A Two-Dimensional Map Derived From An Ordinary Difference Equation of mKdV and Its Properties

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Abstract. The discrete modified Korteweg–de Vries (mKdV) is a class of discrete integrable systems that may be distinguished as integrable partial difference equations (PΔE) and integrable ordinary difference equations (OΔE). By considering traveling wave solutions, the OΔE mKdV can be obtained from PΔE mKdV. Meanwhile, a mapping can be constructed from an OΔE mKdV. In this paper, we will focus on producing a new map using a process (replacement), the interchange of a single parameter, and an integral and investigate its properties.

Keywords: OΔE mKdV, PΔE mKdV, Anti measure-preserving.

1. Introduction

The theory of discrete dynamical system and difference equations has been being developed in the last thirty years of the twentieth century. Recently, there is much application of the discrete dynamical system, and difference equations have appeared in the areas of biology, economics, physics, resource management, and others [1]. One of the discrete dynamical system types is the discrete integrable system. In this type, the system can have an integral or invariant (for a two-dimensional case with invariants of high degree [2]. An example of this type is the discrete modified Korteweg–de Vries (mKdV) equation. The mKdV is a partial differential equation which known to has soliton solution; hence it is also called one of the soliton equation [3]. As a class of discrete integrable systems, discrete mKdV may be distinguished integrable partial difference equations (PΔE) and integrable ordinary difference equations (OΔE). Discrete mKdV is a class of QRT (Quispel-Roberts-Thompson) map[4,5]. The discretization of the mKdV equation has been done in various ways. One of them, the method by describing its Lax-pair, can be found in [3,4,5].

There is a connection between the two classes, namely that many integrable maps can be obtained from integrable PΔE by imposing periodic boundary conditions [6]. By using the staircase method, PΔE

Explicitly, $\tilde{h}_{(\mu_0,\mu_1,\alpha)}$ with the replacement $\alpha=\alpha(x,y)$ yields the map,

$$\begin{aligned}&\hat{h}_{(\mu_0,\mu_1,\beta_0,\beta_1)}:\mathbb{R}^2\longrightarrow\mathbb{R}^2\\&(x,y)\mapsto\left(\frac{\beta_1x(1-\mu_0x)+\mu_1\left(1+\beta_0x^2+x^3+x^2y-x^3y\right)}{(-1+x)x\left(\beta_1xy+\mu_1(x+y)\right)},x\right)\end{aligned} \tag{17}$$

The mapping Eq. (17) have some properties:

- The mapping $\hat{h}_{(\mu_0,\mu_1,\beta_0,\beta_1)}$ has an integral Eq. (15).
- $\hat{h}_{(\mu_0,\mu_1,\beta_0,\beta_1)}$ is anti-measure-preserving, i.e.

$$\left|D\hat{h}_{(\mu_0,\mu_1,\beta_0,\beta_1)}\right|=-\frac{\rho(x,y)}{\rho\left(\frac{\beta_1x(1-\mu_0x)+\mu_1\left(1+\beta_0x^2+x^3+x^2y-x^3y\right)}{(-1+x)x\left(\beta_1xy+\mu_1(x+y)\right)},x\right)}$$

where

$$\begin{aligned}\rho(x,y)&=\frac{1}{xy}\left[\partial_\mu\tilde{H}(x,y)\right]^{-1}\\&=\frac{1}{\mu_1(x+y)+\beta_1xy}.\end{aligned}$$

- There exists a reversing symmetry $L(x,y)=(y,x)$ such that

$$L\circ\hat{h}_{(\mu_0,\mu_1,\beta_0,\beta_1)}\circ L=\hat{h}^{-1}_{(\mu_0,\mu_1,\beta_0,\beta_1)}.$$

It means that $\hat{h}_{(\mu_0,\mu_1,\beta_0,\beta_1)}$ is reversible ($\hat{h}_{(\mu_0,\mu_1,\beta_0,\beta_1)}\circ L\circ\hat{h}_{(\mu_0,\mu_1,\beta_0,\beta_1)}=L$).

CONCLUSIONS

Based on the results in the previous section, we have described in detail that a mapping derived from an OΔE mKdV has (anti) measure-preserving and reversing symmetry properties.

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