

Forecasting Seasonal Time Series Data using The Holt-Winters Exponential Smoothing Method of Additive Models

NURHAMIDAH, NUSYIRWAN, AHMAD FAISOL

Jurusan Matematika, Fakultas MIPA, Universitas Lampung,
Jl. Prof. Dr. Sumantri Brojonegoro No.1 Bandar Lampung 35145
email: nurhamidah247@gmail.com, nusyirwan.1966@fmipa.unila.ac.id,
ahmadfaisol@fmipa.unila.ac.id

Abstract

This study aimed to predict seasonal time series data using the Holt-Winters exponential smoothing additive model. The data used in this study is data on the number of passengers departing at Hasanudin Airport in 2009-2019, the source of the data obtained from the official website of the Central Statistics Agency. The results showed that the Holt-Winters exponential smoothing method on the passenger's number at Hasanudin Airport from 2009 to 2019 contained trend patterns and seasonal patterns by first determining the initial values and smoothing parameters minimize forecasting errors.

Keywords: forecasting, seasonal time series, Holt-Winters, smoothing method, additive models.

1. INTRODUCTION

Forecasting is an activity to predict future events by using and considering data from the past [9]. The use of the forecasting method depends on the data pattern to be analyzed. If the method used is considered correct for forecasting, then the best forecasting method research is based on prediction error [11]. Many statistical methods can forecast time-series data, such as the smoothing method, Box-Jenkins, econometrics, regression, and transfer functions. These methods are expected to identify data used to predict conditions in the future so that the error is as minimal as possible [5].

The exponential smoothing method is forecasting the moving average by weighting it down exponentially to past observations' value (Indriana and Anjasari, 2018). This method consists of two types, including single exponential smoothing and double exponential smoothing. The single exponential smoothing method is used if the time series data does not contain trend and seasonal patterns. In contrast, the double exponential smoothing method is used if the time series data contains trend patterns and does not contain seasonal patterns.

Data patterns can be divided into four types, namely trend patterns, seasonal patterns, cyclic patterns, and irregular patterns [12]. Suppose the stationary and non-stationary data types do not contain seasonal patterns. In that case, forecasting can be done using the moving average method and single and multiple exponential smoothing methods. If the data contains seasonal patterns, the method will produce low forecasts. Therefore, to minimize errors in forecasting results, it is necessary to determine the appropriate seasonal data patterns method. The Holt-Winters smoothing method is one of the appropriate methods to predict data containing seasonal patterns.

In previous research [10], Holt-Winters Additive and Additive Damped smoothing were discussed to predict students' numbers. In this study, a trial was conducted on forecasting to compare the results of the two methods. The test is carried out using the selection of the best model seen from the Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD), Mean Square Deviation (MSD) so that the test results can be seen which method is more appropriate to predict the number of new student enrollments.

Research conducted by [13] has discussed the implementation of the Exponential Smoothing Additive method for predicting sales of office stationery (ATK) on X Stationery, showing that the Triple Exponential Smoothing Additive method is right for making sales analysis in the future. Come off the three-parameter value used; the middle range produces the best accuracy value. The accuracy of the parameter value is 6.8%.

Research conducted by Atalis [2] has discussed comparing the Multiplicative, Additive, and Double Seasonal Holt-Winters methods for predicting car sales, showing that the accuracy of the Holt-Winters forecasting method depends on the model used. The Additive Holt-Winters method is suitable for average models that are relatively constant fluctuating around the average value. In contrast, the Double Seasonal Holt-Winters method is suitable for sales data models with two seasonal patterns. Moreover, research on the Holt-Winters methods can be seen in [1, 8, 15].

This research will discuss seasonal time series data forecasting using the additive Holt-Winters exponential smoothing method. This method has three smoothing stages: actual data smoothing, trend estimation smoothing, and seasonal estimation smoothing. The data used in this research is data the number of passengers departing at Hasanudin Airport in 2009-2019. This data is secondary data obtained from the Central Statistics Agency (BPS).

2. RESEARCH METHODS

This research was conducted by predicting seasonal time series data using the additive Holt-Winters exponential smoothing method, while the steps taken are as follows:

1) Time series data

Time series data is a collection of the observed values of a variable taken at different times. This type of data is collected from certain time intervals, for example, daily, weekly, monthly, and yearly [3].

The time series method is a forecasting method using an analysis of the relationship between the variables to be estimated and the time variable. Things that need to be considered in forecasting are errors that cannot be separated in the forecasting method. Forecasting results will be better if the error value is getting smaller. Time series data can be used as a basis for current decision-making, forecasting the state of time series data in the future, and planning future planning activities.

2) Seasonal data test using a seasonal index

Seasonal variations are periodic variations. That is, there are repetitions at specific periods for each year. Seasonal variations can occur in one year, one month, one week, or in one day [14]. To find the seasonal index, we can use a simple average method, namely the formula

$$\text{Seasonal index} = \left(\frac{\bar{X}_i}{\bar{X}_j} \times 100\% \right) \times 12 \tag{1}$$

Where \bar{X}_i is the average data of the i -th month each year ($i = 1, 2, 3, \dots, 12$) and \bar{X}_j is the average data of each month in year j ($j = 1, 2, \dots, n$).

3) Specifies the initial value

The initial value is the value used to estimate the initial value of the coefficient ($S_0, b_0, ;i_k$).

The following are the methods used to determine the initial value:

- The initial value for exponential smoothing

$$S_0 = \frac{1}{L} (X_1 + X_2 + \dots + X_k) \quad (2)$$

with:

S_0 = The initial value of exponential smoothing

X_k = Data k

L = Seasonal period data ($L = 3, L = 4, L = 6$, or $L = 12$)

- Initial values for trend smoothing

$$b_0 = \frac{1}{k} \left[\frac{X_{L+1} - X_1}{L} + \frac{X_{L+2} - X_2}{L} + \dots + \frac{X_{L+k} - X_k}{L} \right] \quad (3)$$

with:

b_0 = Initial values for trend factors

X_k = Data k

L = Seasonal period data ($L = 3, L = 4, L = 6$, or $L = 12$)

- Initial values for additive model seasonal smoothing

$$I_k = X_k - S_0 \quad (4)$$

with

I_k = Initial value for the k-season factor

k = Seasonal period ($k = 1, 2, \dots, L$)

4) Determine guesswork α, β, γ

The coefficient has a distance between 0 and 1, determined subjectively, or minimizes the estimate's error [7]. The greater the number of constants, the forecasting process will take a long time because it will perform more iterations. This forecasting system will combine 3 (three) parameters to determine the best endless combination to produce the smallest MAPE and MSE.

5) Computes Holt-Winters' exponential smoothing value and forecast

The Holt-Winter method can handle seasonal factors and trends that appear simultaneously in a time series data. The Holt-Winter method can be used for non-stationary data [4].

Holt Winter additive model exponential smoothing the seasonal additive model with the seasonal addition method is suitable for predicting time series with the amplitude (height) of the seasonal pattern independent of the average level, or data size is constant [6].

Additive models are used when there is no trend or sign that the seasonal pattern is dependent on data size. The equations used in the additive model are as follows:

- The equation for calculating Holt-Winters exponential smoothing

$$S_t = \alpha (X_t - I_{t-L}) + (1 - \alpha) (S_{t-1} + b_{t-1}) \quad (5)$$

with:

S_t = Exponential smoothing in year t

S_{t-1} = Exponential smoothing in year t-1

b_t = Smoothing trend elements in year t

b_{t-1} = Element smoothing trend on year t-1

X_t = Data t

α = Exponential smoothing parameter for data ($0 < \alpha < 1$)

I_t = Seasonal factor smoothing

L = Seasonal length ($L = 3, L = 4, L = 6$ or $L = 12$)

- The equation for calculating the trend

$$b_t = \beta (S_t - S_{t-1}) + (1 - \beta) b_{t-1} \quad (6)$$

with:

- S_t = Exponential smoothing in year t
- S_{t-1} = Exponential smoothing in year t-1
- b_t = Element smoothing trend in year t
- b_{t-1} = Element smoothing trend in year t-1
- β = Exponential smoothing parameter for trend ($0 < \beta < 1$)

- The equation for calculating the seasonal smoothing in the additive model

$$I_t = \gamma(X_t - S_t) + (1 - \gamma)I_{t-L} \tag{7}$$

with:

- S_t = Exponential smoothing in year t
- X_t = Data t
- γ = Exponential smoothing parameter for seasonality ($0 < \gamma < 1$)
- I_t = Seasonal factor smoothing

- For the Holt-Winters exponential smoothing forecast value in the additive model, the following equation is used:

$$F_{t+m} = S_t + mb_t + I_{t-L+m} \tag{8}$$

with:

- S_t = Exponential smoothing in year t
- b_t = Element smoothing trend in year t
- F_{t+m} = Results of forecasting - (t + m)
- F_t = The value want to predict
- I_t = Seasonal factor smoothing
- m = The period to be predicted

3. RESULTS AND DISCUSSION

Data shows that the number of passengers departing planes at Hasanudin Airport increases in July and October every year. Figure 1 is a plot of data on the number of passenger aircraft departures at Hasanudin Airport in 2009-2019.

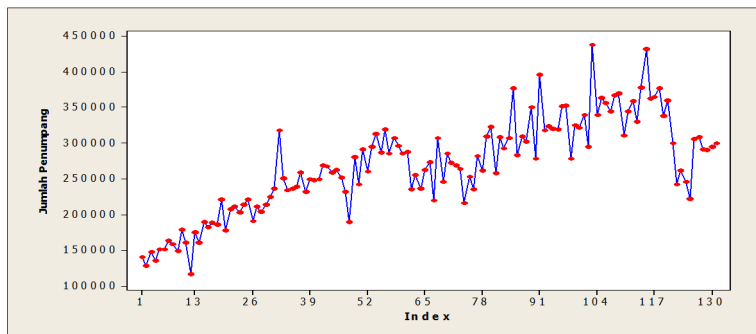


FIGURE 1. The plot of the Number of Aircraft Departure Passengers at Hasanudin Airport in 2009-2019

Results and discussion of the additive Holt-Winters exponential smoothing method begin with creating a data pattern. If the data has an upward trend (trend), it is not stationary to the mean and variance. In Figure 1, it can be seen that the data has a seasonal pattern because it recurs in certain months. It can be seen in the increase in the number of aircraft passengers at Hasanudin Airport every year in July. Next is to determine the length or seasonal period. If the data is based on quarterly, the length or seasonal period is 4. Based on the data, the length or seasonal period is 12 because the data is per month. After the length or seasonal period is obtained, the next step is to determine the initial value that affects the next prediction.

Before forecasting, calculate the initial value for each smoothing with the formula in equations (2.2), (2.3), (2.4). The obtained exponential smoothing initialization value is (S_0) is 148090. Trend initialization (b_0) is 3718.22, and the seasonal factor initialization (I_k) in period 1 to period 12 is respectively $I_1 = -7683$, $I_2 = -20326$, $I_3 = -980$, $I_4 = -12410$, $I_5 = 2972$, $I_6 = 2451$, $I_7 = 15110$, $I_8 = 9606$, $I_9 = 856$, $I_{10} = 30029$, $I_{11} = 12717$, $I_{12} = -32340$. Furthermore, the Holt-Winters exponential smoothing of the additive model is obtained from a combination of the mean (α), trend (β), and seasonal (γ) by trial and error to get the best model.

The number of combinations where the greater the value β and γ the greater the MSE (Mean Square Error) value. The best model parameter needs to be done so that the resulting forecast has the smallest error or is close to the actual value. There are several criteria for selecting the best model, choosing the smallest MSE value.

The greater the number of constants, the forecasting process will take a long time because it will perform more iterations. The calculation of forecasting the Holt-Winters Additive smoothing method is performed repeatedly by combining all three-parameter values. So from the calculation of the values $\alpha, \beta, dan \gamma$ that have been done, obtained 376 combinations of parameter values. Furthermore, from the 376 models formed, the best model was obtained, namely the parameter $\alpha = 0.4$, $\beta = 0$, and $\gamma = 0$ with the MSE value 721794499 with the smallest error value.

Based on the results of the smallest MSE, the model can be expressed with $\alpha = 0.4$, $\beta = 0$, and $\gamma = 0$ is the best model. Based on the results of this analysis, four additive equations for the Holt-Winters exponential smoothing model are obtained in equation (2.5), (2.6), (2.7), (2.8). Exponential smoothing is $S_t = 0.4(X_t - I_{t-12}) + (1 - 0.4)(S_{t-1} + b_{t-1})$. Trend pattern smoothing is $b_t = 0.4(S_t - S_{t-1}) + (1 - 0.4)b_{t-1}$, seasonal patterns smoothing is $I_t = 0.4(X_t - S_t) + (1 - 0.4)I_{t-12}$. Moreover, forecast in the future period, ie. $F_{t+m} = S_t + mb_t + I_{t-L+m}$.

The following is a time series plot for forecasting the number of passengers departing aircraft at Hasanudin Airport.

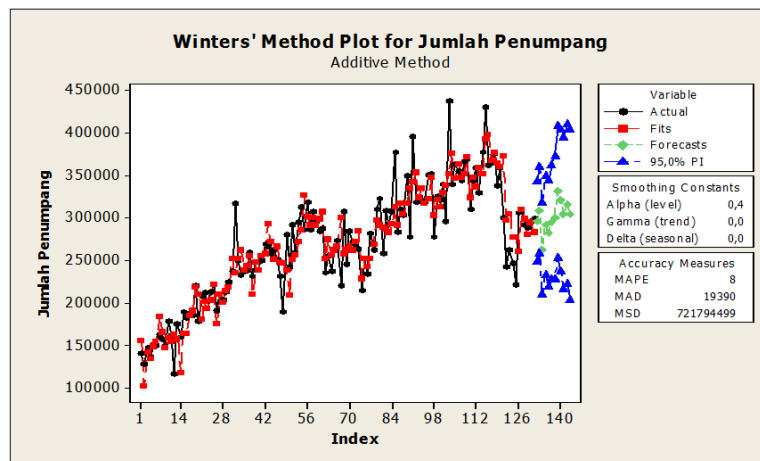


FIGURE 2. Time Series Plot of Passenger Forecasting Aircraft Departures at Hasanudin Airport

It can be seen in Figure 2 that the black line is a plot of data on the number of passengers departing aircraft at Hasanudin Airport from January 2009 to November 2019, while the red line is the data smoothing line. The green line is the plot of the predicted value of the number of passengers departing aircraft at Hasanudin Airport from December 2019 to November 2020.

The following are the results of forecasting the number of aircraft departure passengers at Hasanudin Airport using the Additive Holt-Winters exponential smoothing method for 2020.

TABLE 1. Forecasting the Number of Aircraft Departure Passengers at Hasanudin Airport using the Additive Holt-Winter Method

Month	Forecast
Dec-19	302749
Jan-20	306467
Feb-20	310185
Mar-20	313903
Apr-20	317622
May-20	321340
Jun-20	325058
Jul-20	328776
Aug-20	332495
Sep-20	336213
Oct-20	339931
Nov-20	343649

4. CONCLUSION

Based on the results and discussion that has been carried out, it can be concluded that the Additive Holt-Winters Exponential Smoothing Method in the data on the number of aircraft departing passengers at Hasanudin Airport from 2009 to 2019 contains trend patterns and seasonal patterns by determining the initial values and parameters in advance smoothing that can minimize forecasting errors. The obtained additive Holt-Winters exponential smoothing parameters are $\alpha = 0.4$, $\beta = 0$, $\gamma = 0$ with Mean Square Error (MSE)= 721794499. The results of forecasting the number of passengers departing aircraft at Hasanudin Airport have increased continuously every month. Based on the results and discussions analyzed, Hasanudin Airport should pay attention to the increasing number of passengers departing planes every month and adjusting the required capacity.

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