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Counting the number of vertexes labeled connected graphs of order five with minimum five edges and maximum ten parallel edges

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Counting the number of vertexes labeled connected graphs of order five with minimum five edges and maximum ten parallel edges

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Abstract. If given a graph G(V,E) with n vertices and m edges many graphs can be constructed. The graphs constructed maybe connected graphs (there exists at least one path connecting every pair of vertices in the graph) or disconnected; either simple a (contains loop or parallel edges) or not simple. In ,this paper we will discuss the formula for counting the number of connected vertex labelled graph of order five (n=5) without loops, witof h minimum five edges and maa y contaiof n maximum ten parallel edges.

1. Introduction

Graph theory is a branch of mathematics which discusess about discrete objects and the relationship among those objects. A point in a graph represents an object and an edge (a line) connecting a pair of vertices in graph represents the relationship between that pair of vertices. Points in a graph can represent cities, stations, houses, etc , while edges can represent roads, train track, fresh water pipeline, and so on. There is no a single correct way to draw a graph. A line can be drawn as a straight line, a curve , etc. Therefore, because of its flexibility, graph theoretical concepts are used to represent many real-life problems.

Not only can it represent the real-life problem, graph theoretical concept also can be used to solve it. A comprehensive study about the application of graph theory in networks design is given in [1], while in [2] some applications in operations research problem are given. Enumerative or counting graph is pioneered by Cayley in 1874 who interested to count the isomer of hydrocarbon CnH2n+2 and found that counting problem is related with counting rooted tree in graph. Inspirated by the work of Cayley, Slomenski investigated the additive structural properties of hydrocarbon using the concept of graph [3]. However, theoretically enumerative graph is given in [4-6].

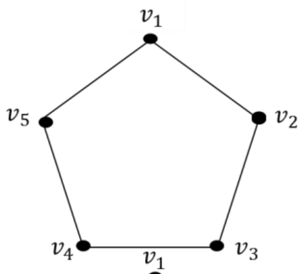
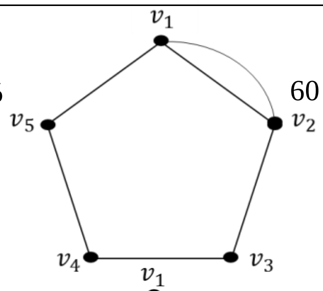
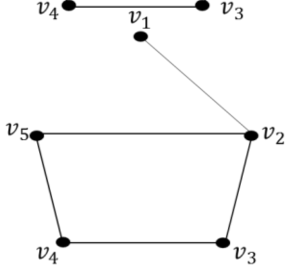
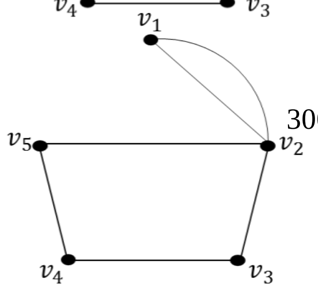
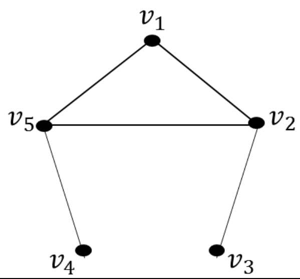
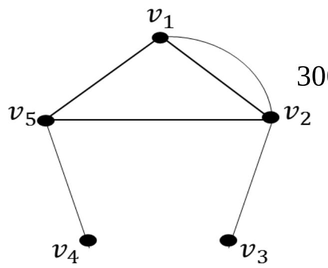
2. Literature review

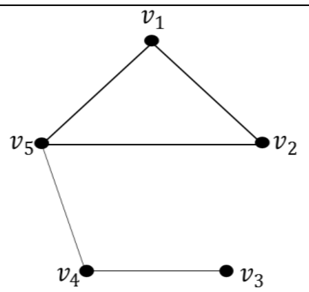
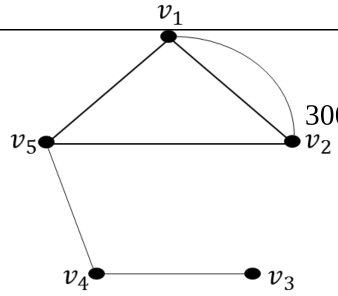
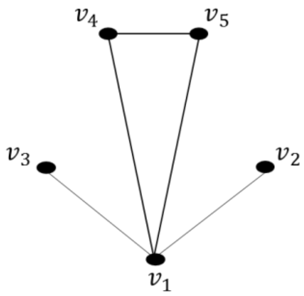
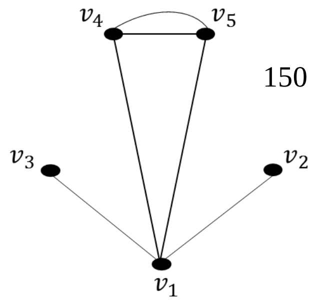
If given graph G(V,E) with n number of vertices and m edges, there are a lot of graphs that can be formed, either simple graph or not simple. A simple graph is a graph which not contains loops nor paralel edges. Moeover, a graph constructed maybe connected or disconnected. To distinguished one graph with another graph with the same form or structure we can use labelling. If the label is given on every vertices, it is called vertex labelling, if the label is given on every edge than is is called as edge labelling, and if the label is given on every vertex and edge then is is called as total labelling The formula for counting vertex labelled graphs is given in [7], but in that formula there are no distinction if the graphs are connected or disconnected. In that formula, all graphs are included except loops which are not allowed to be in the graph. Based on [7] , in 2017 the number of disconnected vertex labelled graphs of order gour is investigated [8], and the number of disonnected vertex labelled graphs of order five without parallel edges is investigated in [9]. The formula for counting disconnected vertex label graphs of order five with maximum six 3-parallel edges and containing no loops is investigated in [10], and the formula for counting the number of connected vertex labelled graph of order five with maximum 5 parallel edges is investigated in [11].

3. Graphs construction and the patterns obtained

Given G(V,E) with the number of vertices is 5(n = 5) and the number of edges is m, $5 \leq m \leq 20$, some patterns obtained are as shown in Table 1, where t is the number of edges that connect different pairs of vertices (parallel edges are counted as one). There are many graphs can be obtained, but due to space limitation, we only provide some patterns and the number of graphs that can be constructed using that pattern.

Table 1. Some connected vertex labelled graphs of order five constructed

Graph info	Patterns	The number of graphs	Graph info	Patterns	The number of graphs
$n = 5$ $t = 5$ $m = 5$		12	$n = 5$ $t = 5$ $m = 6$		60
		60			300
		60			300

	60		300
	30		150
Total	222	Total	1110

Notate t as the number of edges that connect different pairs of vertices (two or more edges that connect the same pair of vertices are counted as one), pi is the number of i-parallel edges, and g is the number of non parallel edges, then t= $\sum_{i=1} p_i + g$, and $m = \sum_{i=1} j.p_i + g\,; j \in N$, where n is the the order of graph and m is the number of edges in the graph. From Figure 1 we can see that n =5, p2 = 2 , g = 5 , t = p2 + g = 2 = 5 = 7 and m = 2 p2 + g = 2 (2) + 5 = 9.

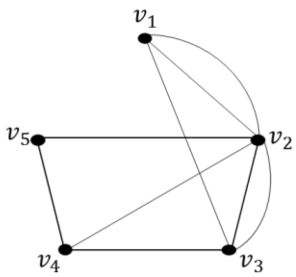


Figure 1. An example of graph wih 2-parallel edges

4. Results and discusion

From construction and observation we get Table 2 as follows:

Table 2. The number of connected vertex labelled graphs of order five with minimum edges is five and maximum ten parallel edges.

m	The number of connected vertex labelled graphs of order five with minimum edges is five and maximum ten parallel edges					
	t					
	5	6	7	8	9	10
5	222					
6	1110	205				
7	3330	1230	110			

8	7770	4305	770	45		
9	15540	11480	3080	360	10	
10	27972	25830	9240	1620	90	1
11	46620	51660	23100	5400	450	10
12		94710	50820	14850	1650	55
13		162360	101640	35640	4950	220
14			188760	77220	12870	715
15			330330	154440	30030	2002
16				289575	64350	5005
17				514800	128700	11440
18					243100	24310
19					437580	48620
20						92378

The following table is derived from Table 2 by observing the patterns obtained on the column of the table.

Table 3. The patterns obtained in every colmn of the table.

m	The number of connected vertex labelled graphs of order five with minimum edges is five and maximum ten parallel edges					
	t					
	5	6	7	8	9	10
5	1 x 222					
6	5 x 222	1 x 205				
7	15 x 222	6 x 205	1 x 110			
8	35 x 222	21 x 205	7 x 110	1 x 45		
9	70 x 222	56 x 205	28 x 110	8 x 45	1 x 10	
10	126 x 222	126 x 205	84 x 110	36 x 45	9 x 10	1 x 1
111	210 x 222	252 x 205	210 x 110	120 x 45	45 x 10	10 x 1
12		462 x 205	462 x 110	330 x 45	165 x 10	55 x 1
13		792 x 205	924 x 110	792 x 45	495 x 10	220 x 1
14			1716 x 110	1716 x 45	1287 x 10	715 x 1
15			3003 x 110	3432 x 45	3003 x 10	2002 x1
16				6435 x 45	6435 x 10	5005 x 1
17				11440 x 45	12870 x10	11440 x 1
18					24310 x 10	24310 x 1
19					43758 x 10	48620 x 1
20						92378 x 1

By grouping the graphs that can be constructed in terms of m and t, we found that the number in every column of the table formed patterns. In every column the numbers can be rewrite as a product of fixed constant with a number, and the numbers in every column makes a sequences.

For t =5 : 1, 5, 15, 35, 70, 126, 210

1 5 15 35 70 126 210
4 10 20 35 56 84
6 10 15 21 28
4 5 6 7
1 1 1

The fixed difference occurs on the fourth level, therefore the polynomial that can represent that sequence is polynomial of order four: $P_4(m) = A_4m^4 + A_3m^3 + A_2m^2 + A_1m + A_0$

Result 1 : Given G (V,E), |V| = 5, |E| = m , $5 \le m \le 20$, t = 5, t is the number of edges that connect different vertices (parallel edges are counted as one), then the number of connected vertex labelled graphs of order five with minimum five edges and maximum ten parallel edges is $N(G_{5,m,t}) = 222 \binom{m-1}{4}$.

Proof :

Substituting the value of the number of graph obtained in the first column we get:

222 = 625 A4 + 125 A3+ 25A2 + 5A1 + A0

1110 = 1296 A4 + 216 A3+ 36A2 + 6A1 + A0

3330 = 2401 A4 + 343A3 + 49A2 + 7A1 + A0

7770 = 4096 A4 + 512 A3+ 64A2 + 8A1 + A0

15540 = 6561 A4 + 729A3 + 81A2 + 9A1 + A0

Solving the above system of equations we get $A4 = \frac{2664}{288}$, $A3 = \frac{-26640}{288}$, $A2 = \frac{93240}{288}$, $A1 = \frac{-133200}{288}$, and

$A0 = \frac{63936}{288}$.

Therefore : $P_4(m) = \frac{2664}{288} m4 - \frac{26640}{288} m3 + \frac{93240}{288} m2 - \frac{133200}{288} m + \frac{63936}{288}$

$= \frac{222}{24} (m4 - 10m3 + 35m2 - 50m + 24)$

$= \frac{222}{24} (m - 1) (m - 2) (m - 3) (m - 4)$

$= 222 \binom{m-1}{4}$.

Therefore $N(G_{5,m,t})$ for t = 5 is $222 \binom{m-1}{4}$.

For t =6 , the sequence of number is 1, 6, 21, 56, 126, 252, 462, 792.

1	6	21	56	126	252	462	792
5	15	35	70	126	210	330	
10	20	35	56	84	120		
10	15	21	28	36			
5	6	7	8				
1	1	1	1				

The fixed difference occurs on the fourth level, therefore the polynomial that can represent that sequence is polynomial of order five: $P_5(m) = A_5m^5 + A_4m^4 + A_3m^3 + A_2m^2 + A_1m + A_0$

Result 2 : Given G (V,E), |V| = 5, |E| = m , $5 \le m \le 20$, t = 6, t is the number of edges that connect different vertices (parallel edges are counted as one), then the number of connected vertex labelled graphs of order five with minimum five edges and maximum ten parallel edges is $N(G_{5,m,t}) = 205 \binom{m-1}{5}$.

Proof :

Substituting the value of the number of graph obtained in the first column we get:

205 = 7776 A5 + 1296A4 + 216 A3+ 36A2 + 6A1 + A0

1230 = 16807A5 + 4012 A4 + 343 A3+ 49A2 + 7A1 + A0

4305 = 32768A5 + 4096A4 + 512A3 + 64A2 + 8A1 + A0

11480 = 59049A5 + 6561A4 + 729 A3+ 81A2 + 9A1 + A0

25850 = 100000A5 + 10000A4 +1000A3 + 100A2 +10A1 + A0

51660 = 161051A5 +114641A4 +1331A3 + 121A2 +11A1 +A0

Solving the above system of equations we get A5 = $\frac{59040}{34560}$, A4 = $\frac{-885600}{34560}$, A3 = $\frac{5016400}{34560}$, A2 = $\frac{-13284000}{34560}$, A1 = $\frac{16176960}{34560}$, and A0 =$\frac{-7084800}{34560}$.

Therefore : P_5(m) = $\frac{59040}{34560}$ m5 – $\frac{885600}{34560}$ m4 + $\frac{5016400}{34560}$ m3 – $\frac{13284000}{34560}$ m2 + $\frac{16176960}{34560}$ m – $\frac{7084800}{34560}$.

= $\frac{205}{120}$ (m5– 15m4+ 85m3 – 225m2 +274m –120)

= $\frac{205}{120}$ (m – 1) (m – 2) (m – 3) (m – 4)(m-5)

= 205 $\binom{m-1}{5}$.

Therefore $N(G_{5,m,t})$ for t = 6 is 205 $\binom{m-1}{5}$.
Based on Results 1 and 2, we get the following results:
Result 3 : Given G (V,E), |V| = 5, |E| = m , 5 ≤ m ≤ 20 , t = 7, t is the number of edges that connect different vertices (parallel edges are counted as one), then the number of connected vertex labelled graphs of order five with minimum five edges and maximum ten parallel edges is $N(G_{5,m,t})$ = 110 $\binom{m-1}{6}$.
Result 4 : Given G (V,E), |V| = 5, |E| = m , 5 ≤ m ≤ 20 , t = 8, t is the number of edges that connect different vertices (parallel edges are counted as one), then the number of connected vertex labelled graphs of order five with minimum five edges and maximum ten parallel edges is $N(G_{5,m,t})$ = 45 $\binom{m-1}{7}$.
Result 5 : Given G (V,E), |V| = 5, |E| = m , 5 ≤ m ≤ 20 , t = 9, t is the number of edges that connect different vertices (parallel edges are counted as one), then the number of connected vertex labelled graphs of order five with minimum five edges and maximum ten parallel edges is $N(G_{5,m,t})$ = 10 $\binom{m-1}{8}$.
Result 6 : Given G (V,E), |V| = 5, |E| = m , 5 ≤ m ≤ 20 , t = 10, t is the number of edges that connect different vertices (parallel edges are counted as one), then the number of connected vertex labelled graphs of order five with minimum five edges and maximum ten parallel edges is $N(G_{5,m,t})$ = $\binom{m-1}{9}$.

5. Conclusion

Based on the above discussion we can conclude that given G (V,E), |V| = 5, |E| = m , 5 ≤ m ≤ 20 , and t, t is the number of edges that connect different vertices (parallel edges are counted as one), then the number of connected vertex labelled graphs of order five with minimum five edges and maximum ten parallel edges $N(G_{5,m,t}$ is :
For t = 5, $N(G_{5,m,t})$ = 222 $\binom{m-1}{4}$,
For t = 6, $N(G_{5,m,t})$ = 205 $\binom{m-1}{5}$,
For t = 7, $N(G_{5,m,t})$ = 110 $\binom{m-1}{6}$,
For t = 8, $N(G_{5,m,t})$ = 45 $\binom{m-1}{7}$,
For t = 9, $N(G_{5,m,t})$ = 10 $\binom{m-1}{8}$,
For t = 10, $N(G_{5,m,t})$ = $\binom{m-1}{9}$.

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