



## The $X[[S]]$ -Sub-Exact Sequence of Generalized Power Series Rings

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### Abstract

Let  $R$  be a ring,  $(S, +, \leq)$  a strictly ordered monoid, and  $K, L, M$  are  $R$ -modules. Then, we can construct the Generalized Power Series Modules (GPSM)  $K[[S]]$ ,  $L[[S]]$ , and  $M[[S]]$ , which are the module over the Generalized Power Series Rings (GPSR)  $R[[S]]$ . In this paper, we investigate the property of  $X[[S]]$ -sub-exact sequence on GPSM  $L[[S]]$  over GPSR  $R[[S]]$ .

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### Introduction

A non-empty set of  $S$  with an associative binary " $*$ " is called a semigroup. If  $S$  has an identity element, then  $(S, *)$  is called a monoid. Furthermore, if each element of  $S$  has an inverse, then  $(S, *)$  is called a group (Howie 1995). A ring  $(R, +, \cdot)$  is a non-empty set of  $R$  with two binary operations.  $(R, +)$  is a commutative group,  $(R, \cdot)$  a semigroup, and satisfies the left and right distributive laws (Adkins and Weintraub 1992).

One example of a ring is the polynomial ring  $R[X]$ , which is defined as the set of all functions from non-negative integers  $\mathbb{N} \cup \{0\}$  to ring  $R$  with finite support. Furthermore, this ring is generalized into the power series ring  $R[[X]]$  by removing the finite support conditions (Hungerford 1974). Furthermore, the polynomial ring  $R[X]$  can be generalized by changing its function domain to any  $S$  semigroup. This ring is then known as the semigroup ring and is denoted by  $R[S]$  (Gilmer 1984).

A partially ordered relation is a binary relation " $\leq$ " on a non-empty set of  $S$  that fulfills reflexive, anti-symmetric, and transitive properties. Furthermore,  $(S, \leq)$  is called a partially ordered set. An order " $\leq$ " is said to be trivial if for any  $s, t \in S, s \leq t$  results in  $s = t$  and is said to be strictly ordered if  $(\forall x, y, s \in S)(x < y \rightarrow x + s < y + s)$ . Furthermore,  $(S, \leq)$  is said to be Artinian if it does not contain any infinite strictly decreasing sequence  $s_1 > s_2 > s_3 > \dots$ , and is said to be narrow if it does not contain an infinite subset consisting of pairwise incomparable elements. (Elliott and Ribenboim 1990).

By using the Artinian and narrow partially ordered set concept, ring semigroup  $R[S]$  can be generalized into a Generalized Power Series Ring (GPSR) by weakening the finite support condition that became Artinian and narrow. Furthermore, this ring is denoted by  $R[[S, \leq]]$  or abbreviated as  $R[[S]]$  (Ribenboim 1990). Furthermore, the research results relating to the properties that apply in GPSR can be seen in ((Ribenboim 1991), (Ribenboim 1992), (Priess-Crampe and Ribenboim 1993), (Benhissi and Ribenboim 1993), (Ribenboim 1994), (Ribenboim 1995).

Furthermore, the structure of GPSR  $R[[S]]$  can be generalized by applying a monoid homomorphism  $\omega: S \rightarrow \text{End}(R)$  to the convolution multiplication operation (Mazurek and Ziembowski 2007). This ring is called the Skew Generalized Power Series Ring (SGPSR), and

it is denoted by  $R[[S, \omega]]$ . The properties related to the structure of SGPSR  $R[[S, \omega]]$  can be seen in ((Mazurek and Ziemkowski 2008); (Mazurek and Ziemkowski 2009); (Mazurek and Ziemkowski 2010), (Faisol 2009), (Faisol 2013), (Faisol 2014), (Faisol, Surodjo, and Wahyuni 2016), (Faisol, Surodjo, and Wahyuni 2018), (Faisol and Fitriani 2019).

It is known that a ring can be seen as a module over itself. Based on this, we can form the Generalized Power Series Module (GPSM)  $M[[S]]$ , which is a module over GPSR  $R[[S]]$  where  $M$  is a module over the ring  $R$  (Varadarajan 2001a). In addition to the GPSM  $M[[S]]$  structure, the necessary and sufficient conditions of  $M[[S]]$  to be Noetherian module over  $R[[S]]$  can be seen in (Varadarajan 2001b). Furthermore, the generalization of Noetherian property on GPSM  $M[[S]]$  can be seen in (Faisol, Surodjo, and Wahyuni 2019a), which is about the necessary and sufficient conditions of GPSM  $M[[S]]$  is a  $T[[S]]$ -Noetherian module. This is obtained by generalizing the necessary and sufficient conditions for the polynomial module  $M[[X]]$  to be  $S[X]$ -Noetherian module over polynomial ring  $R[X]$  (Faisol, Surodjo, and Wahyuni 2019c), and applies the relationship between almost generated module, almost Noetherian module and  $T$ -Noetherian module (Faisol, Surodjo, and Wahyuni 2019b).

The Noetherian properties of an  $R$ -module  $M$  can be investigated through an exact sequence. If there is an exact sequence  $A \xrightarrow{f} B \xrightarrow{g} C$  where  $A$  and  $C$  are Noetherian, then  $B$  is a Noetherian  $R$ -module (Wisbauer 1991). The generalization of the exact sequence in the  $R$ -module is investigated by (Davvaz and Parnian-Garamaleky 1999). This result is obtained by replacing submodule  $0$  with submodule  $U \subseteq C$ , called the  $U$ -exact sequence. Another study related to the properties of the  $U$ -exact sequence can be seen in ((Davvaz and Shabani-Solt 2002) (Anvariye and Davvaz 2005)).

Motivated by the  $U$ -exact sequence definition, the  $X$ -sub-exact sequence concept was introduced in (Fitriani, Surodjo, and Wijayanti 2016), which is a generalization of the exact sequence. Besides that, the generalization of an  $R$ -module generator to become a  $U$ -generator has been reviewed in (Fitriani, Wijayanti, and Surodjo 2018b). Furthermore, by using the concept of sub-linearly independent modules (Fitriani, Surodjo, and Wijayanti 2017), a basis and free module relative to a family of modules over  $R$  can be defined (Fitriani, Wijayanti, and Surodjo 2018a).

It was explained earlier that Varadarajan determines the necessary and sufficient conditions of GPSM  $M[[S]]$  is a Noetherian  $R[[S]]$ -module; this will be easier to do using the exact sequence concept. Therefore, this motivates us to study the exact sequence of  $R[[S]]$ -modules and construct  $X[[S]]$ -sub-exact sequence on GPSM  $M[[S]]$ . Besides, this also provides an opportunity to investigate the properties that satisfy them.

## The Research Methods

The research methods are based on the study of literature. They relate to the concept of partially ordered set, strictly ordered monoid, Artinian and narrow properties, generalized power series rings (GPSR), generalized power series modules (GPSM), exact-sequences, and  $X$ -sub-exact sequences. The results of this study obtained by constructing the exact sequence and  $X[[S]]$ -sub-exact sequence over an  $R[[S]]$ -module, as well as investigating the properties that apply in it.

**The Results of the Research and the Discussion**

Before discussing the definition and properties of the  $X[[S]]$ -sub-exact sequence, the following is explained about the structure of GPSM  $M[[S]]$  over GPSR  $R[[S]]$  as well as the exact and  $X$ -sub-exact sequence definition, which have been explained in ((Ribenoim 1990), (Varadarajan 2001a), (Wisbauer 1991), dan (Fitriani et al. 2016)).

We were given a strictly ordered monoid  $(S, +, \leq)$  and commutative ring  $R$  with unit element 1. Next, is defined as the set  $R[[S]] = \{f: S \rightarrow R \mid \text{supp}(f) \text{ Artin dan } \textit{narrows}\}$ , with  $\text{supp}(f) = \{s \in S \mid f(s) \neq 0\}$ . Against the operation of the addition function:

$$(f + g)(s) = f(s) + g(s)$$

and convolution multiplication operations:

$$(f \cdot g)(s) = \sum_{t+u=s} f(t)g(u),$$

for each  $s \in S, t \in \text{supp}(f), u \in \text{supp}(g)$  and  $f, g \in R[[S]]$ , it can be shown  $(R[[S]], +, \cdot)$  is a ring. Furthermore, this ring is called the Generalized Power Series Ring (GPSR).

Furthermore, if given an  $R$ -module  $M$ , then the set  $M[[S]] = \{\alpha: S \rightarrow M \mid \text{supp}(\alpha) \text{ Artin dan } \textit{narrows}\}$  can be formed. Against the operation of the addition function :

$$(\alpha + \beta)(s) = \alpha(s) + \beta(s)$$

and scalar multiplication operations:

$$(\alpha \cdot f)(s) = \sum_{t+u=s} \alpha(t)f(u),$$

for each  $s \in S, t \in \text{supp}(\alpha), u \in \text{supp}(f), f \in R[[S]]$ , and  $\alpha, \beta \in M[[S]]$ , it can be shown that  $M[[S]]$  is an  $R[[S]]$ -module. This module is called the Generalized Power Series Module (GPSM).

The following is the definition of the exact sequence and  $X$ -sub-exact sequence over an  $R$ -modules. Let  $R$  be a ring and  $M_i$  an  $R$ -module for each  $i$ .  $R$ -module sequence

$$\dots \rightarrow M_{i-1} \xrightarrow{f_i} M_i \xrightarrow{f_{i+1}} M_{i+1} \rightarrow \dots$$

is said to be exact in  $M_i$  if there are  $R$ -homomorphism  $f_i$  and  $f_{i+1}$  that satisfies  $\text{Im}(f_i) = \text{Ker}(f_{i+1})$ . The sequence is said to be exact if it is exact at every  $M_i$ .

Furthermore, this exact sequence is generalized to the  $X$ -sub-exact sequence. Suppose  $K, L, M$  are modules over  $R$  and  $X$  is a submodule of  $L$ . Triple  $(K, L, M)$  is said to be  $X$ -sub-exact over  $L$  if there are  $R$ -homomorphism  $f$  and  $g$  such that the sequence  $K \xrightarrow{f} X \xrightarrow{g} M$  is the exact sequence over  $R$ -modules.

Next, all submodules  $X$  of  $L$  can be collected, so the triple  $(K, L, M)$  is  $X$ -sub-exact over  $L$ . Furthermore, this set is denoted by  $\sigma(K, L, M)$ . In other words,  $\sigma(K, L, M) = \{X \leq L \mid (K, L, M) \text{ } X\text{-sub-exact over } L\}$ .

Now, we define the exact sequence of GPSR.

**Definition 1.** Let  $R$  be a ring,  $(S, \leq)$  a strictly ordered monoid, and  $M_i$  modules over  $R$  for every  $i$ . Given GPSR  $R[[S]]$  and GPSM  $M_i[[S]]$ . An  $R[[S]]$ -module sequence

$$\dots \rightarrow M_{i-1}[[S]] \xrightarrow{\mu_i} M_i[[S]] \xrightarrow{\mu_{i+1}} M_{i+1}[[S]] \rightarrow \dots$$

is said to be exact in  $M_i[[S]]$  if there are  $R[[S]]$ -homomorphisms  $\mu_i$  and  $\mu_{i+1}$  that satisfy  $\text{Im}(\mu_i) = \text{Ker}(\mu_{i+1})$ . Furthermore, this sequence is said to be exact if it is exact at every  $M_i[[S]]$ .

It is known that, if  $X$  is the submodule of  $M$  over  $R$ , then the set  $X[[S]] = \{\alpha \in M[[S]] \mid \alpha(s) \in X; \forall s \in S\}$  is the submodule of  $M[[S]]$  over  $R[[S]]$ . The following is the definition of  $X[[S]]$ -sub-exact sequence of GPSR.

**Definition 2.** Let  $R$  be a ring,  $(S, \leq)$  a strictly ordered monoid, and  $K, L, M$  are the modules over  $R$ . Given GPSR  $R[[S]]$  and GPSM  $K[[S]], L[[S]]$  and  $M[[S]]$ . If  $X$  is a submodule of  $L$ , the triple  $(K[[S]], L[[S]], M[[S]])$  is said to be  $X[[S]]$ -sub-exact over  $R[[S]]$  if there are  $R[[S]]$ -homomorphisms  $\mu$  and  $\rho$  so that the sequence  $K[[S]] \xrightarrow{\mu} X[[S]] \xrightarrow{\rho} M[[S]]$  is the exact sequence over  $R[[S]]$ .

Based on Definition 2, we can set all  $R[[S]]$ -submodules  $X[[S]]$  of  $L[[S]]$  so that triple  $(K[[S]], L[[S]], M[[S]])$  is  $X[[S]]$ -sub-exact over  $R[[S]]$ . This set is then denoted by  $\sigma(K[[S]], L[[S]], M[[S]])$  or written as  $\sigma(K[[S]], L[[S]], M[[S]]) = \{X[[S]] \leq L[[S]] \mid (K[[S]], L[[S]], M[[S]]) \text{ is } X[[S]]\text{-sub-exact over } R[[S]]\}$ .

Next, the  $X[[S]]$ -sub-exact characteristics of GPSR are given as the main results in this study.

**Proposition 3.** For  $i = 1, 2$ , let  $K_i, L_i, M_i$  are the modules over  $R$ ,  $X_i$  a submodule of  $L_i$ , and  $(S, \leq)$  a strictly ordered monoid. If  $X_1[[S]] \in \sigma(K_1[[S]], L_1[[S]], M_1[[S]])$  and  $X_2[[S]] \in \sigma(K_2[[S]], L_2[[S]], M_2[[S]])$ , then  $X_1[[S]] \times X_2[[S]] \in \sigma(K_1[[S]] \times K_2[[S]], L_1[[S]] \times L_2[[S]], M_1[[S]] \times M_2[[S]])$ .

**Proof:** Because it is known that  $X_1[[S]] \in \sigma(K_1[[S]], L_1[[S]], M_1[[S]])$  and  $X_2[[S]] \in \sigma(K_2[[S]], L_2[[S]], M_2[[S]])$ , then clearly there is  $R[[S]]$ -homomorphism  $\mu_1, \rho_1, \mu_2$ , and  $\rho_2$  so that  $K_1[[S]] \xrightarrow{\mu_1} X_1[[S]] \xrightarrow{\rho_1} M_1[[S]]$  and  $K_2[[S]] \xrightarrow{\mu_2} X_2[[S]] \xrightarrow{\rho_2} M_2[[S]]$  are exact sequences.

Next is defined function  $\mu: K_1[[S]] \times K_2[[S]] \rightarrow X_1[[S]] \times X_2[[S]]$ , where  $\mu((\alpha_1, \alpha_2)) = (\mu_1(\alpha_1), \mu_2(\alpha_2))$ , for each  $(\alpha_1, \alpha_2) \in K_1[[S]] \times K_2[[S]]$  and  $\rho: X_1[[S]] \times X_2[[S]] \rightarrow M_1[[S]] \times M_2[[S]]$ , where  $\rho((\beta_1, \beta_2)) = (\rho_1(\beta_1), \rho_2(\beta_2))$ , for each  $(\beta_1, \beta_2) \in X_1[[S]] \times X_2[[S]]$ .

Based on the definitions of the functions  $\mu$  and  $\rho$ , it can be shown easily that the functions  $\mu$  and  $\rho$  are  $R[[S]]$ -homomorphisms. Therefore, the sequence  $K_1[[S]] \times K_2[[S]] \xrightarrow{\mu} X_1[[S]] \times X_2[[S]] \xrightarrow{\rho} M_1[[S]] \times M_2[[S]]$  is an exact sequence. In other words,  $X_1[[S]] \times X_2[[S]] \in \sigma(K_1[[S]] \times K_2[[S]], L_1[[S]] \times L_2[[S]], M_1[[S]] \times M_2[[S]])$ . □

As a direct result of Proposition 3, the following properties are obtained for a set of indexes  $\Delta$ .

**Corollary 4.** Let  $K_\delta[[S]], L_\delta[[S]], M_\delta[[S]]$  are a family of  $R[[S]]$ -module and  $X_\delta[[S]]$  is a submodule of  $L_\delta[[S]]$  for every  $\delta \in \Delta$ . If  $X_\delta[[S]] \in \sigma(K_\delta[[S]], L_\delta[[S]], M_\delta[[S]])$  for every  $\delta \in \Delta$ , then  $\prod_{\delta \in \Delta} X_\delta[[S]] \in \sigma(\prod_{\delta \in \Delta} K_\delta[[S]], \prod_{\delta \in \Delta} L_\delta[[S]], \prod_{\delta \in \Delta} M_\delta[[S]])$

The following properties show that if triple  $(0, L[[S]], M[[S]])$   $X_1[[S]]$ -sub-exact and dan also  $X_2[[S]]$ -sub-exact, then triple  $(0, L[[S]], M[[S]])$  is  $(X_1[[S]] \cap X_2[[S]])$ -sub-exact over  $R[[S]]$ .

**Proposition 5.** Suppose that  $L$  and  $M$  are modules over  $R$  and  $(S, \leq)$  a strictly ordered monoid. Given  $R[[S]]$ -modules  $L[[S]]$  and  $M[[S]]$ , and  $X_1[[S]], X_2[[S]]$  are submodules of  $L[[S]]$ . If  $X_1[[S]], X_2[[S]] \in \sigma(0, L[[S]], M[[S]])$ , then  $X_1[[S]] \cap X_2[[S]] \in \sigma(0, L[[S]], M[[S]])$ .

**Proof:** Since  $X_1[[S]], X_2[[S]] \in \sigma(0, L[[S]], M[[S]])$ , then there are  $R[[S]]$ -homomorphisms  $\rho_1$  and  $\rho_2$  such that  $0 \rightarrow X_1[[S]] \xrightarrow{\rho_1} M[[S]]$  and  $0 \rightarrow X_2[[S]] \xrightarrow{\rho_2} M[[S]]$  are exact sequences. Therefore,  $\rho_1$  and  $\rho_2$  are  $R[[S]]$ -monomorphisms. Next, it is defined as  $\rho = \rho_1|_{X_1[[S]] \cap X_2[[S]]}$ . Then,  $\rho$  is an  $R[[S]]$ -monomorphism. Therefore,  $0 \rightarrow X_1[[S]] \cap X_2[[S]] \xrightarrow{\rho} M[[S]]$  is an exact sequence. So, it is proved that  $X_1[[S]] \cap X_2[[S]] \in \sigma(0, L[[S]], M[[S]])$ .  $\square$

The properties described in Proposition 5 cause the following properties to take the consequence.

**Corollary 6.** Suppose  $L$  and  $M$  are modules over  $R$ , and  $(S, \leq)$  is strictly ordered monoid. Given GPSM  $L[[S]]$  and  $M[[S]]$  over GPSR  $R[[S]]$ , and  $X_\delta[[S]]$  is a submodule of  $M[[S]]$  for every  $\delta \in \Delta$ . If  $X_\delta[[S]] \in \sigma(0, L[[S]], M[[S]])$  for each  $\delta \in \Delta$ , then  $\bigcap_{\delta \in \Delta} X_\delta[[S]] \in \sigma(0, L[[S]], M[[S]])$

**Example 7.** After the properties related to  $X[[S]]$ -sub-exact sequence of GPSR are given, here are examples:

1. Triple  $(R[X], R[X], 0)$  is  $R$ -sub-exact on  $R[X]$ , where  $R$ -homomorphism  $f: R[X] \rightarrow R$  is defined by

$$f(a_0 + a_1x + \dots + a_nx^n) = a_0$$

and  $g$  is zero mappings, such that  $R[X] \xrightarrow{f} R \xrightarrow{g} 0$  is an exact sequence.

2. Triple  $(R[X], R[X], R[X])$  is a 0-sub-exact on  $R[X]$ , because  $R[X] \xrightarrow{g} 0 \xrightarrow{i} R[X]$  is an exact sequence, where the zero mapping  $g$  and inclusion  $i$  are  $R$ -homomorphisms.

3. If  $I[[S]]$  is ideal of  $R[[S]]$ , then we can form the exact sequence

$$I[[S]] \xrightarrow{i} R[[S]] \xrightarrow{\pi} R[[S]]/I[[S]], \text{ where } i \text{ is an identity and } \pi \text{ a natural homomorphism.}$$

### Conclusion and Suggestion

If given GPSM  $K[[S]], L[[S]], M[[S]]$  over GPSR  $R[[S]]$ , then we can form a set of all submodule  $X[[S]]$  of  $L[[S]]$  so that triple  $(K[[S]], L[[S]], M[[S]])$  is  $X[[S]]$ -sub-exact.

If  $X_\delta[[S]] \in \sigma(K_\delta[[S]], L_\delta[[S]], M_\delta[[S]])$  for each  $\delta \in \Delta$ , then  $\bigcap_{\delta \in \Delta} X_\delta[[S]] \in \sigma(\bigcap_{\delta \in \Delta} K_\delta[[S]], \bigcap_{\delta \in \Delta} L_\delta[[S]], \bigcap_{\delta \in \Delta} M_\delta[[S]])$ . If  $X_\delta[[S]] \in \sigma(0, L[[S]], M[[S]])$  for each  $\delta \in \Delta$ , then  $\bigcap_{\delta \in \Delta} X_\delta[[S]] \in \sigma(0, L[[S]], M[[S]])$ .

In this paper, there are still many opportunities to investigate the characterization of the  $X[[S]]$ -sub-exact sequence of GPSR  $R[[S]]$ . Also, investigating the necessary and sufficient conditions for  $X[[S]]$  to be a Noetherian module over  $R[[S]]$ , where  $K[[S]], M[[S]]$  are Noetherian modules, but  $L[[S]]$  is not Noetherian.

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