

ON SUB-EXACT SEQUENCES

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ON SUB-EXACT SEQUENCES

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Abstract

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We introduce and study the notion of a sub-exact sequence.

1. Introduction

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Let R be a ring and let $A \xrightarrow{f} B \xrightarrow{g} C$ be an exact sequence of R -modules,

i.e.,

$$\text{Im } f = \text{Ker } g (= g^{-1}(0)).$$

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Davvaz and Parnian-Garamaleky [1] introduced the concept of quasi-exact sequences by replacing the submodule 0 by a submodule $U \subseteq C$. A sequence

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of R -modules and R -homomorphisms $A \xrightarrow{f} B \xrightarrow{g} C$ is quasi-exact at B or U -exact at B if there exists a submodule U in C such that $Im f = g^{-1}(U)$.

Anvariye and Davvaz [2] proved further results about quasi-exact sequences and introduced a generalization of Schanuel lemma. Moreover, they obtained some relationships between quasi-exact sequences and superfluous (or essential) submodules.

Furthermore, Davvaz and Shabani-Sol [3] introduced a generalization of some notions in the homological algebra. They gave a generalization of the Lambek lemma, Snake lemma, connecting homomorphism and exact triangle and they established new basic properties of the U -homological algebra. In [4], Anvariye and Davvaz studied U -split sequences and established several connections between U -split sequences and projective modules.

In this paper, we introduce a new notion of an exact sequence which is called a *sub-exact sequence*. A sub-exact sequence is a generalization of an exact sequence. Let K, L, M be R -modules and X be a submodule of L . The triple (K, L, M) is said to be *X -sub-exact at L* if there is a homomorphism making $K \rightarrow X \rightarrow M$ exact at X . We collect all submodules X of L such that the triple (K, L, M) is X -sub-exact at L , which we denote by $\sigma(K, L, M)$. In this paper, we investigate whether $\sigma(K, L, M)$ is closed under submodules, products and extensions. Moreover, we provide necessary condition for $\sigma(K, L, M)$ so that it has a maximal element.

2. Main Result

Definition. Let K, L, M be R -modules and X be a submodule of L . Then the triple (K, L, M) is said to be *X -sub-exact at L* if there exist R -homomorphisms f and g such that the sequence of R -modules and R -homomorphisms

$$K \xrightarrow{f} X \xrightarrow{g} M$$

is exact.

Example 2.1. Let $K = 4\mathbb{Z}$, $L = \mathbb{Z}$ and $M = \mathbb{Z}/4\mathbb{Z}$ be \mathbb{Z} -modules. Then the triple $(4\mathbb{Z}, \mathbb{Z}, \mathbb{Z}/4\mathbb{Z})$ is $4\mathbb{Z}$ -sub-exact at \mathbb{Z} since there are the identity $i : 4\mathbb{Z} \rightarrow 4\mathbb{Z}$ and canonical homomorphism (projection) $\pi : 4\mathbb{Z} \rightarrow \mathbb{Z}/4\mathbb{Z}$ such that the sequence $4\mathbb{Z} \xrightarrow{i} 4\mathbb{Z} \xrightarrow{\pi} \mathbb{Z}/4\mathbb{Z}$ is exact at $4\mathbb{Z}$.

Now, we give an example where the sequence $K \rightarrow L \rightarrow M$ is not exact, but the triple (K, L, M) is X -sub-exact, for some submodule X of L .

Example 2.2. Let $K = \mathbb{Z}_2$, $L = \mathbb{Z}_2 \oplus \mathbb{Z}_3$ and $M = 0$ be \mathbb{Z} -modules. Then the triple $(\mathbb{Z}_2, \mathbb{Z}_2 \oplus \mathbb{Z}_3, 0)$ is \mathbb{Z}_2 -sub-exact at $\mathbb{Z}_2 \oplus \mathbb{Z}_3$ since for the homomorphism $i : \mathbb{Z}_2 \rightarrow \mathbb{Z}_2 \oplus \mathbb{Z}_3$, given by $i(a) = (a, 0)$, for every $a \in \mathbb{Z}_2$, the sequence

$$\mathbb{Z}_2 \xrightarrow{i} \mathbb{Z}_2 \oplus \mathbb{Z}_3 \rightarrow 0$$

is sub-exact at $\mathbb{Z}_2 \oplus \mathbb{Z}_3$.

But, we cannot define an epimorphism p from \mathbb{Z}_2 to $\mathbb{Z}_2 \oplus \mathbb{Z}_3$.

Remark 2.3. Since the sequence $K \rightarrow \{0\} \rightarrow M$ is exact, the triple (K, L, M) is $\{0\}$ -sub-exact for any R -modules K, L, M .

Remark 2.4. Let K be an R -module.

(a) Since there are the identity $i : K \rightarrow K$ and zero homomorphism $\theta : K \rightarrow K$ such that the sequence $K \xrightarrow{i} K \xrightarrow{\theta} K$ is exact at K , the triple (K, K, K) is K -sub-exact at K .

(b) Since the identity $i : K \rightarrow K$ is surjective, the sequence $K \xrightarrow{i} K \xrightarrow{\theta} 0$ is exact at K . So, the triple $(K, K, 0)$ is K -sub-exact at K .

(c) Let V be a direct summand of K . We can define an epimorphism $p : K = V \oplus V' \rightarrow V$ such that the sequence $K \xrightarrow{p} V \rightarrow 0$ is exact at V . Hence, the triple $(K, K, 0)$ is V -sub-exact at K .

5 (d) Let U be a submodule of K . Then the triple $(U, K, K/U)$ is K -sub-exact and U -sub-exact at K .

30 (e) The triples $(K, 0, K)$ and $(0, 0, K)$ are 0 -sub-exact at 0 .

(f) The triple $(0, 0, K)$ is V -sub-exact at K , for every submodule V of K

since there is the inclusion $i : V \rightarrow K$ such that the sequence $0 \rightarrow V \xrightarrow{i} K$ is exact at V .

5 Let K, L, M be R -modules. We define

$$\sigma(K, L, M) = \{X \leq L \mid (K, L, M) \text{ is } X\text{-sub-exact at } L\}.$$

32 Then $\sigma(K, L, M) \neq \emptyset$ since $0 \in \sigma(K, L, M)$.

Proposition 2.5. Let $K_i, L_i, M_i, i = 1, 2$ be families of R -modules. If $X_1 \in \sigma(K_1, L_1, M_1)$ and $X_2 \in \sigma(K_2, L_2, M_2)$, then $X_1 \times X_2 \in \sigma(K_1 \times K_2, L_1 \times L_2, M_1 \times M_2)$.

Proof. Since $X_1 \in \sigma(K_1, L_1, M_1)$ and $X_2 \in \sigma(K_2, L_2, M_2)$, there are R -homomorphisms f_1, g_1, f_2 and g_2 such that the sequences $K_1 \xrightarrow{f_1} X_1 \xrightarrow{g_1} M_1$ and $K_2 \xrightarrow{f_2} X_2 \xrightarrow{g_2} M_2$ are exact. We define:

$$\mathbf{18} \quad f : K_1 \times K_2 \rightarrow X_1 \times X_2,$$

where $f((k_1, k_2)) = (f_1(k_1), f_2(k_2))$, for every $(k_1, k_2) \in K_1 \times K_2$ and

$$g : X_1 \times X_2 \rightarrow M_1 \times M_2,$$

24 where $g((x_1, x_2)) = (g_1(x_1), g_2(x_2))$, for every $(x_1, x_2) \in X_1 \times X_2$. So, the sequence

$$K_1 \times K_2 \xrightarrow{f} X_1 \times X_2 \xrightarrow{g} M_1 \times M_2$$

is exact. Therefore, $X_1 \times X_2 \in \sigma(K_1 \times K_2, L_1 \times L_2, M_1 \times M_2)$. \square

As a corollary, for any index set Λ , we obtain:

Corollary 2.6. *Let $K_\lambda, L_\lambda, M_\lambda$ be families of R -modules and X_λ be a submodule of L_λ , for every $\lambda \in \Lambda$. If $X_\lambda \in \sigma(K_\lambda, L_\lambda, M_\lambda)$, for every $\lambda \in \Lambda$, then $\prod_{\lambda \in \Lambda} X_\lambda \in \sigma(\prod_{\lambda \in \Lambda} K_\lambda, \prod_{\lambda \in \Lambda} L_\lambda, \prod_{\lambda \in \Lambda} M_\lambda)$.*

Proof. We define

$$f = \prod_{\lambda \in \Lambda} f_\lambda : \prod_{\lambda \in \Lambda} K_\lambda \rightarrow \prod_{\lambda \in \Lambda} X_\lambda$$

and

$$g = \prod_{\lambda \in \Lambda} g_\lambda : \prod_{\lambda \in \Lambda} X_\lambda \rightarrow \prod_{\lambda \in \Lambda} M_\lambda.$$

Hence, the sequence $\prod_{\lambda \in \Lambda} K_\lambda \xrightarrow{f} \prod_{\lambda \in \Lambda} X_\lambda \xrightarrow{g} \prod_{\lambda \in \Lambda} M_\lambda$ is exact.

Therefore, $\prod_{\lambda \in \Lambda} X_\lambda \in \sigma(\prod_{\lambda \in \Lambda} K_\lambda, \prod_{\lambda \in \Lambda} L_\lambda, \prod_{\lambda \in \Lambda} M_\lambda)$. □

In case $K = 0$, 13 we have the following properties:

Proposition 2.7. Let L, M be two R -modules and X_1, X_2 be submodules of L . If $X_1, X_2 \in \sigma(0, L, M)$, then $X_1 \cap X_2 \in \sigma(0, L, M)$.

Proof. Since $X_1, X_2 \in \sigma(0, L, M)$, there are R -homomorphisms f_1

and f_2 such that the sequences: $0 \rightarrow X_1 \xrightarrow{f_1} M$ and $0 \rightarrow X_2 \xrightarrow{f_2} M$ are exact. So, f_1 and f_2 are monomorphisms. We define $f = f_1|_{X_1 \cap X_2}$.

Hence, f is a monomorphism. So, the sequence $0 \rightarrow X_1 \cap X_2 \xrightarrow{f} M$ is exact.

Therefore, $X_1 \cap X_2 \in \sigma(0, L, M)$. □

As a corollary, we obtain:

6 **Corollary 2.8.** Let L, M be two R -modules and X_λ be a submodule of L , for every $\lambda \in \Lambda$. If $X_\lambda \in \sigma(0, L, M)$, for every $\lambda \in \Lambda$, then $\bigcap_{\lambda \in \Lambda} X_\lambda \in \sigma(0, L, M)$.

Proof. We define $f : \bigcap_{\lambda \in \Lambda} X_\lambda \rightarrow M$, where $f = f_\mu|_{\bigcap_{\lambda \in \Lambda} X_\lambda}$, for some $\mu \in \lambda$. Hence, by Proposition 2.7, the sequence

$$0 \rightarrow \bigcap_{\lambda \in \Lambda} X_\lambda \xrightarrow{f} M$$

is exact. Therefore, $\bigcap_{\lambda \in \Lambda} X_\lambda \in \sigma(0, L, M)$. \square

Following example shows that if $X_1 \in \sigma(K, L, M)$ and $X_2 \subset X_1$, then X_2 does not necessarily belong to $\sigma(K, L, M)$.

Example 2.9. Let \mathbb{Q} be a \mathbb{Z} -module. Since there is the identity $i : \mathbb{Q} \rightarrow \mathbb{Q}$, where $i(a) = a$, for every $a \in \mathbb{Q}$, the sequence $\mathbb{Q} \xrightarrow{i} \mathbb{Q} \rightarrow 0$ is exact. Hence, $\mathbb{Q} \in \sigma(\mathbb{Q}, \mathbb{Q}, 0)$. But, we already know that the only \mathbb{Z} -module homomorphism from \mathbb{Q} to \mathbb{Z} is zero homomorphism, then there is no homomorphism f such that the sequence $\mathbb{Q} \xrightarrow{f} \mathbb{Z} \rightarrow 0$. Hence, $\mathbb{Z} \notin \sigma(\mathbb{Q}, \mathbb{Q}, 0)$.

Proposition 2.10. Let K, L, M be R -modules and X_1, X_2 be submodules of L , where $X_2 \subset X_1$. If $X_1 \in \sigma(K, L, M)$ and X_2 is a direct summand of X_1 , then $X_2 \in \sigma(K, L, M)$.

Proof. Since $X_1 \in \sigma(K, L, M)$, there are R -homomorphisms f_1 and g_1 such that the sequence

$$K \xrightarrow{f_1} X_1 \xrightarrow{g_1} M$$

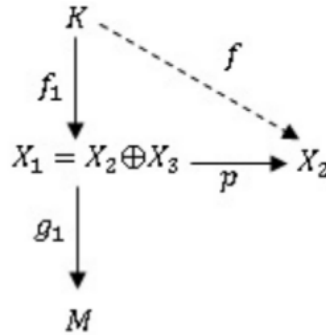
is exact.

Since X_2 is a direct summand of X_1 , there exists X_3 a submodule of X_1 such that $X_1 = X_2 \oplus X_3$. Hence, for every $x_1 \in X_1$, $x_1 = x_2 + x_3$, for some $x_2 \in X_2$ and $x_3 \in X_3$. Then we define R -homomorphism

$$p : X_1 = X_2 \oplus X_3 \rightarrow X_2,$$

where $p(x_1) = p(x_2 + x_3) = x_2 \in X_2$.

So, we construct a homomorphism $f : K \rightarrow X_2$, where $f = p \circ f_1$. We can see this in the following commutative diagram:



Now, let $g = g_1|_{X_2}$. We will show that $\text{Ker } g = \text{Im } f$.

(a) Let $x \in \text{Ker } g \subseteq X_2$. Then $g(x) = g_1(x) = 0$. Hence, $x \in \text{Ker } g_1$. Since $\text{Im } f_1 = \text{Ker } g_1$, there is $k \in K$ such that $f_1(k) = x$. Then $f(k) = (p \circ f_1)(k) = p(f_1(k)) = p(x) = x$. This implies, $\text{Ker } g \subseteq \text{Im } f$.

(b) Let $x \in \text{Im } f \subseteq X_2$. We have $k \in K$ such that $f(k) = x$. Then $x = f(k) = (p \circ f_1)(k) = f_1(k)$. Hence, $x \in \text{Im } f_1 = \text{Ker } g_1$. Therefore, $g_1(x) = 0$. Since $x \in X_2$, $g(x) = g_1(x) = 0$. So that $x \in \text{Ker } g$. Hence, $\text{Im } f \subseteq \text{Ker } g$.

We conclude that $\text{Im } f = \text{Ker } g$. So, the sequence $K \xrightarrow{f} X_2 \xrightarrow{g} M$ is exact. Therefore, $X_2 \in \sigma(K, L, M)$. □

As a corollary of Proposition 2.10, we obtain:

Corollary 2.11 Let K, L, M be \mathbb{R} -modules and L be a semisimple \mathbb{R} -module. If $L \in \sigma(K, L, M)$, then $X \in \sigma(K, L, M)$, for any submodule X of L .

Proof. Let X be any submodule of L . Since L is a semisimple module, X is complemented. Hence, there is a submodule X' of L such that $X \oplus X' \cong L$. Since $L \in \sigma(K, L, M)$, by Proposition 2.10, $X \in \sigma(K, L, M)$. \square

Proposition 2.12. *If there are R -homomorphisms f and g such that the sequence $K \xrightarrow{f} L \xrightarrow{g} M$ is exact, then L is the maximal element in $\sigma(K, L, M)$, i.e., for every $C \in \sigma(K, L, M)$, if $H \subseteq C$, then $H = C$.*

Proof. It is obvious. \square

This example illustrates Proposition 2.12.

Example 2.13. Let $8\mathbb{Z}, \mathbb{Z}$ be \mathbb{Z} -modules. We define $f : 8\mathbb{Z} \rightarrow \mathbb{Z}$, where $f(8a) = a$, for every $8a \in 8\mathbb{Z}$, and $g : \mathbb{Z} \rightarrow 0$ is zero homomorphism.

We have the exact sequence $8\mathbb{Z} \xrightarrow{f} \mathbb{Z} \xrightarrow{g} 0$. Hence, $\mathbb{Z} \in \sigma(8\mathbb{Z}, \mathbb{Z}, 0)$. So, \mathbb{Z} is the maximal element of $\sigma(8\mathbb{Z}, \mathbb{Z}, 0)$.

This proposition shows the relation between maximal submodule of L and maximal element of $\sigma(K, L, M)$.

Proposition 2.14. *Let K, L, M be R -modules. We assume that $L \notin \sigma(K, L, M)$. Consider the following assertions:*

(1) *There exists a maximal submodule $H \subset L$ such that $H \in \sigma(K, L, M)$.*

(2) *There exists $H \in \sigma(K, L, M)$ such that H is the maximal element in $\sigma(K, L, M)$ (i.e., for every $C \in \sigma(K, L, M)$, if $H \subseteq C$, then $H = C$).*

Then (1) \Rightarrow (2).

Proof. Let H be a maximal submodule of L . Assume that $H \notin \sigma(K, L, M)$. Since H is a maximal submodule of L , for every $C \in \sigma(K, L, M)$, if $H \subseteq C$, then $H = C$. Hence, H is the maximal element in $\sigma(K, L, M)$. \square

3 But, the converse is not always true. For example, let $K = M = 0$ and $L = \mathbb{Z}_6$ be \mathbb{Z} -modules. We get $\sigma(0, \mathbb{Z}_6, 0) = \{0\}$. So, $0 \subset \mathbb{Z}_6$ is the maximal element in $\sigma(0, \mathbb{Z}_6, 0)$. But, 0 is not a maximal submodule of \mathbb{Z}_6 .

The properties of Noetherian module are in [5]. 11 M is Noetherian if and only if every non-empty set of (finitely generated) submodules of M has a maximal element.

6 **Proposition 2.15.** Let K, L, M be R -modules and L be Noetherian. If $U \in \sigma(K, L, M)$, then there is a maximal element W in $\sigma(K, L, M)$ which contains U .

3 **Proof.** Let $U \in \sigma(K, L, M)$. If U is a maximal element in $\sigma(K, L, M)$, then it is clear.

If not, let

$$U \subset U' \subset U'' \subset \dots$$

3 be an ascending chain of submodules of a module L in $\sigma(K, L, M)$. Since L is Noetherian, there is a maximal element $W \in \sigma(K, L, M)$ which contains U . □

1 Let M be an R -module. A finite chain of submodules

$$0 = M_0 \subset M_1 \subset \dots \subset M_k = M, \quad k \in \mathbb{N} \tag{1}$$

is called a normal series of M . A normal series (1) is a composition series of M if all factors M_i/M_{i-1} are simple modules. The number k is said to be the length of the normal series and the factor modules M_i/M_{i-1} , $1 \leq i \leq k$ are called its factors [5]. So, any finitely generated semisimple module has a finite length or equivalently, it is Noetherian. As a corollary of Proposition 2.15, we obtain:

12 **Corollary 2.16.** Let K, L, M be R -modules and L be a finitely generated semisimple module. If $U \in \sigma(K, L, M)$, then there is a maximal element W in $\sigma(K, L, M)$ which contains U .

Proof. Let K, L, M be R -modules and L be a finitely generated semisimple module. Since any finitely generated semisimple module is Noetherian, by Proposition 2.15, if $U \in \sigma(K, L, M)$, then there is a maximal element W in $\sigma(K, L, M)$ which contains U . \square

However, $\sigma(K, L, M)$ may have more than one maximal element.

Example 2.17. Let $A = \{2a \mid a \in \mathbb{Z}_6\} = \{0, 2, 4\}$ and $B = \{3a \mid a \in \mathbb{Z}_6\} = \{0, 3\}$ be \mathbb{Z} -modules. If we take $K = 0$, $L = \mathbb{Z}_6$ and $M = A \times B$ as \mathbb{Z} -modules, then $\sigma(K, L, M) = \{0, \{0, 2, 4\}, \{0, 3\}\}$. Since we cannot define a monomorphism from \mathbb{Z}_6 to M , $\mathbb{Z}_6 \notin \sigma(K, L, M)$. So, the maximal elements of $\sigma(K, L, M)$ are $\{0, 2, 4\}$ and $\{0, 3\}$. Furthermore, $\{0, 2, 4\}$ is not isomorphic to $\{0, 3\}$. So, we can conclude that two elements of $\sigma(K, L, M)$ are not necessarily unique up to isomorphism.

3. Conclusion

Let K, L, M be R -modules. The collection of all submodules X of L such that the triple (K, L, M) is X -sub-exact denoted by $L(\sigma(K, L, M))$ is not closed under submodules. But, if a submodule of L is a direct summand of any element of $\sigma(K, L, M)$, then this submodule is contained in $\sigma(K, L, M)$. Therefore, if L is semisimple and $L \in \sigma(K, L, M)$, then any submodule of L is contained in $\sigma(K, L, M)$. Moreover, $\sigma(K, L, M)$ is not closed under extensions.

If there are R -module homomorphisms f and g such that the sequence $K \xrightarrow{f} L \xrightarrow{g} M$ is exact, then $\sigma(K, L, M)$ has a maximal element. If not, then the set $\sigma(K, L, M)$ has a maximal element if L is Noetherian. Furthermore, $\sigma(K, L, M)$ may have more than one maximal element. But, any two elements of $\sigma(K, L, M)$ are not necessarily unique up to isomorphism.

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