $\underline{N^2}$ unknown T_{id} 's to estimated from a set of $\underline{L}$ simultaneous linear equations (1) where $\underline{L}$ is the total number of traffic (passenger) counts. In principle, $\underline{N^2}$ independent and consistent traffic counts are required in order to determine uniquely the O-D matrix [T_{id}]. In practice, the number of observed traffic counts is much less than the number of unknown T_{id} 's.

In gravity model, trip distribution is described as accessibility, production and attraction from origin zone to destination zone. Description of accessibility to reach the destination zone in this model is expressed in function of traveling cost or impedance function. The Model is inspired by analogy with Newton's law of gravitational force. Gravity model can be expressed as:

$$T_{id} = \sum_m \left(O_i^m . D_d^m . A_i^m . B_d^m . f_{id}^m \right) \tag{2}$$

where: A_i^m and B_d^m = the balancing factors expressed as:

$$A_i^m = \left[\sum_d \left(B_d^m . D_d^m . f_{id}^m \right) \right]^{-1} \text{ and } B_d^m = \left[\sum_i \left(A_i^m . O_i^m . f_{id}^m \right) \right]^{-1} \tag{3}$$

With exponential deterrence function as bellow:

Power function: $\boldsymbol{f_{id} = C_{id}^{-\alpha}}$ (4)

Exponential function: $\boldsymbol{f_{id} = e^{-\beta C_{id}}}$ (5)

Tanner function: $\boldsymbol{f_{id} = C_{id}^{-\alpha} . e^{-\beta C_{id}}}$ (6)

By subtituting equation (2) to equation (1), then **the fundamental equation** for the estimation of a transport demand model from traffic counts can be expressed as:

$$\boldsymbol{V_l = \sum_i \sum_d \left(O_i . D_d . A_i . B_d . f_{id} . p_{id}^l \right)} \tag{7}$$

This process would be iterated until the values of A_i^m and B_d^m converge to certain unique values. The fundamental equation (7) has been used by many literatures not only to estimate the O-D matrices but also to calibrate the transport demand models from traffic count information (see Tamin, 1988,1989,2008).

The Logit model estimates the proportion of trips by a special mode ***m*** according to the relative utility of each mode as a summation of each modal attribute. The most general and simplest mode choice model (Multi-Nomial Logit Model) was used in this study. It can be expressed as:

$$\boldsymbol{T_{id}^k = T_{id} \cdot \frac{\exp\left(-\gamma . C_{id}^k\right)}{\sum_m \exp\left(-\gamma . C_{id}^m\right)}} \tag{8}$$

By substituting equations (2)-(8) to equation (1), then '**the fundamental equation**' for the estimation of a combined transport demand model from traffic counts is:

$$\boldsymbol{V_l^k = \sum_d \sum_i \left[O_i^k . D_d^k . A_i^k . B_d^k . f_{id}^k . p_{id}^{lk} \frac{\exp\left(-\gamma . C_{id}^k\right)}{\sum_m \exp\left(-\gamma . C_{id}^m\right)} \right]} \tag{9}$$

Theoritically, having known the values of $\boldsymbol{\hat{V}_l}$ and $\boldsymbol{p_{id}^l}$, $\boldsymbol{T_{id}}$ can then be estimated. Equation (9) is a system of ***L*** simultaneous equations with only (**2**) unknown parameters β and γ need to be estimated. The problem now is how to estimate the unknown parameters so that the model reproduces the estimated traffic flows as close as possible to the observed traffic counts. The main idea of this method is to estimate the unknown parameter which minimize the sum of the squared differences between the estimated and observed traffic counts. The problem now is:

$$\text{to minimize} \quad \boldsymbol{S} = \sum_{\boldsymbol{l}} \left[\boldsymbol{V_l^{+k} - V_l^k} \right]^2 \tag{10}$$

$\boldsymbol{\hat{V}_l^k}$ = observed traffic flows for mode k $\boldsymbol{V_l^k}$ = estimated traffic flows for mode k

Having substituted (9) to (10), the following set of equation is required in order to find a set of unknown parameter β which minimize equation (11) and (12):

$$\frac{\delta \boldsymbol{S}}{\delta \beta} = \sum_{l} \left[\left(2 \sum_{i} \sum_{d} \boldsymbol{T}_{id}^{k} \cdot \boldsymbol{p}_{id}^{lk} - \boldsymbol{V}_{l}^{k} \right) \left(\frac{\sum_{i} \sum_{d} \delta \boldsymbol{T}_{id}^{k}}{\delta \beta \cdot \boldsymbol{p}_{id}^{lk}} \right) \right] = 0 \tag{11}$$

$$\frac{\partial \boldsymbol{S}}{\partial \gamma} = \sum_{l} \sum_{k} \left[\frac{2}{\dot{\boldsymbol{V}}_{l}^{k}} \left(\sum_{i} \sum_{d} \boldsymbol{T}_{id}^{k} \boldsymbol{p}_{id}^{lk} - \hat{\boldsymbol{V}}_{l}^{k} \right) \left(\sum_{i} \sum_{d} \frac{\partial \boldsymbol{T}_{id}^{k}}{\partial \gamma} \boldsymbol{p}_{id}^{lk} \right) \right] = 0 \tag{12}$$

$$\frac{\partial^{2} \boldsymbol{S}}{\partial \beta^{2}} = \sum_{l} \sum_{k} \left[\frac{2}{\dot{\boldsymbol{V}}_{l}^{k}} \left\{ \left(\sum_{i} \sum_{d} \boldsymbol{T}_{id}^{k} \boldsymbol{p}_{id}^{lk} - \hat{\boldsymbol{V}}_{l}^{k} \right) \left(\sum_{i} \sum_{d} \frac{\partial^{2} \boldsymbol{T}_{id}^{k}}{\partial \beta^{2}} \boldsymbol{p}_{id}^{lk} \right) + \left(\sum_{i} \sum_{d} \frac{\partial \boldsymbol{T}_{id}^{k}}{\partial \beta} \boldsymbol{p}_{id}^{lk} \right)^{2} \right\} \right] \tag{13}$$

$$\frac{\partial^{2} \boldsymbol{S}}{\partial \gamma^{2}} = \sum_{l} \sum_{k} \left[\frac{2}{\dot{\boldsymbol{V}}_{l}^{k}} \left\{ \left(\sum_{i} \sum_{d} \boldsymbol{T}_{id}^{k} \boldsymbol{p}_{id}^{lk} - \hat{\boldsymbol{V}}_{l}^{k} \right) \left(\sum_{i} \sum_{d} \frac{\partial^{2} \boldsymbol{T}_{id}^{k}}{\partial \gamma^{2}} \boldsymbol{p}_{id}^{lk} \right) + \left(\sum_{i} \sum_{d} \frac{\partial \boldsymbol{T}_{id}^{k}}{\partial \gamma} \boldsymbol{p}_{id}^{lk} \right)^{2} \right\} \right] \tag{14}$$

$$\begin{aligned} \frac{\partial^{2} \boldsymbol{S}}{\partial \beta \partial \gamma} = \sum_{l} \sum_{k} \Bigg[\frac{2}{\dot{\boldsymbol{V}}_{l}^{k}} \Bigg\{ & \left(\sum_{i} \sum_{d} \boldsymbol{T}_{id}^{k} \boldsymbol{p}_{id}^{lk} - \hat{\boldsymbol{V}}_{l}^{k} \right) \left(\sum_{i} \sum_{d} \frac{\partial^{2} \boldsymbol{T}_{id}^{k}}{\partial \beta \partial \gamma} \boldsymbol{p}_{id}^{lk} \right) \\ & + \left(\sum_{i} \sum_{d} \frac{\partial \boldsymbol{T}_{id}^{k}}{\partial \beta} \boldsymbol{p}_{id}^{lk} \right) \left(\sum_{i} \sum_{d} \frac{\partial \boldsymbol{T}_{id}^{k}}{\partial \gamma} \boldsymbol{p}_{id}^{lk} \right) \Bigg\} \Bigg] \end{aligned} \tag{15}$$

Those equations have two (2) unknown parameters $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ need to be estimated. Then it is possible to determine uniquely all the parameters, provided that L>1. Newton–Raphson's method combined with the Gauss–Jordan Matrix Elimination technique can then be used to solve this equation

3. VOLUME DELAY FUNCTION

The separation distance is referred to as the deterrence or impedance function and is often measured by travel time. All current user and system equilibrium assignment formulations require that the relationships of link travel time and link flow be known a priori and be a function. Most researchers have assumed a BPR (Bureau of Public Roads, in Heinz Spiess**,** 1997) form of the travel time – flow relationship. However, for congested traffic networks, this relationship often does not accurately reflect the travel times experienced by drivers. This error is particularly evident when considering the problem of estimating dynamic O-D demands.

$$\boldsymbol{S}_{a}(\boldsymbol{v}_{a}) = \boldsymbol{d}_{a} \boldsymbol{t}_{0} \left[1 + \alpha \left(\boldsymbol{v}_{a} / \boldsymbol{c}_{a} \right)^{\beta} \right] \tag{16}$$

where

d_a = the link length,

v_a = the link volume,

c_a = the practical capacity of the link

t_o can be obtained from the road network data.

4. NEWTON–RAPHSON'S COMBINE WITH GAUSS-JOURDAN

Newton–Raphson method is an efficient algorithm for finding approximations to the zeros (or roots) of a real-valued function. As such, it is an example of a root-finding algorithm.

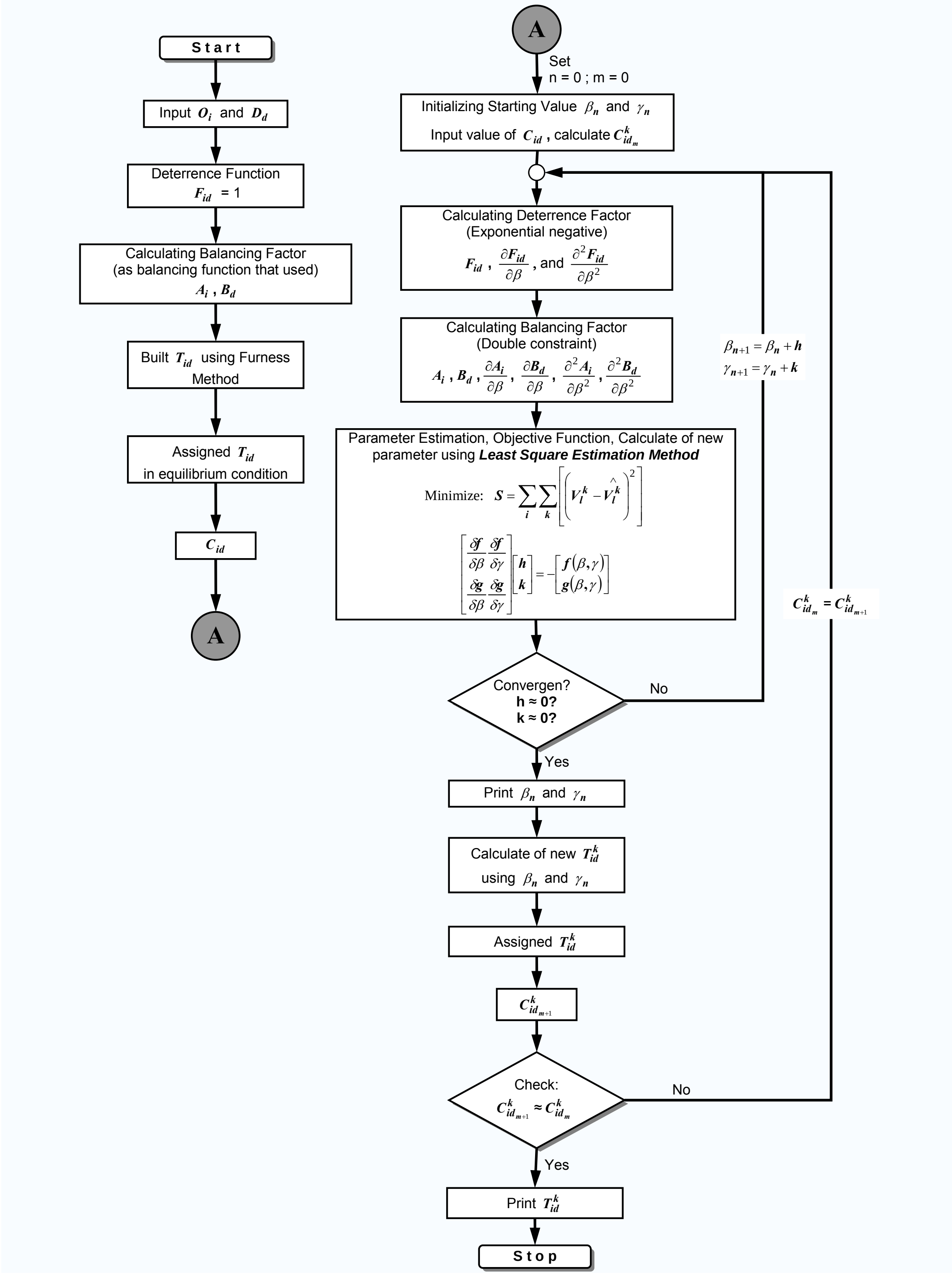


Figure 1 Newton–raphson's combine with the gauss–jourdan matrix elimination technique

It produces iteratively a sequence of approximations to the root, their rate of convergence to the root is quadratic. It can also be used to find a minimum or maximum of such a function, by finding a zero in the function's first derivative. (Weisstein and Eric W.,2008). For further explanation, is shown as appendix bellow.

5. ASSIGNMENT PROCEDURE

The all-or-nothing assignment allocates the entire trips for all O-D pairs to their relative shortest paths, which are calculated based on free flow link travel time. The flow on a link is the sum of a1ll O-D flows that include this link on their shortest paths. All-or-nothing assignment assumes that all drivers traveling between a pair of zones select the same path. Obviously, it is not realistic. In reality, when all trips are assigned to a path included in shortest paths, it is initially the best one. This path may increase travel time because of the assignment trips, so some of trips on this path will have an incentive to select a new less travel time path.

The widely used traffic assignment models perform equilibrium assignment. In an equilibrium assignment, there are usually several equally good paths through the network for each original-destination pair. Although various equilibrium assignment methods can be found in technical literature, all of them were developed based on two traffic assignment principles developed by Wardrop. (in Boyce D and Florian M, 2005). Wardrop’s first principle can be stated as that drivers may try to choose the paths to minimize their own travel times through the network. This is also called user optimization principle or user equilibrium principle. Wardrop’s second principle can be stated as that traffic is assigned in such a manner as to minimize the total travel time of all drivers in the whole network. This is also called system optimization principle.

During the regular equilibrium auto assignment, the additional demand matrix is added to the auto demand matrix, after the latter has been converted into vehicles using the (optional) auto occupancy matrix. The additional demand matrix is optional, and indeed, many applications do not use it. The additional volumes are used to model a fixed background volume on the auto network which is independent of the auto route choice. Additional volumes are computed in the preparation phase of the assignment (module 5.11) and originate from a user link data item and/or the number of transit vehicles on the auto network, converted to auto equivalents (EMME/3 auto assignment). The results of the auto assignment are: auto volumes on links and turns, auto times on links and turns and auto times per OD pair.

6. METHODOLOGY

The methodology of this research can be seen on **figure 2.**

a. Problem identification and the needs for the model
b. Model Development including review of literature, determination of model specifications and development of model structures as well as programmes
    - Preliminary Study
    - Variable Input
    - Modeling Process

    Which basically is the combination, modification and extension of the existing models.
c. Verification
    - Verification in artificial network

- Preliminary testing

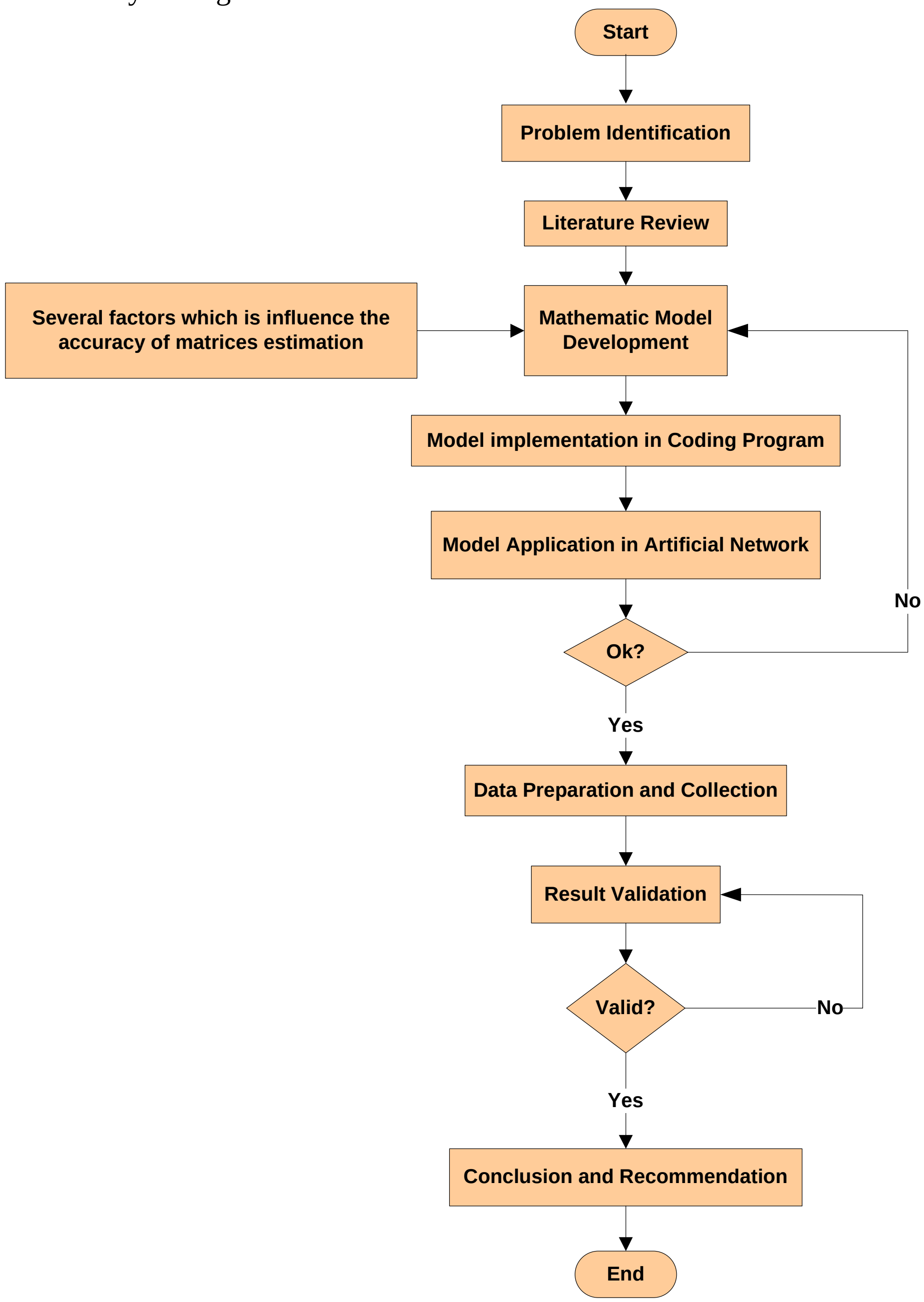


Figure 2 Research methodology

d. Model Implementation
Concerned with the model's parameters setting and computation of actual cases. Parameter settings of the model, followed by the application of the model for several cases depending on the model's objective. Further extension and implementation of the model is described and discussed

e. Conclusion and Recommendation
Concludes the study with a brief summary of the modelling process and implementation results, along with the recommendation for further researches.

7. APPLICATION IN ARTIFICIAL NETWORK

The model has been tested in artificial network consisting of four zones and 42 links representing the road network. The region is divided into a set of zones, with each zone represented by a centroid node.Therefore, there will be (5x5=25) number of p_{id}^{l} for each link and (42x25=1050) number of p_{id}^{l} for total number of available links within the study area. There are two mode, bus and car. The data as input for estimation process are: O_i, D_d, and network system.

Table 1 $C_{id}1$ Car

Zone	901	902	903	904	1001	1002
901	0	6	10	7	8	14
902	6	0	5	11	12	10
903	9	5	0	8	15	6
904	7	11	8	0	9	12
1001	8	12	15	9	0	19
1002	14	10	6	12	19	0

Table 2 $C_{id}2$ Transit

Zone	901	902	903	904	1001	1002
901	0	7	10	5	9	16
902	5	0	8	13	14	12
903	9	6	0	9	15	8
904	8	12	9	0	12	14
1001	7	12	9	12	0	14
1002	15	11	8	14	20	0

Table 3 O_i and D_d value

Zone	O_i	D_d
901	575	720
902	395	605
903	785	745
904	610	600
1001	485	495
1002	835	520
Total	**3685**	**3685**

Two modes, auto (au) and transit (tr), are considered to operate over independent networks, so that the generalized costs associated with each mode are separate. The costs of the transit mode are fixed and given by a timetable and fare schedule.

Giving value for β and γ and using *Gravity* (GR) model with double constraint, we built *Original O-D Matrices*. This matrix is used to get traffic volume in each link through assignment and to get (p_{id}^{l}) value. We give β value and γ value to built matrices and observed traffic count.

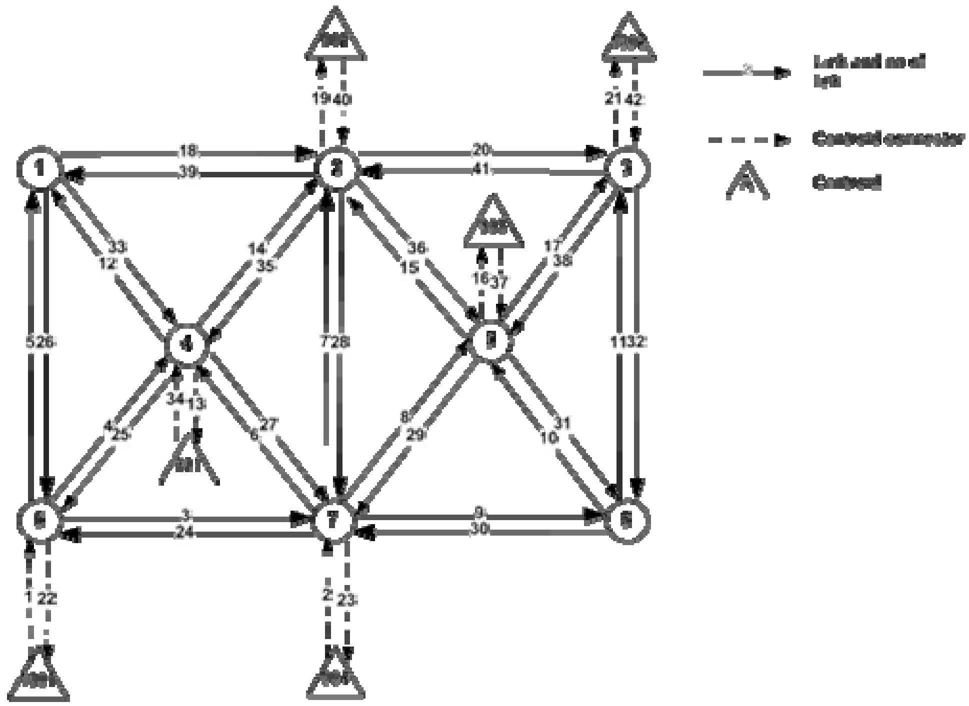


Figure 3 The hypothetical network of car

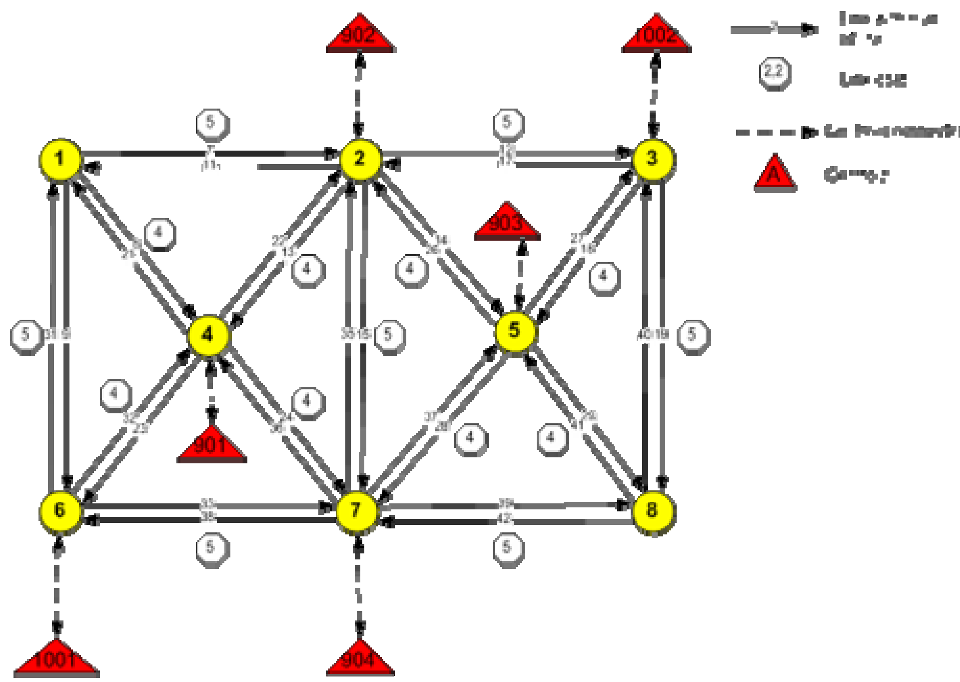


Figure 4 The hypothetical network of bus

8. RESULT

The mathematic model as above is applied in artificial network using write some coding program with *macro* language in EMME/3. The major advantage of EMME/3 is its incorporation of multimodal equilibrium: in all applications both auto and transit-related characteristics can be modeled simultaneously, which closely approximates real-world conditions (i.e. auto and transit modes are competing in an urban environment). This property not only offers the ability to assess the impact of transit services on road networks, but also aids in the identification of more efficient routes for transit services.

Traffic Volume is the result of assignment procedure from the matrices using equilibrium assignment. $\boldsymbol{p}_{id}^{l}$ value is determine in the same stage with procedure of parameter estimated. For artificial data we create traffic observed ($\hat{V}_l$) with giving error factor ± 10 % in traffic volume above. Using Traffic Volume observed and others data (zoning system, network system, Generation and Attraction Data) we run estimation process to get new parameter value.

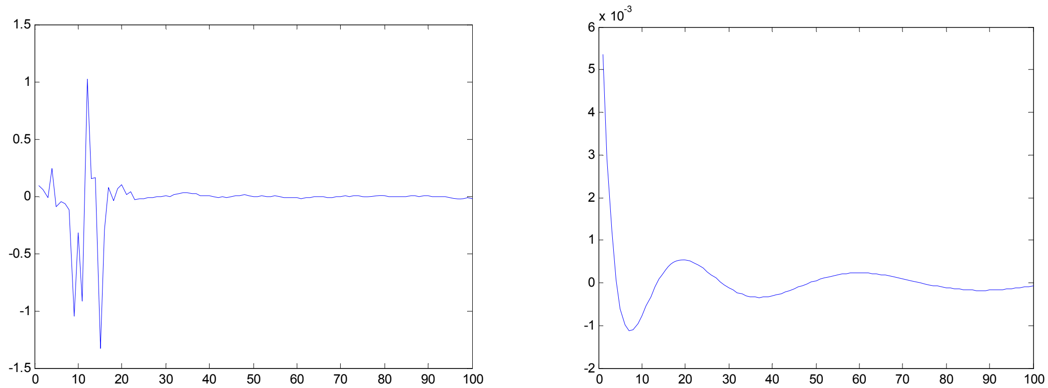
Table 4 Traffic volume observed for car

No	Link No	Link		Error 0 %	Error ± 10 %	
		From	To	Traffic Volume	Traffic Volume	% Error
1	13	2	3	53	50	-6
2	2	2	4	72	77	7
3	9	2	5	70	75	7
4	11	2	7	9	10	11
5	14	3	2	110	103	-6
6	20	3	5	325	318	-2
7	1	4	2	3	3	0
8	5	4	6	4	4	0
9	3	4	7	19	18	-5
10	10	5	2	184	202	10
11	19	5	3	220	239	9
12	17	5	7	38	40	5
13	6	6	4	7	7	0
14	24	6	7	242	244	1
15	12	7	2	12	11	-8
16	4	7	4	19	17	-11
17	18	7	5	31	28	-10
18	23	7	6	262	260	-1

Table 5 Traffic volume observed for transit (bus)

No	Link No	Link		Error 0 %	Error ± 10 %	
		From	To	Traffic Volume	Traffic Volume	% Error
1	6	6	4	237	239	8
2	1	4	2	9	9	0
3	9	2	5	112	112	-7
4	10	5	2	286	266	2
5	2	2	4	79	76	4
6	5	4	6	229	212	9

Iterative solution algorithms are proposed to solve each of the model formulations. It is shown, by way of the application of these iterative algorithms to several example networks, that the estimated O-D demands, which result from these iterative solution techniques, are consistent with the model formulations and with the analytical solutions. Furthermore, it is shown for several examples that, when multiple solutions exist which each exactly replicate the observed link flows. The methods proposed in this research are based on the least-squares (LS) estimation technique. In the formulation of the problem in a transportation network, link-flow proportions play a key role. The model that is proposed assumes the sum of the squared deviations between the observed and estimated link flows as the error function that is to be minimized.



h value   **k** value

Figure 5 Convergence of h and k value

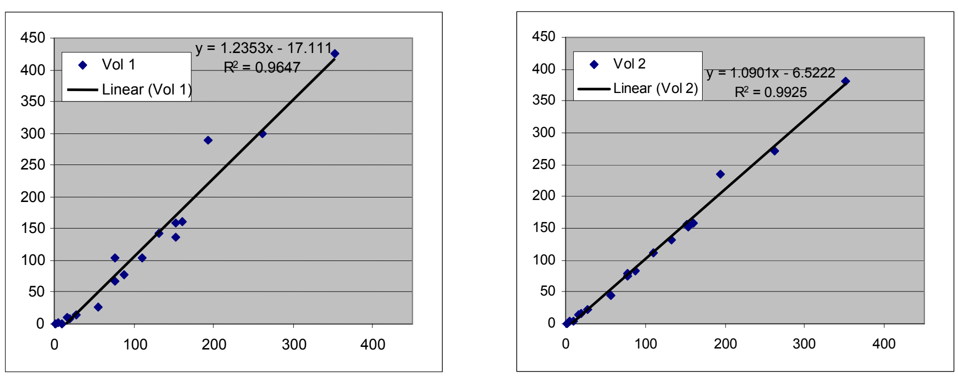


Figure 6 Statistical value of traffic volume observed Vs modeled

The next stage of this research is estimated O-D demands which result from these iterative solution techniques, are consistent with the model formulations and with the analytical solutions. It is hope, by way of the application of these iterative algorithms to several example networks, that the estimated O-D demands, which result from these iterative solution techniques, are consistent with the model formulations and with the analytical solutions.

9. CONCLUSION

In this paper, we have developed a general approach to the explanation of transport flows that combines into a consistent format the trip distribution (gravity), mode choice (multinomial-logit) and trip assignment (equilibrium condition). **Non-Linear-Least- Squares (NLLS)** estimation methods were used to calibrate two unknown parameters of the combined model. We developed a two mode (private car and transit) network equilibrium model where the most important features are the distinction between the flow of vehicles and flow of transit passengers and the means of modeling the interaction between both types of vehicles that use the same road links of the network. It is found that by having the information of passenger flows using bus, we can obtain the O-D matrices for private and bus.

The equilibrium is found by solving a sequence of problems while parametrically varying the equilibrium travel costs of the other mode (transit) whose assignment is determined by an all-or-nothing technique. The number of observed traffic (passenger) counts required is at least as many as the number of parameters. The more link flows you have, the faster the estimation method will converge and also the more accurate the estimated O-D matrix we have.

It is also shown that the TDMC model with negative exponential deterrence function produced estimation parameter (β and γ) for NLLS estimation methods. This result is very important in terms of time and money for estimating the demand of public transport and also for forecasting purposes.

Limitations of time and budget, as well as lack of perfect foresight, tended to reduce the quality of the results below what might have been accomplished otherwise. Nevertheless, the model was estimated and validated in a completely new way from the viewpoint of urban transportation research and practice. Even so, more detailed model testing and implementation studies remain to be accomplished before this model can be regarded as ready for use in practice.

Finally, for such a model formulation and estimation procedure to be applied in professional practice, improved software systems need to be devised. Although much progress has been made in the last decade in software systems for transportation planning, the requirements for this model go well beyond existing systems. Nevertheless, we are optimistic that ongoing improvements will be made available to the professional community in the future.

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APPENDIX

1. Mathematical Modeling

Minimize $S = \sum_{l}\sum_{k}\left[\frac{1}{\dot{V}_l^k}\left(V_l^k - \hat{V}_l^k\right)^2\right]$

f (β , γ) = $\frac{\delta S}{\delta\beta} = \frac{\delta f}{\delta\beta}$ .h + $\frac{\delta f}{\delta\gamma}$ .k

g (β , γ) = $\frac{\delta S}{\delta\gamma} = \frac{\delta g}{\delta\beta}$ .h + $\frac{\delta g}{\delta\gamma}$ .k

$$\frac{\delta f}{\delta\beta} = \frac{\delta^2 S}{\delta\beta^2}\ ;\quad \frac{\delta f}{\delta\gamma} = \frac{\delta^2 S}{\delta\beta\delta\gamma}\ ;\ \frac{\delta g}{\delta\beta} = \frac{\delta^2 S}{\delta\beta\delta\gamma}\ ;\quad \frac{\delta g}{\delta\gamma} = \frac{\delta^2 S}{\delta\gamma}$$

$$T_{id}^k = T_{id}\frac{e^{-\gamma C_{id}^k}}{\sum_m e^{-\gamma C_{id}^m}} \qquad \frac{\partial T_{id}^k}{\partial\beta} = \frac{\partial T_{id}}{\partial\beta}\frac{e^{-\gamma C_{id}^k}}{\sum_m e^{-\gamma C_{id}^m}} \qquad \frac{\partial^2 T_{id}^k}{\partial\beta^2} = \frac{\partial^2 T_{id}}{\partial\beta^2}\frac{e^{-\gamma C_{id}^k}}{\sum_m e^{-\gamma C_{id}^m}}$$

$$\frac{\partial T_{id}^1}{\partial\gamma} = T_{id}.e^{-\gamma C_{id}^1}.e^{-\gamma C_{id}^2}.\frac{(C_{id}^2 - C_{id}^1)}{\left(e^{-\gamma C_{id}^1} + e^{-\gamma C_{id}^2}\right)^2} \qquad \frac{\partial T_{id}^2}{\partial\gamma} = T_{id}.e^{-\gamma C_{id}^1}.e^{-\gamma C_{id}^2}.\frac{(C_{id}^1 - C_{id}^2)}{\left(e^{-\gamma C_{id}^1} + e^{-\gamma C_{id}^2}\right)^2}$$

$$\frac{\delta^2 T^1{}_{id}}{\delta\gamma^2} = T_{id}.\left(C_{id}^2 - C_{id}^1\right)^2.\left(e^{-\gamma C_{id}^1}.e^{-\gamma C_{id}^2}\right).\frac{\left(e^{-\gamma C_{id}^2} - e^{-\gamma C_{id}^1}\right)}{\left(e^{-\gamma C_{id}^1} + e^{-\gamma C_{id}^2}\right)^3}$$

$$\frac{\delta^2 T^2{}_{id}}{\delta\gamma^2} = T_{id}.\left(C_{id}^1 - C_{id}^2\right)^2.\left(e^{-\gamma C_{id}^1}.e^{-\gamma C_{id}^2}\right).\frac{\left(e^{-\gamma C_{id}^1} - e^{-\gamma C_{id}^2}\right)}{\left(e^{-\gamma C_{id}^1} + e^{-\gamma C_{id}^2}\right)^3}$$

$$\frac{\delta^2 T^1{}_{id}}{\delta\beta\delta\gamma} = \frac{\delta T_{id}}{\delta\beta}.\frac{\left(e^{-\gamma C_{id}^1}.e^{-\gamma C_{id}^2}\right)\left(C_{id}^2 - C_{id}^1\right)}{\left(e^{-\gamma C_{id}^1} + e^{-\gamma C_{id}^2}\right)^2} \qquad \frac{\delta^2 T^2{}_{id}}{\delta\beta\delta\gamma} = \frac{\delta T_{id}}{\delta\beta}.\frac{\left(e^{-\gamma C_{id}^1}.e^{-\gamma C_{id}^2}\right)\left(C_{id}^1 - C_{id}^2\right)}{\left(e^{-\gamma C_{id}^1} + e^{-\gamma C_{id}^2}\right)^2}$$

$$\frac{\partial T_{id}}{\partial\beta} = [O_i D_d]\left\{A_i f_{id}\frac{\partial B_d}{\partial\beta} + B_d f_{id}\frac{\partial A_i}{\partial\beta} + A_i B_d\frac{\partial f_{id}}{\partial\beta}\right\}$$

$$\frac{\partial^2 T_{id}}{\partial\beta^2} = [O_i D_d]\left\{\left(B_d f_{id}\frac{\partial^2 A_i}{\partial\beta^2}\right) + 2\left(\frac{\partial B_d}{\partial\beta}f_{id} + B_d\frac{\partial f_{id}}{\partial\beta}\right)\frac{\partial A_i}{\partial\beta} + \left(B_d\frac{\partial^2 f_{id}}{\partial\beta^2} + 2\frac{\partial B_d}{\partial\beta}\frac{\partial f_{id}}{\partial\beta} + \frac{\partial^2 B_d}{\partial\beta^2}f_{id}\right)A_i\right\}$$

$$\frac{\partial A_i}{\partial\beta} = -(A_i)^2\left[\sum_d\left\{D_d\left(B_d\frac{\partial f_{id}}{\partial\beta} + f_{id}\frac{\partial B_d}{\partial\beta}\right)\right\}\right] \qquad \frac{\partial B_d}{\partial\beta} = -(B_d)^2\left[\sum_i\left\{O_i\left(A_i\frac{\partial f_{id}}{\partial\beta} + f_{id}\frac{\partial A_i}{\partial\beta}\right)\right\}\right]$$

$$\frac{\partial^2 A_i}{\partial\beta^2} = 2(A_i)^3\left[\sum_d\left\{D_d\left(B_d\frac{\partial f_{id}}{\partial\beta} + f_{id}\frac{\partial B_d}{\partial\beta}\right)\right\}\right]^2 - (A_i)^2\left[\sum_d\left\{D_d\left(B_d\frac{\partial^2 f_{id}}{\partial\beta^2} + 2\frac{\partial B_d}{\partial\beta}\frac{\partial f_{id}}{\partial\beta} + f_{id}\frac{\partial^2 B_d}{\partial\beta^2}\right)\right\}\right]$$