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Atsnaita Yasrina, and Doni Andra

Theoretical Study of Maxwell's Solution of Magnetic Field Dynamics around Neutron Star in the ZAMO (Zero Angular Momentum Observers) Frame: Accreting and Rapidly Rotating Cases

Atsnaita Yasrina[1, *)], and Doni Andra[2]

[1]*Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Malang, Jl. Semarang No. 5, Malang 65145, Indonesia*
[2]*Physics Education Study Program, Faculty of Teacher Training and Education, Universitas Negeri Lampung, Lampung, Indonesia*

*)Corresponding author: atsnaita.yasrina.fmipa@um.ac.id

Abstract. The neutron star accretes in a binary system. Accretion is hypothesized to cause a magnetic field to decrease in a neutron star. Equation dynamics of neutron star magnetic fields are needed to formulate the relativistic Maxwell equations. Magnetic field dynamic equations are differential equations that require solutions for each radial, polar and azimuthal component. Maxwell's equation has a solution that is assumed to be a bipolar magnetic field. The equation obtained is the equation of the dynamics of the magnetic field in the ZAMO (Zero Angular Momentum Observers) framework by assuming the neutron star is a rapidly rotating and accreting neutron star. This equation corresponds to the slowly rotating neutron star.

INTRODUCTION

A massive star is a star that has a mass $M_* \gtrsim 8M_\odot$ ($M_\odot$ is the mass of the sun). A massive star will explode called a supernova in the final phase of its life. A neutron star is the final phase of a supernova, the core of the star collapses, cool, and has a high density so that it contains many degenerated neutrons [1]. Characteristics of neutron stars have mass and radius $M_* \sim 1-2\ M_\odot$; $R_* \approx 10-14$ km sequentially [2]. Therefore the neutron star has a density of $\rho \simeq (2-3)\rho_o$. The normal density of the nucleus is $\rho_o = 2.8\ \times 10^{14}$ g cm^{-3}[3].

Neutron stars have very large density values, causing the gravitational field on the surface and around the neutron star to become very large. Therefore, the study of neutron stars cannot ignore the relativistic aspects (general relativistic). For example, if we are going to study the phenomenon of diffusion around neutron stars, then we must refer to the general theory of relativity formulated by Einstein. [4,5]. Another example is the observation of electromagnetic fields around slowly-rotating neutron stars [6]. In this context, the general theory of relativity formulated by Einstein, becomes very important. The compactness of a neutron star is characterized by the compactness value of $x_g \ll 1$. In addition, neutron stars include compact objects that rotates rapidly. There are neutron stars that rotate very fast, called pulsars [7].

Pulsars can be known from the emission of electromagnetic waves that is from X-rays and radio waves. The neutron star's magnetic field can be known from the emission of electromagnetic wave radiation [8]. Neutron stars have a strong gravitational field, besides neutron stars is also known as a compact object that has the strongest magnetic field in the universe, for example, the pulsar magnetic field can reach of $B{\sim}10^{15}$G [9]. While in a netutron star, the magnitude of the magnetic field is estimated to reach $\sim 10^{15}G$, some astrophysicists also predict it reaches $\sim 10^{18}G$ [2, 10].

Neutron stars have magnetic fields that can be reduced in value, for example from $\sim 10^{12}$ G to $\sim 10^{8}$ G over 5×10^{6} years [11]. A neutron star in a double system is found to have a reduced magnetic field, such that it is assumed to be

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caused by an accreting neutron star [12-18]. This hypothesis arises from the discovery that in a binary system, the magnetic field decreases in neutron stars [9]. This is supported by the fact that in a binary system, neutron stars carry out the accretion process.

Theoretical study of the effect of accretion on decreasing magnetic fields has been carried out by Cumming, and produced an equation of decreasing the magnetic field that accretes with Ohmic diffusion [14]. However, the study conducted by Cumming is still in the non-relativistic region. While a relativistic study was carried out, and this study produces stationary electromagnetic fields in the Schwarzchild spacetime [19]. Studies related to electric fields in the Schwarzschild metric on neutron stars were carried out, and the results showed that the solution obtained was not a solution to the Maxwell equation [20]. In line with previous findings of Ohmic diffusion, it was obtained that the speed of Ohmic decay in the Schwarschild spacetime does not provide a solution for the internal electromagnetic field of stars in general relativity. Therefore, we need another metric besides the Schwarzschild metric, which is the rotating neutron star metric. The results of Muslimov and Tsygan's studies show that the effect of general relativity is caused by star rotation in the context of slow-rotating neutron stars [21]. Maxwell's equation solution has been obtained for slowly-rotating neutron stars and measured in the ZAMO (Angle Zero Momentum Observer) framework by assuming the neutron star is not accreting. Studies using ZAMO are needed for rotating neutron stars [7,22]. Equation dynamics of the magnetic field for slowly-rotating neutron stars have been obtained, by stuttering accreting neutron stars [23].

A neutron star is the rapidly spinning stars, for example Pulsar. Therefore, it is necessary to formulate a magnetic field dynamic equation in fast-rotating neutron stars. The formulation of the magnetic field dynamic equations in fast-spinning neutron stars has been obtained. The equation obtained has a covariant shape with an equation of the dynamics of the neutron star's rotating magnetic field slowly [24]. The magnetic field dynamics equation is a differential equation, so it requires a solution to show each component of the magnetic field. The solution obtained by assuming the neutron star's magnetic field is dipolar. The equation is obtained by assuming a magnetic field in the ZAMO framework.

METHODS

The first step done in this research was defining a rapidly rotating neutron star metrics (ds^2) to obtain tetrad ($e_{\hat{\mu}}$), 1-form ($w^{\hat{\mu}}$), and four-velocity (u^{μ}). Both of these quantities were used to formulate conductor four-velocity ($\mathcal{w}^{\mu}$), and electromagnetic tensors ($F_{\hat{\mu}\hat{\nu}}$). The conductor four-velocity ($\mathcal{w}^{\mu}$) for formulating a four-current ($J^{\hat{\mu}}$). Electromagnetic tensors and four-current were used to formulate Maxwell's equations in the ZAMO framework. Equation of magnetic field dynamics was obtained by formulating Maxwell's equations. The magnetic field dynamic equation is a differential equation, so it is necessary to formulate the value of each magnetic component ($B^{\hat{r}}, B^{\hat{\theta}}, B^{\hat{\varphi}}$) to obtain a solution of the magnetic field dynamic equation. The procedure of this study is as shown in Fig. 1.

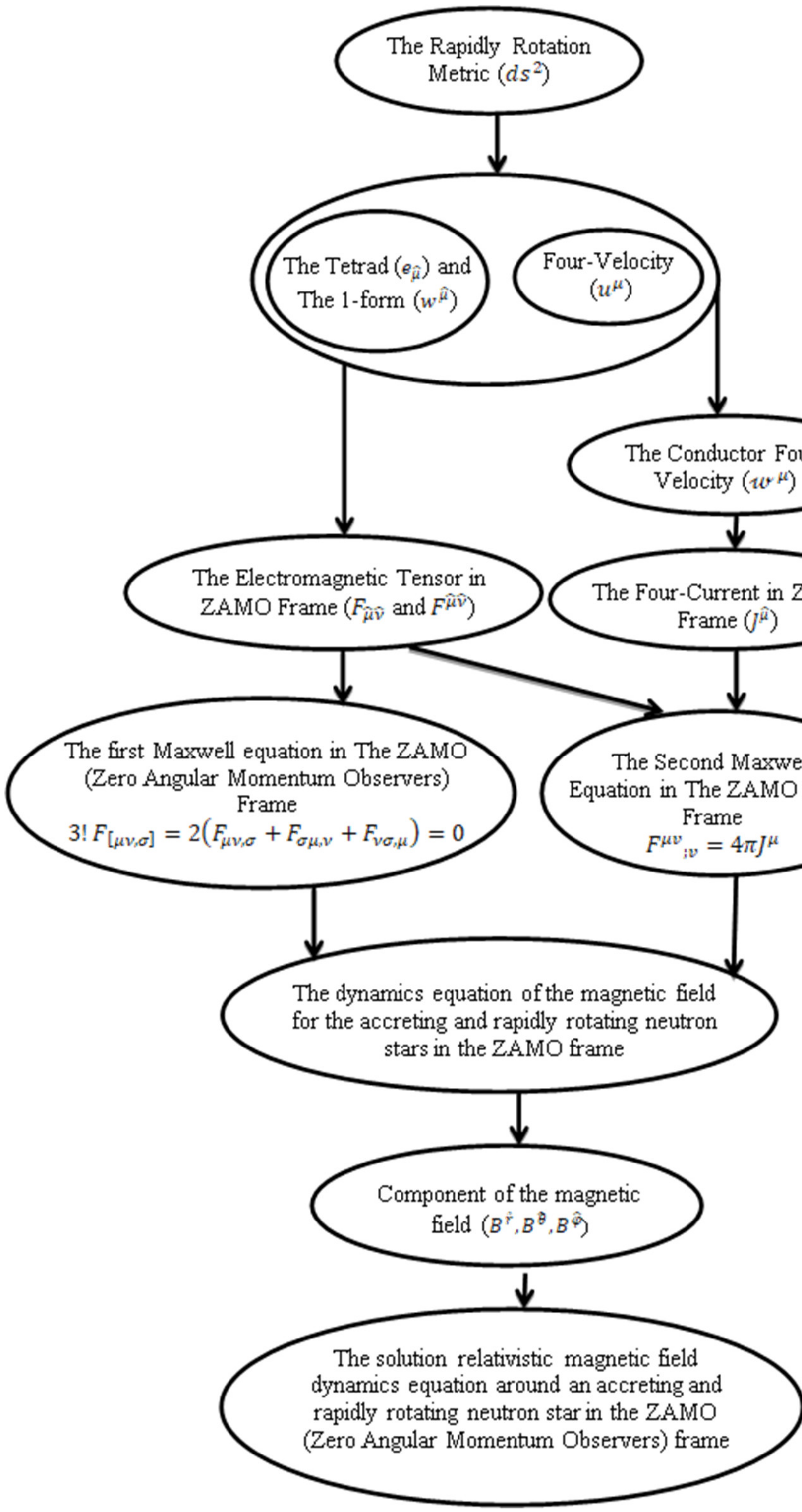

FIGURE 1. Research procedures to obtain a solution to Maxwell equations in a rapidly rotating neutron star

RESULTS AND DISCUSSION

In our previous article, we have built equations for rapidly-rotating and acceleration neutron stars which are differential equations of magnetic field dynamics in the ZAMO framework [24]. The relativistic Maxwell's first and second equations for rapidly-rotating and acceleration neutron stars are used to formulate the magnetic field dynamics in the ZAMO framework. Maxwell's relativistic first and second equation is shown in our previous work [24].

The rapid rotation metric for a rotating relativistic star in the coordinate system (t, r, θ, φ) (for radial (r), polar (θ) and azimuthal (φ)) is

$$ds^2 = -e^{2\phi}dt^2 + e^{2\lambda}r^2\sin^2\theta\,(d\varphi - \omega dt)^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2). \quad (1)$$

The function of ϕ, λ, ω, and α depend on r and θ. While $\omega(r)$ is the angular velocity of the inertial reference frame [25]. Based on our work before we have built Maxwell's relativistic first and second equations in the ZAMO framework for rapidly-rotating neutron stars [24]. The first relativistic Maxwell equations in the ZAMO framework are [24]

$$\left(-r\,e^{\alpha+\phi}\,(1-W^2)^{-\frac{1}{2}}\right)\left(E^{\hat{\theta}}\right)_{,\varphi} + \left(\omega\,r^2\sin\theta\;e^{\lambda+\alpha}\,(1-W^2)^{-\frac{1}{2}}\right)\left(B^{\hat{r}}\right)_{,\varphi}$$
$$+r\left(\zeta\sin\theta\,e^{\lambda}\,(1-W^2)^{-\frac{1}{2}}\,E^{\hat{\varphi}}\right)_{,\theta} + \left(r^2\sin\theta\,e^{\lambda+\alpha}(1-W^2)^{-\frac{1}{2}}\right)\frac{\partial B^{\hat{r}}}{\partial t} = 0, \quad (2a)$$

$$-\left((1-W^2)^{-\frac{1}{2}}\,e^{\alpha+\phi}\right)\left(E^{\hat{r}}\right)_{,\varphi} - \left(\omega\,r^2\sin\theta\,(1-W^2)^{-\frac{1}{2}}e^{\lambda+\alpha}\right)\left(B^{\hat{\theta}}\right)_{,\varphi} + \sin\theta\,\left(r\,\zeta\,(1-W^2)^{-\frac{1}{2}}\;e^{\lambda}E^{\hat{\varphi}}\right)_{,r}$$
$$-\left(r\sin\theta\,e^{\lambda+\alpha}\,(1-W^2)^{-\frac{1}{2}}\right)\frac{\partial B^{\hat{\theta}}}{\partial t} = 0, \quad (2b)$$

$$-e^{\alpha+\phi}\left((1-W^2)^{-\frac{1}{2}}E^{\hat{r}}\right)_{,\varphi} - \omega\,r\left(\sin\theta\,e^{\lambda+\alpha}\,(1-W^2)^{-\frac{1}{2}}B^{\hat{\theta}}\right)_{,\theta} + e^{\alpha+\phi}\left(r\,(1-W^2)^{-\frac{1}{2}}E^{\hat{\theta}}\right)_{,r}$$
$$-\sin\theta\left(\omega\,r^2e^{\lambda+\alpha}\,(1-W^2)^{-\frac{1}{2}}\,B^{\hat{r}}\right)_{,r} + \left(r\,\zeta e^{2\alpha-\phi}(1-W^2)^{-\frac{1}{2}}\right)\frac{\partial B^{\hat{\varphi}}}{\partial t} = 0, \quad (2c)$$

$$(1-W^2)^{-\frac{1}{2}} = \left(1-\left(r^2\sin^2\theta\;(\Omega-\omega)^2e^{2\lambda} + (v^r)^2e^{2\alpha} + r^2\left(v^{\theta}\right)^2e^{2\alpha}\right)e^{2\phi}\right)^{-\frac{1}{2}}, \quad (3)$$

$$\zeta(r,\theta) = \left(e^{2\phi} - \omega^2r^2\sin^2\theta\,e^{2\lambda}\right)^{\frac{1}{2}}. \quad (4)$$

Meanwhile, in the ZAMO framework, the second relativistic Maxwell equation is [24]

$$\left(r^2\sin\theta\,e^{\lambda+\alpha}\,(1-W^2)^{\frac{1}{2}}\,E^{\hat{r}}\right)_{,r} + \left(r\sin\theta\,e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\,E^{\hat{\theta}}\right)_{,\theta} + \left(r\,e^{2\alpha-\phi}(1-W^2)^{\frac{1}{2}}\,\zeta\,E^{\hat{\varphi}}\right)_{,\varphi} =$$
$$4\pi\Gamma\rho_e r^2\sin\theta\,e^{\lambda+2\alpha+2\phi} + \left(4\pi\Gamma\,r^3\sin^2\theta\,(\Omega-\omega)e^{\lambda+2\alpha}(1-W^2)^{\frac{1}{2}}\right)\,E^{\hat{\varphi}}, \quad (5a)$$

$$\left(-r^2\sin\theta\,e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\,\frac{\partial E^{\hat{r}}}{\partial t}\right)_{,r} + r\left(\sin\theta\,\zeta\,e^{\lambda}(1-W^2)^{\frac{1}{2}}B^{\hat{\varphi}}\right)_{,\theta} - \left(r\,e^{\alpha-\phi}\right)\left(B^{\hat{\theta}}\right)_{,\varphi} - \left(\omega\,r^2\sin\theta\,e^{\lambda+\alpha}\right)\left(E^{\hat{r}}\right)_{,\varphi}$$
$$= \left(4\pi\Gamma\rho_e\,r^2\sin\theta\,e^{\lambda+2\alpha+2\phi}(1-W^2)^{-\frac{1}{2}}\right)E^{\hat{r}} - \left(4\pi\Gamma\,r^3\sin^2\theta\,(\Omega-\omega)e^{\lambda+2\alpha}(1-W^2)^{-\frac{1}{2}}\right)B^{\hat{\theta}}, \quad (5b)$$

$$\left(-r\sin\theta\,e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\,\frac{\partial E^{\hat{\theta}}}{\partial t}\right)_{,r} - \sin\theta\left(r\,\zeta\,e^{\alpha}\,(1-W^2)^{\frac{1}{2}}B^{\hat{\varphi}}\right)_{,r} + \left(e^{\phi+\alpha}\right)\left(B^{\hat{r}}\right)_{,\varphi} - \left(\omega\,r\sin\theta\,e^{\lambda+\alpha}\right)\left(E^{\hat{\theta}}\right)_{,\varphi}$$
$$= \left(4\pi\Gamma\rho_e\;r^2\sin\theta\,e^{\lambda+2\alpha+2\phi}(1-W^2)^{-\frac{1}{2}}\right)E^{\hat{\theta}} + \left(4\pi\Gamma\,r^3\sin^2\theta\,(\Omega-\omega)e^{\lambda+2\alpha+\phi}(1-W^2)^{-\frac{1}{2}}\right)B^{\hat{r}}, \quad (5c)$$

$$\left(-r\,\zeta\,e^{\alpha-\phi}(1-W^2)^{\frac{1}{2}}\frac{\partial E^{\hat{\varphi}}}{\partial t}\right)_{,r} + \left(e^{\alpha+\phi}rB^{\hat{\theta}}\right)_{,r} - \left(e^{\alpha+\phi}\right)\left(B^{\hat{r}}\right)_{,\theta} - \left(\omega e^{\lambda+\alpha}rE^{\hat{r}}\right)_{,r} + (r\,\omega)\left(e^{\lambda+\alpha}E^{\hat{\theta}}\right)_{,\theta}$$
$$= 4\pi\Gamma\rho_e\,r^3\sin^2\theta\,\zeta^{-1}(\Omega-\omega)e^{\lambda+2\alpha+2\phi}(1-W^2)^{-\frac{1}{2}} + \left(4\pi\sigma\Gamma\,r^4\sin^4\theta\,e^{\lambda+2\alpha+\phi}(1-W^2)^{-\frac{1}{2}}\right)E^{\hat{\varphi}}. \quad (5d)$$

Furthermore, through Maxwell's first and second equations are relativistic, we obtain the dynamics equation of the magnetic field in the ZAMO framework for rapidly rotating and accreting neutron stars. [24]

$$\frac{\partial B^{\hat{r}}}{\partial t} = -\left(r^2 \sin\theta\, e^{\lambda+\alpha}\right)^{-1}(1-W^2)^{\frac{1}{2}}\Bigg\{(4\pi\Gamma r^2 \sin\theta)^{-1} e^{-(\lambda+\alpha+\phi)}\Bigg[\left(\left(r^2 \sin\theta\, e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\right)\left(\frac{\partial E^{\hat{r}}}{\partial t}\right)\right)_{,\varphi}$$
$$+\sin\theta\left(\left(r\,\zeta e^{\alpha}\,(1-W^2)^{\frac{1}{2}}\, B^{\hat{\varphi}}\right)_{,r}\right)_{,\varphi} - \left(e^{\phi+\alpha}\right)\left(\left(B^{\hat{r}}\right)_{,\varphi}\right)_{,\varphi}\Bigg] - \mathcal{O}(\Omega)_{,\varphi}$$
$$+\left(\omega\, r^2 \sin\theta\, e^{\lambda+\alpha}(1-W^2)^{-\frac{1}{2}}\right)\left(B^{\hat{r}}\right)_{,\varphi} + \Bigg(r \sin\theta\, \zeta\left(4\pi\sigma\Gamma e^{2\alpha+2\phi}\right)^{-1}$$
$$\left[-\left(r\,\zeta e^{\alpha-\phi}\,(1-W^2)^{\frac{1}{2}}\right)\frac{\partial E^{\hat{\varphi}}}{\partial t} + \left(r\, e^{\alpha+\phi} B^{\hat{\theta}}\right)_{,r} - \left(e^{\alpha+\phi} B^{\hat{r}}\right)_{,\theta}\right] + \mathcal{O}(\Omega)\Bigg)_{,\theta}\Bigg\}, \tag{6a}$$

$$\frac{\partial B^{\hat{\theta}}}{\partial t} = -\left(r \sin\theta\, e^{\lambda+\alpha}\right)^{-1}(1-W^2)^{\frac{1}{2}}\Bigg\{(4\pi\Gamma r^2 \sin\theta)^{-1} e^{-(\lambda+\alpha+\phi)}\Bigg[\left(\left(r^2 \sin\theta\, e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\right)\left(\frac{\partial E^{\hat{r}}}{\partial t}\right)\right)_{,\varphi}$$
$$-(r\, e^{\alpha})\left(\left(\sin\theta\, \zeta(1-W^2)^{\frac{1}{2}}\, B^{\hat{\varphi}}\right)_{,\theta}\right)_{,\varphi} + \left(e^{\alpha+\phi}\right)\left(\left(B^{\hat{\varphi}}\right)_{,\theta}\right)_{,\varphi}\Bigg] - \mathcal{O}(\Omega)_{,\varphi}$$
$$+\left(\omega\, r \sin\theta\, e^{\lambda+\alpha}(1-W^2)^{-\frac{1}{2}}\right)\left(B^{\hat{\theta}}\right)_{,\varphi} - \sin\theta\Bigg(r\,\zeta\,\left(4\pi\sigma\Gamma e^{2\alpha+2\phi}\right)^{-1}$$
$$\left[-\left(r\,\zeta e^{\alpha-\phi}\,(1-W^2)^{\frac{1}{2}}\right)\frac{\partial E^{\hat{\varphi}}}{\partial t} + \left(r\, e^{\alpha+\phi} B^{\hat{\theta}}\right)_{,r} - \left(e^{\alpha+\phi} B^{\hat{r}}\right)_{,\theta}\right] + \mathcal{O}(\Omega)\Bigg)_{,\theta}\Bigg\}. \tag{6b}$$

$$\frac{\partial B^{\hat{\varphi}}}{\partial t} = -\left(r\zeta e^{2\alpha-\phi}\right)^{-1}(1-W^2)^{\frac{1}{2}}\Bigg\{e^{\alpha+\phi}\Bigg((4\pi\Gamma\, r^2 \sin\theta)^{-1}\Bigg[\left(r^2 \sin\theta\, e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\right)\frac{\partial E^{\hat{r}}}{\partial t}$$
$$-r\, e^{\alpha}\left(\sin\theta\,\zeta\,(1-W^2)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,\theta} + \left(r\, e^{\alpha+\phi}\right)\left(B^{\hat{\theta}}\right)_{,\varphi} - \mathcal{O}(\Omega)\Bigg)_{,\theta} - \omega\, r\left(\sin\theta\, e^{\lambda+\alpha}(1-W^2)^{-\frac{1}{2}} B^{\hat{\theta}}\right)_{,\theta}$$
$$-e^{\alpha+\phi}\Bigg(\left(4\pi\sigma\Gamma\, r \sin\theta\, e^{2\alpha+2\phi}\right)^{-1}\Bigg[-\left(r \sin\theta\, e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\right)\frac{\partial E^{\hat{\theta}}}{\partial t} + \left(e^{\alpha+\phi}\right)\left(B^{\hat{r}}\right)_{,\varphi}$$
$$-\sin\theta\,\left(r e^{\alpha}\zeta\,(1-W^2)^{\frac{1}{2}}\, B^{\hat{\varphi}}\right)_{,r}\Bigg] + \mathcal{O}(\Omega)\Bigg)_{,r} - \sin\theta\left(\omega\, r^2 e^{\lambda+\alpha}(1-W^2)^{-\frac{1}{2}}\right)_{,r}\Bigg\}. \tag{6c}$$

To get a more visible picture of the dynamics equation, we looked for a general solution of the differential equation. To get a solution for the dynamic equation, the first step we do was to separate the variable of the magnetic field below [22]

$$B^{\hat{r}} = f(r)\,\psi_1(\theta,\varphi,\gamma,t), \tag{7a}$$
$$B^{\hat{\theta}} = g(r)\,\psi_2(\theta,\varphi,\gamma,t), \tag{7b}$$
$$B^{\hat{\varphi}} = h(r)\,\psi_3(\theta,\varphi,\gamma,t), \tag{7c}$$
$$\psi_1(\theta,\varphi,\gamma,t) = \sin\theta\sin\gamma\cos\lambda(t) + \cos\theta\cos\gamma, \tag{7d}$$
$$\psi_2(\theta,\varphi,\gamma,t) = -\sin\gamma\cos\theta\cos\lambda(t) + \cos\gamma\sin\theta, \tag{7e}$$
$$\psi_3(\theta,\varphi,\gamma,t) = \sin\gamma\cos\lambda(t). \tag{7f}$$

In the ZAMO frame, the solution of the relativistic dynamic field equation for rapidly-rotating and accreting neutron stars, that is

$$\frac{\partial \psi_1}{\partial t} = -\left(r^2 \sin\theta\, e^{\lambda+\alpha} f(r)\right)^{-1}(1-W^2)^{\frac{1}{2}}\Bigg\{(4\pi\Gamma\, r^2 \sin\theta)^{-1} e^{-(\lambda+\alpha+\phi)}\Bigg[\left(r^2 \sin\theta\, e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\right)\left(\frac{\partial E^{\hat{r}}}{\partial t}\right)_{,\varphi}$$
$$+\sin\theta\left(\left(r\zeta\, e^{\alpha}\,(1-W^2)^{\frac{1}{2}}\, h(r)\right)_{,r}\psi_3\right)_{,\varphi} - e^{\alpha+\phi}\, f(r)\left((\psi_1)_{,\varphi}\right)_{,\varphi}\Bigg] - \left(\mathcal{O}(\Omega)\right)_{,\varphi} + \omega\, r^2 \sin\theta\;\, e^{\lambda+\alpha}$$
$$(1-W^2)^{-\frac{1}{2}} f(r)(\psi_1)_{,\varphi} + \Bigg(r \sin\theta\,\zeta\left(4\pi\sigma\Gamma\, e^{2\alpha+2\phi}\right)^{-1}\Bigg[-r\, e^{\alpha-\phi}\zeta\,(1-W^2)^{\frac{1}{2}}\,\frac{\partial E^{\hat{\varphi}}}{\partial t}$$
$$+\psi_2\left(e^{\alpha+\phi} r G\right)_{,r} - F\left(e^{\phi+\alpha}\psi_1\right)_{,\theta}\Bigg]\Bigg) + \mathcal{O}(\Omega)\Bigg)_{,\theta}\Bigg\}, \tag{8a}$$

$$\frac{\partial\,\psi_2}{\partial t} = -\left(r\sin\theta\, e^{\lambda+\alpha} g(r)\right)^{-1}(1-W^2)^{\frac{1}{2}}\Bigg\{(4\pi\Gamma\, r^2\sin\theta)^{-1} e^{-(\lambda+\alpha+\phi)}\Bigg[-\left(r^2\sin\theta\, e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\right)\left(\frac{\partial E^{\hat{r}}}{\partial t}\right)_{,\varphi}$$
$$+r\, e^{\alpha} h(r)\left(\left(\sin\theta\,\zeta\,(1-W^2)^{-\frac{1}{2}}\psi_3\right)_{,\theta}\right)_{,\varphi} - \left(e^{\phi+\alpha}\right)\left(\left(B^{\hat{\varphi}}\right)_{,\theta}\right)_{,\varphi}\Bigg] - \mathcal{O}(\Omega)_{,\varphi} + \omega r\sin\theta\, e^{\lambda+\alpha}$$
$$(1-W^2)^{-\frac{1}{2}} g(r)(\psi_2)_{,\varphi} - \sin\theta\left(r\zeta\left(4\pi\sigma\Gamma\, e^{2\alpha+2\phi}\right)^{-1}\Bigg[-r\, e^{\alpha-\phi}\zeta\,(1-W^2)^{\frac{1}{2}}\frac{\partial E^{\hat{\varphi}}}{\partial t}\right.$$
$$\left. +\left(e^{\phi+\alpha} rB^{\hat{\theta}}\right)_{,r} - \left(e^{\phi+\alpha}B^{\hat{r}}\right)_{,\theta}\Bigg] + \mathcal{O}(\Omega)\right)_{,\theta}\Bigg\}, \tag{8b}$$

$$\frac{\partial\psi_3}{\partial t} = -\left(r\zeta\, h(r) e^{2\alpha-\phi}\right)^{-1}(1-W^2)^{\frac{1}{2}}\Bigg\{e^{\alpha+\phi}\Bigg((4\pi\Gamma\, r^2\sin\theta)^{-1}\Bigg[r^2\sin\theta\, e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\frac{\partial E^{\hat{r}}}{\partial t}$$
$$-re^{\alpha}h(r)\left(\sin\theta\,\zeta(1-W^2)^{\frac{1}{2}}\psi_3\right)_{,\theta} + e^{\alpha+\phi}\, r\, g(r)(\psi_2)_{,\varphi}\Bigg] - \mathcal{O}(\Omega)\Bigg)_{,\theta} + \omega r g(r)$$
$$\left(\sin\theta\, e^{\lambda+\alpha}(1-W^2)^{-\frac{1}{2}}\psi_2\right)_{,\theta} - e^{\phi+\alpha}\Bigg(\left(4\pi\sigma\Gamma r\sin\theta\; e^{2\alpha+2\phi}\right)^{-1}\Bigg[-r\sin\theta\, e^{\lambda+\alpha}(1-W^2)^{\frac{1}{2}}\frac{\partial E^{\hat{\theta}}}{\partial t}$$
$$+e^{\alpha+\phi} g(r)\,(\psi_2)_{,\varphi} - \sin\theta\;\psi_1\left(r\zeta\; e^{\alpha}\,(1-W^2)^{\frac{1}{2}} f(r)\right)_{,r}\Bigg] + \mathcal{O}(\Omega)\Bigg)_{,r} - \sin\theta\left(\omega r^2\, e^{\lambda+\alpha}(1-W^2)^{-\frac{1}{2}}\right)_{,r}\Bigg\}. \tag{8c}$$

The set of equations (8a-8c) if assumed to be $\partial E^{\hat{\imath}}/\partial t = 0$, then had the same shape as a neutron star that rotates slowly [6]. The equation for slow-rotating neutron stars is too complex to be solved analytically, and this is the same for fast-rotating neutron stars. Therefore, this equation must be solved by numerical. The set of equations (8a-8c) for the component of the magnetic field function $(\theta,\varphi,\gamma,t)$ shows that there is a decrease in the magnetic field. This result is in accordance with the hypothesis that if the neutron star accretes, the magnetic field can be reduced.

CONCLUSION

The solution of differential equations that describe the dynamics of the relativistic magnetic field in the ZAMO framework for rapidly-rotating and accreting neutron stars has been formulated. The solution of the magnetic field dynamic equation was obtained from the Maxwell relativistic equation and described for each component of the magnetic field. The set of equations (8a-8c) was too complex to be solved analytically; thus, it was solved numerically. Thes equations correspond to the slowly rotating neutron star. The set of equations (8a-8c) for the component of the magnetic field function $(\theta,\varphi,\gamma,t)$ shows that there is a decrease in the magnetic field. This result is in accordance with the hypothesis that if the neutron star accretes, the magnetic field can be reduced.

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