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A. Yasrina, D. Andra, N. Widianingrum, E. Latifah, and N. A. Pramono

Solution for the Relativistic Magnetic Field Dynamics Equation of Neutron Stars as Magnetized, Accreting, and Rapidly Rotating Conductors in Zero Angular Momentum Observers (ZAMO) Frame

A Yasrina[1,a)], D Andra[2], N Widianingrum[1], E Latifah[1], and N A Pramono[1]

[1]Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Malang, Jl. Semarang 5, Malang 65145, Indonesia.
[2]Departmen of Physiscs, Faculty of Matematics and Natural Sciences, Universitas Negeri Lampung, Jl. Prof. Soemantri Brodjonegoro No.1 Gedong Meneng Rajabasa, Bandar Lampung 35144 Indonesia

[a)] Corresponding author: atsnaita.yasrina.fmipa@um.ac.id

Abstract. Solution for the relativistic magnetic field dynamics equation of neutron stars as magnetized conductors has been formulated by assuming the magnetic field as magnetized, rapidly rotating conductor, and accreting in Zero Angular Momentum Observers (ZAMO) frame. The magnetic field in a neutron star is assumed to be bipolar. The relativistic magnetic field dynamics equation of neutron stars as magnetized is obtained by formulating the contravariant tensor, the current density of four, and the relativistic second Maxwell equation. The form of the equation is the differential equation with the solution being the radial, polar, azimuth, and time functions. The equation obtained has the same pattern as a slow-rotating neutron star. But if it is assumed that the neutron star does not perform accretion, then the star's rotation speed affects the magnetic field by showing the presence of polar, azimuthal, and time components.

INTRODUCTION

Neutron star is one of the final phase of stars which have s mass of $M_* \gtrsim 8M_\odot$. The mass of the neutron star is $M_* \sim 1 - 2\, M_\odot$. While the radius is $R_* \approx 10 - 14$ km [1]. If the mass of a neutron star is $M_* \sim 1{,}5\, M_\odot$, then the radius is $R_* \sim 3$ km [2]. Neutron stars have a star density of $\rho \simeq (2-3)2{,}8 \times 10^{14}$ g cm^{-3} [3]. A neutron star is also a rapidly rotating celestial object called pulsar [4]. Strong gravitational field and rotation of a neutron star signify that a neutron star is a relativistic object which is proven by the compactness parameter value $x_g \ll 1$. The compactness factor determines the effect of general relativity on celestial objects, with the compactness factor is

$$x_g = r_g/R. \tag{1}$$

The quantity r_g is radius of Schwarzschild. A Neutron star has a compactness factor $x_g \ll 1$, so it is relativistic object.

Neutron stars can be detected by the electromagnetic wave emissions. The magnitude of the neutron star's magnetic field can be known from radio waves or X-rays emitted [5]. Pulsar has a magnetic field of $B{\sim}10^{15}$G [6]. Inside the neutron star reaches $\sim 10^{15} G$ [7] or 10^{18} G [1]. Neutron stars have the strongest magnetic field in the universe [6].

Unfortunately, the neutron star's magnetic field has decreased. The phenomenon is found in binary systems which indicate that neutron stars is accreting [8-14]. For the example, a magnetic field of $\sim 10^{12}$G decreases to $\sim 10^{8}$G for 5×10^6 years [15].

A reduced magnetic field equation requires a magnetic field dynamic equation, and has been obtained for non-relativitic equations [10]. Whereas, relativistic studies is done, and the result is the electromagnetic fields are stationary

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in Schwarzchild spacetime [16]. Studies about the effects of the general relativity due to rotation of stars are conducted by assuming that the neutron stars rotate slowly [17]. The solution for Maxwell's equations with the assumption that the stars are slowly rotating non-accreting neutron stars which are measured by Zero Angular Momentum Observers (ZAMO) observer is obtained [18]. The Magnetic field dynamic equations have been obtained for slow-rotating and acceleration neutron stars by not ignoring the value of changes in the electric field [19]. Equations of magnetic field dynamics for slow-rotating have a covariant form with rapidly-rotating for acceleration neutron stars [20]. Inasmuch as neutron stars can be considered as rotating conductors [18]. Equations of magnetic field dynamics are needed to explain the phenomenon of decreasing magnetic fields. The assumption that the magnetic field of neutron stars is dipolar. Neutron stars are conductors and in ZAMO frame.

RESEARCH METHODS

The research procedure in this study begins by defining fast-rotating neutron star metrics. From the metric obtained tetrad, 1-form, and four velocity vectors. The next step is to formulate a conductor's four velocity. Four current densities and electromagnetic field tensors are formulated to formulate Maxwell's second equation and formulate a solution. The research is shown in Fig. 1 below

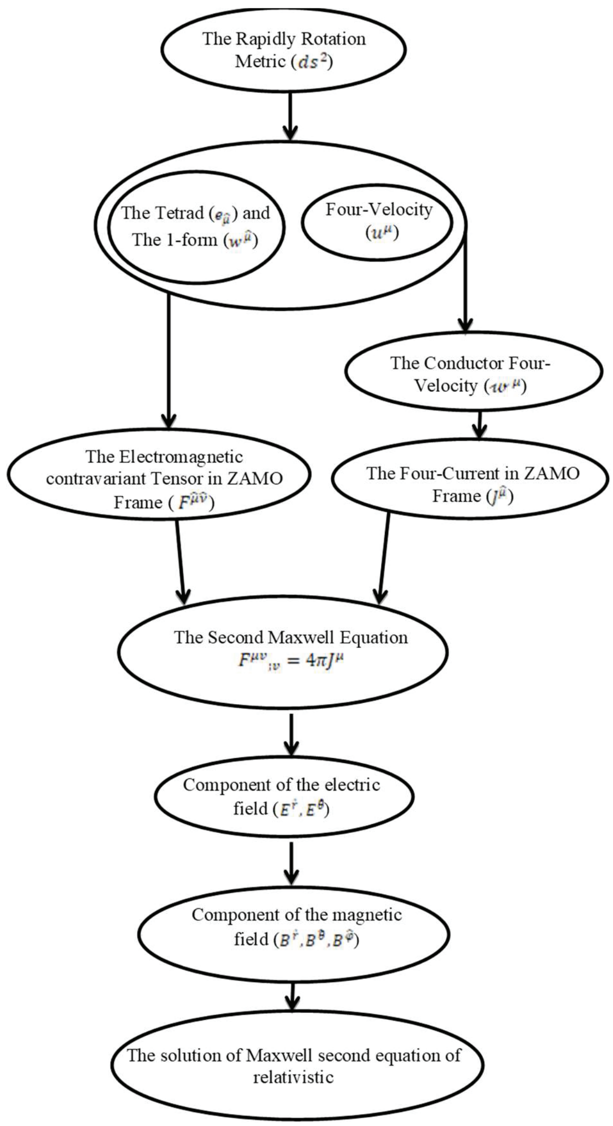

FIGURE 1. The research procedure obtains the solution of Maxwell second equation of relativistic

RESULT AND DISCUSSION

The Rapidly Rotating Metric, Four-Velocity, Four-Velocity Vector of Conductor, Four-Current in ZAMO Frame

Rapidly rotation metric for a rotating relativistic neutron star is

$$ds^2 = -\left(e^{2\phi} - \sin^2\theta\, r^2 e^{2\lambda}\omega^2\right)dt^2 + e^{2\alpha}\, dr^2 + r^2 e^{2\alpha}\, d\theta^2$$
$$-2\sin^2\theta\, r^2 e^{2\lambda}\,\omega\, dt\, d\varphi + \sin^2\theta\, r^2 e^{2\lambda}\, d\varphi^2, \tag{2}$$

for radial (r), polar (θ) and azimuthal (φ) components. The function of λ, α, and ϕ depends on r and θ [21]. The angular velocity of inertial reference frame $\omega(r)$ depends on r. The four velocity vector of the metric is [22]

$$u^\mu = \frac{e^{-\phi}}{\sqrt{1-W^2}}(1,0,0,\omega), \tag{3}$$

$$W^2 = e^{2\phi}\left[\sin^2\theta\; r^2\, e^{2\lambda}\; (\Omega-\omega)^2 + e^{2\alpha}\,(v^r)^2 + r^2\, e^{2\alpha}\;\left(v^\theta\right)^2\right]. \tag{4}$$

The conductor four-velocity vector is

$$w^\alpha = \chi\left(1, e^{-\alpha}\,\delta v^{\hat{r}},\; \frac{e^{-\alpha}}{r}\,\delta v^{\hat{\theta}}, \frac{\left(\sin^2\theta\, r^2\, e^{2\lambda}\omega^2 - e^{2\phi}\right)^{\frac{1}{2}}}{\sin\theta\, r\, e^{\lambda+\phi}}\;\delta v^{\hat{\varphi}}\right), \tag{5}$$

$$\chi \approx \left(e^{2\phi} - \sin^2\theta\, r^2\, e^{2\lambda}\;\omega^2\right)^{-\frac{1}{2}}. \tag{6}$$

The four-current density is

$$J^\mu = j^\mu + \rho_c\, w^\nu, \tag{7}$$

where ρ_c is charge density. Whereas in the ZAMO frame, the four-current density ($J^{\hat{\mu}}$) is [20]

$$J^{\hat{0}} = e^{\phi}\rho_c\,\chi + \sigma\;\sin\theta\, r\, e^{\lambda-2\phi}\;\mathcal{L}\,(\Omega-\omega)\,E^{\hat{\varphi}}, \tag{8}$$

$$J^{\hat{r}} = \sigma\,\chi\,\left[\mathcal{L}\,E^{\hat{r}} - \kappa\frac{e^{-2\alpha}}{r}(\Omega-\omega)B^{\hat{\theta}}\right], \tag{9}$$

$$J^{\hat{\theta}} = \sigma\,\chi\,\left[\mathcal{L}\,E^{\hat{\theta}} + \kappa\;\frac{e^{-2\alpha}}{r}(\Omega-\omega)\,B^{\hat{r}}\right], \tag{10}$$

$$J^{\hat{\varphi}} = \chi\left[\rho_c\,(\Omega-\omega)\frac{\left(\sin\theta\, r\, e^{\lambda+\phi}\right)}{\left(e^{2\phi} - \sin^2\theta\, r^2\, e^{2\lambda}\;\omega^2\right)^{\frac{1}{2}}} + \sigma\sin^2\theta\, r^2\,\mathcal{L}\,E^{\hat{\varphi}}\right], \tag{11}$$

where

$$\kappa = \frac{\sin\theta\, r^2\, e^{\lambda+2\alpha}}{\sqrt{1-W^2}};\; \mathcal{L} = \frac{e^{\phi}}{\sqrt{1-W^2}}. \tag{12}$$

The Electromagnetic Tensor and Maxwell's Equation in A Rapidly Rotating Spacetime in ZAMO Frame

The component of the electromagnetic field tensor ($F_{\alpha\beta}$) is

$$F_{\alpha\beta} \equiv \eta_{\alpha\beta\gamma\delta}\, u^\gamma\, B^\delta + 2u_{[\alpha}E_{\beta]} \tag{13}$$

where $\eta_{\alpha\beta\gamma\delta}$ is pseudo tensor. The components of contravariant tensor of electromagnetic field in ZAMO frame for the rapidly rotating neutron stars are

- $\alpha = \beta$

$$F^{\hat{\alpha}\hat{\alpha}} = 0, \tag{13 a}$$

- $\alpha \neq \beta$

$$F^{\hat{0}\hat{1}} = -\sqrt{1-W^2}\, e^{-(\alpha+\phi)}\, E^{\hat{r}} = -F^{\hat{1}\hat{0}}, \tag{13 b}$$

$$F^{\hat{0}\hat{2}} = \sqrt{1-W^2}\, \frac{e^{-(\alpha+\phi)}}{r}\, E^{\hat{\theta}} = -F^{\hat{2}\hat{0}}, \tag{13 c}$$

$$F^{\hat{0}\hat{3}} = \sqrt{1-W^2}\, \frac{(e^{2\phi} - \sin^2\theta\, r^2\, e^{2\lambda}\, \omega^2)^{\frac{1}{2}}}{\sin\theta\, r\, e^{\lambda+2\phi}}\, B^{\hat{\varphi}} = -F^{\hat{3}\hat{0}}, \tag{13 d}$$

$$F^{\hat{1}\hat{2}} = -\sqrt{1-W^2}\, \frac{(e^{2\phi} - \sin^2\theta\, r^2\, e^{2\lambda}\, \omega^2)^{\frac{1}{2}}}{r\, e^{\lambda+\phi+\alpha}}\, B^{\hat{\varphi}} = -F^{\hat{2}\hat{1}}, \tag{13 e}$$

$$F^{\hat{1}\hat{3}} = -\sqrt{1-W^2}\left[\frac{E^{\hat{\theta}}}{\sin\theta\, r\, e^{\lambda+r}} - \frac{\omega}{e^{\alpha+\phi}}\, B^{\hat{r}}\right] = -F^{\hat{3}\hat{1}}, \tag{13 f}$$

$$F^{\hat{2}\hat{3}} = -\left[\frac{B^{\hat{\theta}}}{\sin\theta\, r^2\, e^{\lambda+\alpha}} + \frac{\omega}{e^{\alpha+\phi}}\, E^{\hat{r}}\right] = -F^{\hat{3}\hat{2}}. \tag{13 g}$$

The relativistic Maxwell's second equation is

$$F^{\mu\nu}{}_{;\nu} = 4\pi J^{\mu}. \tag{14}$$

While both Maxwell's equations for the relativistic rapidly rotating neutron star within the ZAMO frame are [20]

$$\left(\sqrt{1-W^2}\sin\theta\, r^2\, e^{\lambda+\alpha}\, E^{\hat{r}}\right)_{,r} + \left(\sqrt{1-W^2}\sin\theta\, r\, e^{\lambda+\alpha}\, E^{\hat{\theta}}\right)_{,\theta} + \left(\sqrt{1-W^2}\, r\, e^{2\alpha-\phi}\left(e^{2\phi} - \sin^2\theta\, r^2\, e^{2\lambda}\omega^2\right)^{\frac{1}{2}} E^{\hat{\varphi}}\right)_{,\varphi} = 4\pi\sin\theta\, r^2 e^{\lambda+2\alpha+2\phi}\, J^{\hat{0}}, \tag{15 a}$$

$$\left(-\sqrt{1-W^2}\sin\theta\, r^2\, e^{\lambda+\alpha}\, \frac{\partial E^{\hat{r}}}{\partial t}\right)_{,r} + r\left(\sqrt{1-W^2}\sin\theta\, e^{\lambda}\left(e^{2\phi} - \sin^2\theta\, r^2\, e^{2\lambda}\, \omega^2\right)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,\theta} - r\, e^{\alpha-\phi}\left(B^{\hat{\theta}}\right)_{,\varphi} - \sin\theta\, r^2\, e^{\lambda+\alpha}\, \omega\left(E^{\hat{r}}\right)_{,\varphi} = 4\pi\sin\theta\, r^2 e^{\lambda+2\alpha+2\phi} J^{\hat{r}}, \tag{15 b}$$

$$\left(-\sqrt{1-W^2}\sin\theta\, r\, e^{\lambda+\alpha}\, \frac{\partial E^{\hat{\theta}}}{\partial t}\right)_{,r} - \sin\theta\left(\sqrt{1-W^2}\, r\, e^{\alpha}\left(e^{2\phi} - \sin^2\theta\, r^2\, e^{2\lambda}\, \omega^2\right)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,r} + e^{\alpha+\phi}\left(B^{\hat{r}}\right)_{,\varphi} - \sin\theta\, r\, e^{\alpha+\lambda}\, \omega\left(E^{\hat{\theta}}\right)_{,\varphi} = 4\pi\sin\theta\, r^2 e^{\lambda+2\alpha+2\phi} J^{\hat{\theta}}, \tag{15 c}$$

$$\left(-\sqrt{1-W^2}\, r\, e^{\alpha-\phi}\left(e^{2\phi} - \sin^2\theta\, r^2 e^{2\lambda}\omega^2\right)^{\frac{1}{2}}\, \frac{\partial E^{\hat{\varphi}}}{\partial t}\right)_{,r} + \left(r\, e^{\alpha+\phi}\, B^{\hat{\theta}}\right)_{,r} - \left(\sin\theta\, r\, e^{\phi+\alpha}\, B^{\hat{r}}\right)_{,\theta} - \left(r\, e^{\lambda+\alpha}\, \omega\, E^{\hat{r}}\right)_{,r} + r\,\omega\left(e^{\alpha+\lambda}\, E^{\hat{\theta}}\right)_{,\theta} = 4\pi\sin\theta\, r^2 e^{\lambda+2\alpha+2\phi} J^{\hat{\varphi}}. \tag{15 d}$$

Solution for The Relativistic Magnetic Field Dynamics Equation of Neutron Stars as Magnetized, Rapidly Rotating, Accreting, and Conductors in ZAMO Frame

The solutions of relativistic magnetic field dynamics equations with variable separation are [18]

$$B^{\hat{r}} = F(r)\, \psi_1(\theta,\varphi,\xi,t), \tag{16 a}$$

$$B^{\hat{\theta}} = G(r)\, \psi_2(\theta,\varphi,\xi,t), \tag{16 b}$$

$$B^{\hat{\varphi}} = H(r)\, \psi_3(\theta,\varphi,\xi,t), \tag{16 c}$$

$$\psi_1(\theta,\varphi,\xi,t) = \sin\theta\sin\xi\cos\lambda(t) + \cos\theta\cos\xi, \tag{16 d}$$

$$\psi_2(\theta,\varphi,\xi,t) = -\cos\theta\sin\xi\cos\lambda(t) + \sin\theta\cos\xi, \tag{16 e}$$

$$\psi_3(\theta,\varphi,\xi,t) = \sin\xi\cos\lambda(t). \tag{16 f}$$

If the equations (15 a) - (15 d) are assumed to be $\frac{\partial E^{\hat{r}}}{\partial t} = \frac{\partial E^{\hat{\theta}}}{\partial t} = \frac{\partial E^{\hat{\varphi}}}{\partial t}$, then the equations (15 a) - (15 d) may be written as

$$\sin\theta\, r^2 e^{\lambda+\alpha} E^{\hat{r}} = \frac{1}{4\pi\sigma\,\chi\, e^{\alpha+\phi}}\left[r\, e^{\alpha}\left(\sqrt{1-W^2}\,\sin\theta\left(e^{2\phi}-\sin^2\theta\; r^2\, e^{2\lambda}\;\omega^2\right)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,\theta} - r\, e^{\alpha+\phi}\left(B^{\hat{\theta}}\right)_{,\varphi}\right] + \mathcal{O}(\omega), \tag{17}$$

$$\sin\theta\, e^{\alpha+\lambda}\, r\, E^{\hat{\theta}} = \frac{1}{4\pi\sigma\,\chi\, e^{\alpha+\phi}}\left[-\sin\theta\left(\sqrt{1-W^2}\, r\, e^{\alpha}\;\left(e^{2\phi}-\sin^2\theta\; r^2\, e^{2\lambda}\;\omega^2\right)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,r} + e^{\alpha+\phi}\left(B^{\hat{r}}\right)_{,\varphi}\right] + \mathcal{O}(\omega) \tag{18}$$

By subtituting equations (16 a) - (21) into equations (15 a) - (15 d) we finally obtain

$$\sin\xi\, r\, e^{\alpha}\;\left[H\left(K(r,\theta)\right)_{,\theta} - \cos\theta\;\, G e^{\phi}\right]\left[\sin\lambda - \frac{\omega}{e^{\alpha+2\phi}}\cos\lambda\right] = 4\pi\sin\theta\, r^2 e^{\lambda+2\alpha+2\phi} J^{\hat{r}}, \tag{19}$$

$$\left[F\, e^{\phi+\alpha} + (L(r,\theta)h)_{,r}\right]\sin\chi\sin\theta\left[-\sin\lambda + \frac{\omega}{e^{2\phi+\alpha}}\cos\lambda\right] = 4\pi\sin\theta\, r^2 e^{\lambda+2\alpha+2\phi} J^{\hat{\theta}}, \tag{20}$$

$$(-\cos\theta\sin\xi\cos\lambda(t) + \sin\theta\cos\xi)\left[\left(e^{\alpha+\phi} r\; G\right)_{,r} + \left(e^{\phi+\alpha}F\right)\right] - (\sin\theta\sin\xi\cos\lambda(t) + \cos\theta\cos\xi)(e^{\phi+\alpha}F)_{,\theta}$$
$$-(\sin\xi\cos\lambda(t))\left[F\;(M(r,\theta)\cos\theta)_{,\theta} + \left(\frac{1}{r\sin\theta}M(r,\theta)G\right)_{,r} - \left(\frac{1}{\sin\theta\; r\, e^{\phi}}M(r,\theta)\left(\sin\theta\, N(r,\theta)\right)_{,\theta}\right)_{,r}\right.$$
$$\left. + \left(\frac{\sin\theta}{e^{\alpha}}M\;(r,\theta)\;(e^{\alpha} r\; N\;(r,\theta)H)_{,r}\right)_{\theta}\right] = 4\pi\sin\theta\, r^2 e^{\lambda+2\alpha+2\phi} J^{\hat{\varphi}}. \tag{21}$$

The equation (19) – (21) obtained has the same pattern as a slow-rotating neutron star. But if it is assumed that the neutron star does not perform accretion ($J^{\hat{r}} = J^{\hat{\theta}} = J^{\hat{\varphi}} = 0$), then the star's rotation speed affects the magnetic field by showing the presence of polar, azimuthal, and time components.

CONCLUSION

The relativistic magnetic field dynamics equation of neutron stars as magnetized is obtained by formulating the contravariant tensor, the current density of four, and the relativistic second Maxwell equation. The form of the equation is the differential equation with the solution being the radial, polar, azimuth, and time functions. The neutron stars is assumed to have a bipolar magnet, rapidly rotating conductor, and accreting. The equation obtained has the same pattern as a slow-rotating neutron star. But if it is assumed that the neutron star does not perform accretion, then the star's rotation speed affects the magnetic field by showing the presence of polar, azimuthal, and time components.

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