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Simulation Study of Kalman-Bucy filter Based Optimal Yaw Rate Control System for Autonomous Tractor

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Abstract. Unstructured agricultural field environment and varying jobs need to be done by a tractor bring the autonomous tractor subjected into the changes of its system dynamics. Due to this condition, development of autonomous tractor yaw rate dynamics control system is a challenging study. An observer based optimal controller is employed to control the autonomous tractor yaw rate dynamics control system in this simulation study. Linear quadratic regulator (LQR) is used as the optimal control algorithm, while the Kalman-Bucy filter is used as the state observer of the autonomous tractor. This Kalman based LQR method works by combination of optimization and state estimation approaches. Based on the proposed method, the LQR controller provides satisfactory yaw rate controller results. The yaw rate estimation error which is ranged at ± 0.05 deg/sec proves that Kalman-Bucy filter provides satisfactory estimation results.

1. Introduction

As the human population continues to grow while the availability space for food production is limited, the efficient usage of the agricultural resources such as biomass and machinery becomes the primary concern. A solution proposed by researchers are smart farming where autonomous tractor is one of the solution to improve the agricultural machinery usage efficiency. Developing an autonomous tractor is a challenging study. Steering and trajectorycontrol become one of the difficulties in the autonomous tractor development. The tractor generally subjected into unstructured agricultural field environments. There are several factors that can be associated with tractor working environment such as soil types, soil irregularities, varying driving speed according to the type of tractor job, and varying implement loads in which tractors have many types of implement depended on which job is tractor operating [1].

Trying to overcome these challenges, a tractor adaptive steering controller is developed. A model reference adaptive control (MRAC) is developed on a tractor to compensate the yaw rate dynamics by using adaptive the feed-forward yaw rate control system [2]. A self-tuning regulator is also developed to control the tractor yaw rate dynamics with variations in speed and implement forces [1]. The method proposed in the study in [1] is developed based on the pole placement control system design with a minimum-degree pole placement. Understanding the yaw rate dynamics model is important so that in such way a controller can be designed properly. Tractor yaw rate dynamics are modelled and identified in order to develop a tractor speed controller [3] and to understand the relationship between tractor and the agricultural field conditions [4]. New approach of tractor yaw rate modelling is proposed by considering also the implement carried by the tractor during the job [1].

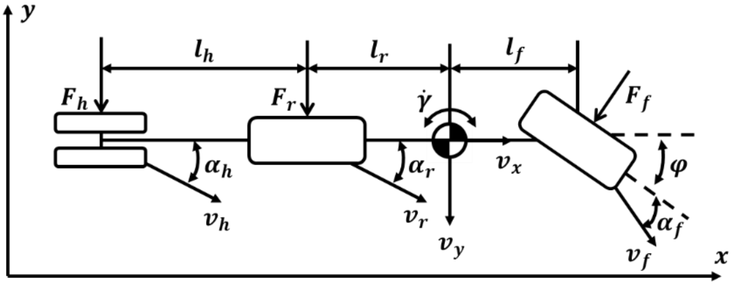

Figure 1. Tractor-implement bicycle free body diagram

A novel approach of yaw rate dynamic control system is proposed by using an optimal control algorithm based on state observer. Linear quadratic regulator is proposed as the optimal control algorithm while the Kalman-Bucy filter is proposed as the autonomous tractor states observer. This paper consists of four more sections. The second section describes the dynamic modelling and open-loop step response analyses. The third section describes the proposed methodology in this study. The fourth section describes the results and discussion. While the fifth section describes the conclusion from this study.

2. Tractor-Implement Analysis

2.1. Tractor Yaw Rate Dynamic Modelling

Understanding the tractor dynamics and kinematics is important to develop a proper controller. The tractor-implement model is shown in figure 1. Where, $\dot{\gamma}$ is the yaw rate tractorin centre of gravity, φ is the steering angle, α_f is the front wheel slip angle, α_r is the rear wheel slip angle, and α_h is the implement slip angle. $l_f$and l_r are distances from front and rear axis to the tractor center of gravity respectively, while l_h is distance from rear axis to the implement. F_f,F_r, and F_h are lateral force at front, rear, and implement respectively. In this condition the longitudinal velocity v_x is constant, therefore no longitudinal acceleration $a_x$in which the longitudinal forces are neglected. Hence, the tractor yaw rate dynamics can be expressed by equation of motions as follows,

$$\sum F_y = m\, a_y \tag{1}$$

$$\sum M_{CG} = I_{zz}\ddot{\gamma} \tag{2}$$

as the tractor longitudinal acceleration is null, the lateral acceleration is expressed as,

$$a_y = \dot{v}_y + \dot{\gamma} v_x \tag{3}$$

assuming constant and proportional to the slip angles, the lateral forces are described as follows,

$$F_f = -C_{\alpha f}\alpha_f \tag{4}$$

$$F_r = -C_{\alpha r}\alpha_r \tag{5}$$

$$F_h = -C_{\alpha h}\alpha_h \tag{6}$$

with$C_{\alpha f}$,$C_{\alpha r}$, and $C_{\alpha h}$ are the front wheel, rear wheel, and implement cornering stiffness withvalues are varying depended on the working environment and the agricultural job performed.

Based on figure 1 with assumption that the tractor model as a rigid body, the relationship between slip angle, steering angles, yaw rate, and linear velocity can be described as follows,

$$\tan(\alpha_f + \varphi) = \frac{v_y + \dot{\gamma} l_f}{v_x} \tag{7}$$

$$\tan(\alpha_r) = \frac{v_y - \dot{\gamma} l_r}{v_x} \tag{8}$$

$$\tan(\alpha_h) = \frac{v_y - \dot{\gamma}(l_r + l_h)}{v_x} \tag{9}$$

Using small angle approximation, the nonlinear terms of tractor model can be linearized in which the slip angles are described as follows,

$$\alpha_f = \frac{v_y + \dot{\gamma} l_f}{v_x} - \varphi \tag{10}$$

$$\alpha_r = \frac{v_y - \dot{\gamma} l_r}{v_x} \tag{11}$$

$$\alpha_h = \frac{v_y - \dot{\gamma}(l_r + l_h)}{v_x} \tag{12}$$

Substituting eq. (3) to (6) and eq. (10) to (12) into eq. (1) and (2), the equation of motion can be expanded as,

$$m\left(\dot{v}_y + \dot{\gamma} v_x\right) = -C_{\alpha f}\left(\frac{v_y + \dot{\gamma} l_f}{v_x} - \varphi\right) - C_{\alpha r}\left(\frac{v_y - \dot{\gamma} l_r}{v_x}\right) - C_{\alpha h}\left(\frac{v_y - \dot{\gamma}(l_r + l_h)}{v_x}\right) \tag{13}$$

$$I_{zz}\ddot{\gamma} = -l_f C_{\alpha f}\left(\frac{v_y + \dot{\gamma} l_f}{v_x} - \varphi\right) + l_r C_{\alpha r}\left(\frac{v_y - \dot{\gamma} l_r}{v_x}\right) + (l_r + l_h)\, C_{\alpha h}\left(\frac{v_y - \dot{\gamma}(l_r + l_h)}{v_x}\right) \tag{14}$$

Based on eq. (13) and (14) the state space representation can be described as follows,

$$\dot{x}(t) = Ax(t) + Bu(t) + Bw(t) \tag{15}$$

$$y = Cx(t) + Du(t) + v(t) \tag{16}$$

where$w(t)$ is disturbance, $v(t)$ is measurement noise, while $x(t), u(t), A, B, C, D$ are as follows,

$$x(t) = [v_y\,, \dot{\gamma}]^T \tag{17}$$

$$u(t) = \varphi \tag{18}$$

$$A = \begin{bmatrix} \frac{-(C_{\alpha f} + C_{\alpha r} + C_{\alpha h})}{m\, v_x} & \frac{-l_f C_{\alpha f} + l_r C_{\alpha r} + (l_r + l_h)\, C_{\alpha h}}{m\, v_x} - v_x \\ \frac{-l_f C_{\alpha f}\alpha_f + l_r C_{\alpha r}\alpha_r + (l_r + l_h)\, C_{\alpha h}\alpha_h}{I_{zz} v_x} & \frac{{l_f}^2 C_{\alpha f} + l_r C_{\alpha r} + (l_r + l_h)\, C_{\alpha h}}{I_{zz} v_x} \end{bmatrix} \tag{19}$$

$$B = \begin{bmatrix} \frac{C_{\alpha f}}{m} \\ \frac{l_f\, C_{\alpha f}}{I_{zz}} \end{bmatrix} \tag{20}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{21}$$

$$D = [0] \tag{22}$$

Based on the state space matrix described in eq. (17) to (22) and by using the Laplace Transform, the tractor steering angle and yaw rate continuous time transfer function can be derived as follows,

$$G(s) = \frac{\dot{\gamma}(s)}{\varphi(s)} = \frac{B_{21}s + (B_{11}A_{21} - B_{21}A_{11})}{s^2 - (A_{11} + A_{22})s + (A_{11}A_{22} - A_{12}A_{21})} \tag{23}$$

Where A_{ij} with i = 1,2 and j = 1,2 andB_{mn} with m = 1 and n = 1,2 are the matrix elements of state space matrix in eq. (19) and (20).

2.2. Open-loop System Step Response Analysis

As it is described before that the autonomous tractor system implement cornering stiffness $C_{\alpha h}$ is changed dynamically according to the working environment and the agricultural job performed, the open-loop autonomous tractor transfer function step response analysis helps to understand how the system behaves according to the condition described. In this simulation, the open-loop step response analysis is done based on the condition that the implement cornering stiffness and tractor longitudinal velocity are varying. By using eq. (23), the analysis is done by using two longitudinal velocity (tractor working speed)v_x variations such as, 2.5 m/s and 10 m/s representing the low and high working speed, and eight cornering stiffness $C_{\alpha h}$ variations generated by random integer number between 0 to 5000

N/deg representing the variations of tractor agricultural jobs. Approximated geometric and dynamic parameters of a 130-170 HP tractor and implementare used in this simulation shown in table 1.

Table 1.Tractor-Implement Parameters.

Tractor Parameters		
Data	**Value**	**Unit**
l_f	1.57	m
l_r	3.2	m
l_h	2.1	m
m	11000	Kg
I_{zz}	18500*	Kg.m
$C_{\alpha f}$	2400*	N/deg
$C_{\alpha r}$	5000*	N/deg
$C_{\alpha h}$	0-5000	N/deg
v_x	0-10	m/s

*parametersareobtained from [2]

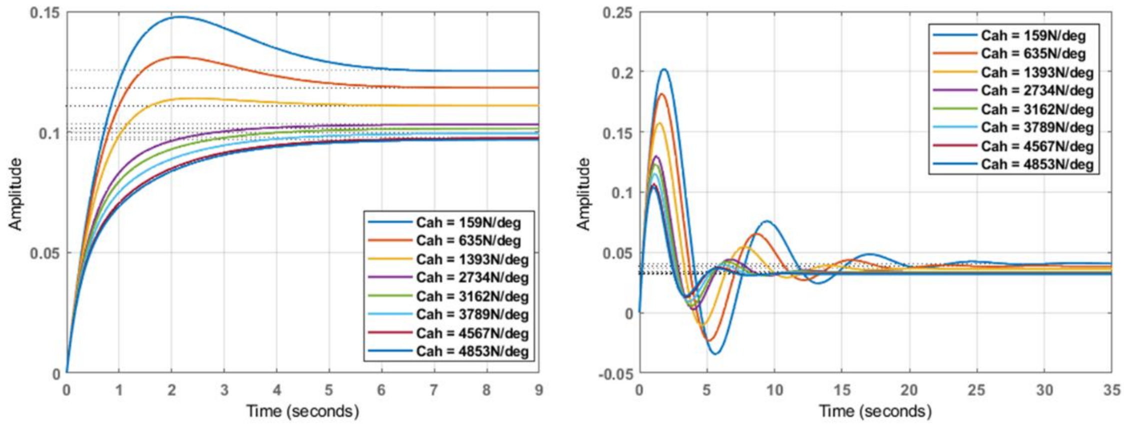

Figure 2. Tractor open-loop system step response: low (left) and high (right) working speed.

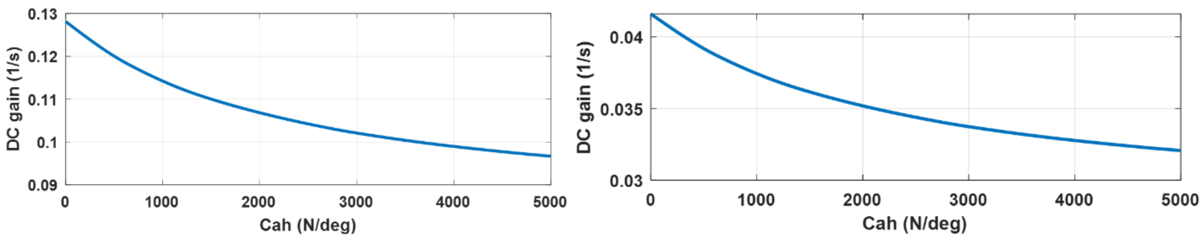

Figure 3. DC gain vs $C_{\alpha h}$: low (left) and high (right) working speed.

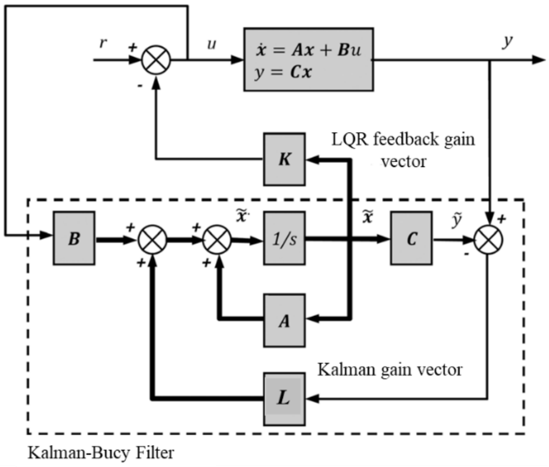

Figure 4. Kalman-Bucy filter based LQR control diagram.

Based on the open-loop step response in figure 2, at both low and high working speed the higher the cornering stiffness the higher the steady state error. While at the high working speed the system transient response showing the high oscillatory behavior. A plot in figure 3 shows the dc gain of the open-loop system, transfer function response with respect to the cornering stiffness variations. According to the figure 2 and 3, it can be concluded that the yaw rate response is directly affected by the variations of cornering stiffness. This simply can be translated as the tractor maneuverability is affected by the types of working environment and agricultural job performed. Hence, according to the described phenomenon, an optimal control method is proposed with Kalman-Bucy filter to observe the changing autonomous tractor states due to the changing of the working environment and the agricultural job performed.

3. Methodology

The yaw rate control systemhas to be designed withsatisfactory performance, fast response, no overshoot or oscillatory behaviour, and accurate response with minimum steady-state error. To achieve those requirements, a linear quadratic regulator (LQR) algorithm with Kalman-Bucy filter is proposed. Figure4 shows the Kalmanbased LQR control diagram. This combination estimator state-feedback controller is adapted from [5].

3.1. Linear Quadratic Regulator

LQR is an optimal control strategy that allows us to apply a full-state feedback to a dynamic system in such that the satisfactory performance is achieved. The objective of LQR is to place the poles of the system in optimal location so that the system closed-loop will minimize the cost function below,

$$J = \int (x^T Qx + u^T Ru)dt \tag{24}$$

where $Q \in \mathbb{R}^{n\,x\,n}$ is a positive semidefinite matrix as the system states weight, and $R \in \mathbb{R}^{m\,x\,m}$ is a positive definite matrix as the system controller input weight. The system states weight is obtained by $Q = CC^T$, and the system controller input weight is obtained by $R = \lambda I$ with $\lambda > 0$ and I as the identity matrix. In order to tune the Q and R weight there is a trade-off. If in some cases the system states are more considered then the Q should be tuned higher than R, and this is called cheap control method. While in some cases that the system control effort is more considered then the R should be tuned higher than Q, and this is called as expensive control method.

The input controller is described as follows,

$$u(t) = -Kx(t) \tag{25}$$

where K is LQR gain matrix obtained by using described equation,

$$K = R^{-1}B^T P \tag{26}$$

with *P*is obtained by solving this following Algebraic Riccati Equations,

$$\dot{P} = A^T P + PA - PBR^{-1}B^T P + Q \tag{27}$$

Matrix *A* and *B*are the dynamic system model state-space in eq. (19) and (20).

3.2. *Kalman-Bucy filter*

In this study, the dynamic of the system keeps changing according to working environment conditions and types of agricultural job, and this condition may bring a reality that the system states will not always be measurable. So, a Kalman-Bucy filter is employed to estimate the full states ($x(t)$) of the system and provide accurate estimated states for the control effort by the LQR. The Kalman-Bucy filter is described by the following equation,

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + L(y(t) - \tilde{y}(t)) \tag{28}$$

where the Kalman gain L can be obtained by this following equation,

$$L = PC^T R_v^{-1} \tag{29}$$

with $R_v$is a measurement noises covariance and *P*is obtained by solving this following Algebraic Riccati Equations,

$$\dot{P} = AP + PA^T - PC^T R_v^{-1} CP + BR_w B^T \tag{30}$$

with R_w is a disturbance covariance. Both R_w and R_v are tuned manually. Matrix A, B, and C are the dynamic system model state-space in Eq. (19), (20), and (21).

4. Results and Discussions

As our simulation study, two sets of tractor working condition simulation are performed: one is low working speed at 2.5 m/s with two variations of cornering stiffness at 500 N/deg and 4500 N/deg, and one is high working speed at 10 m/s with two variations of cornering stiffness at 500 N/deg and 4500 N/deg. Based on these sets of simulation closed-loop step response analyses are done. Figure 5 and 6 show the closed-loop step response of the tractor yaw rate dynamics at low and high working speed respectively. It can be seen that the LQR control algorithm results track the reference with satisfactory, while the Kalman-Bucy filter can estimate the response also with satisfactory. Both at low and high working speed with low and high cornering stiffness show the stable response without overshoot oscillation in transient response and with minimum steady state error.

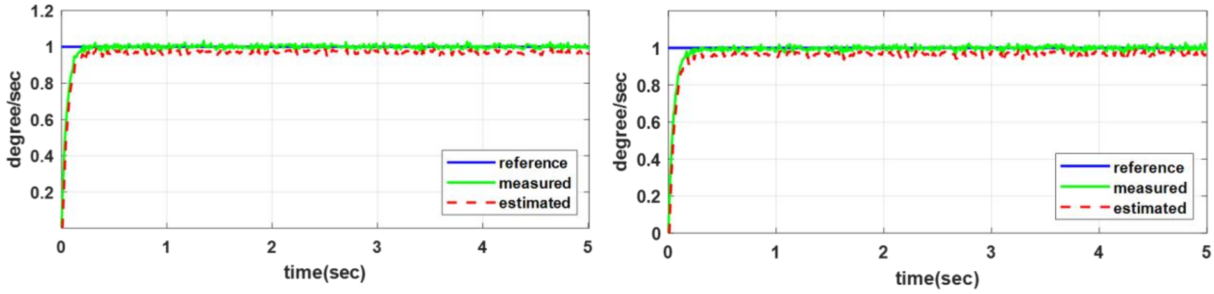

Figure 5. Closed-loop yaw rate controller step response at low working speed: $C_{\alpha h}$ = 500 N/deg (left) and $C_{\alpha h}$ = 4500 N/deg (right).

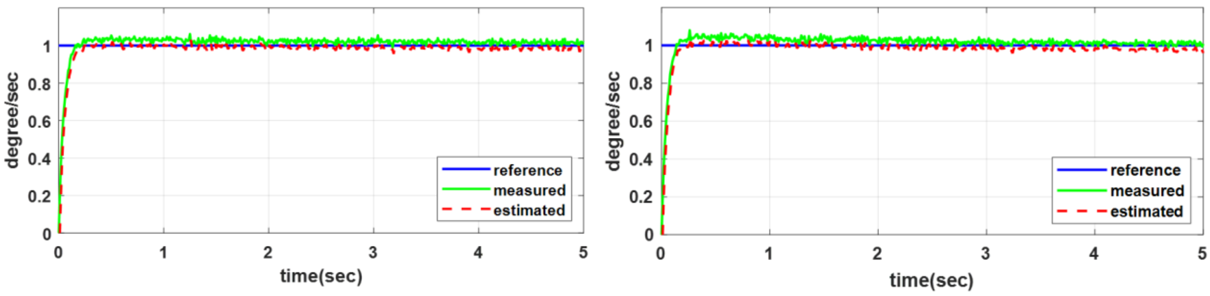

Figure 6. Closed-loop yaw rate controller step response at high working speed: $C_{\alpha h} = 500$ N/deg (left) and $C_{\alpha h} = 4500$ N/deg (right).

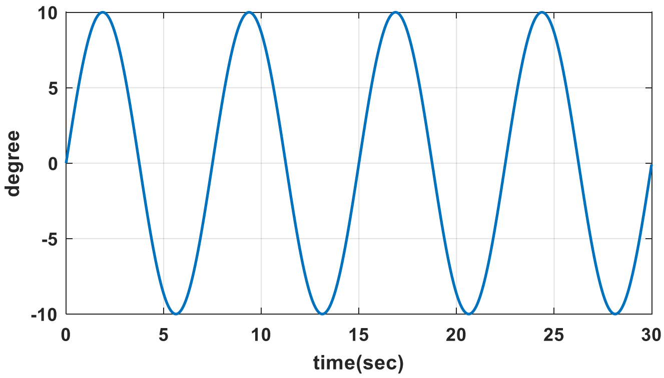

Figure 7. Tractor steering angle as controller input.

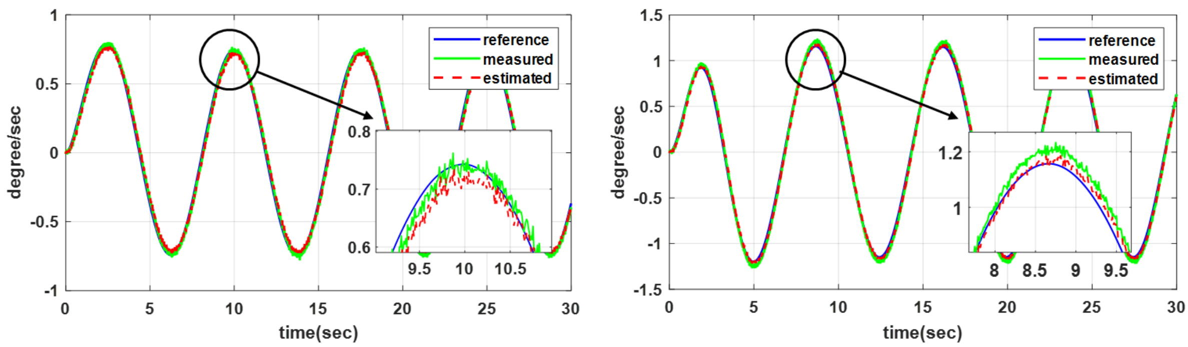

Figure 8. Yaw rate controller output tracking response at $C_{\alpha h} = 4500$ N/deg: low working speed at $v_x = 2.5$m/s (left) and high working speed at $v_x = 10$ m/s (right).

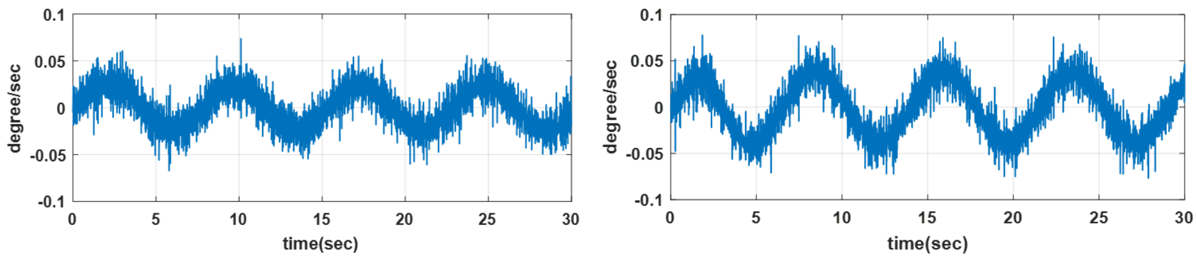

Figure 9. Yaw rate controller estimation error response at $C_{\alpha h} = 4500$ N/deg: low working speed at $v_x = 2.5$m/s (left) and high working speed at $v_x = 10$ m/s (right).

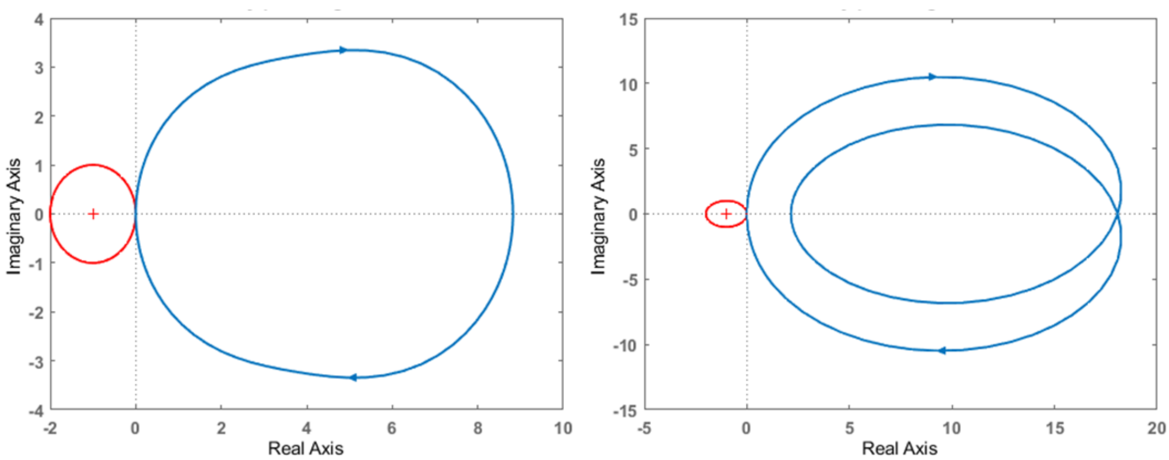

Figure 10. Nyquist plot of yaw rate controller: low working speed at $v_x = 2.5$m/s (left) and high working speed at $v_x = 10$ m/s (right)

Besides the step response, the tracking response analysis of tractor yaw rate dynamics is done as well. In this tracking response low and high working speed are performed with only high cornering stiffness at 4500 N/deg. This tracking analysis is done by giving a steering angle as the controller input shown in Figure 7. The tracking responses are shown in figure 8. The tracking responses show that the LQR control algorithm provides satisfactory tracking result with respect to the reference signal. Figure 9 shows the yaw rate controller estimation error. Based on the figure 9, the Kalman-Bucy filter provides satisfactory estimation with respect to the measurement signal. The stability of proposed controller method is shown by the Nyquist plot in figure 10. In which the Nyquist plot for both low and high working speed avoid the unit circle centered in -1.

5. Conclusions

A Kalman-Bucy filter linear quadratic regulator control algorithm is implemented to control the autonomous tractor yaw rate dynamics as the simulation based study.By using the small angle approximation, the nonlinear terms of tractor dynamics model are linearized in which the linear control theory advantages can be applied. Since the tractor states are subjected to the dynamic changing with respect to the working environment and its types of agricultural job performed, a cheap control method is used for the linear quadratic regulator control algorithm in which the states of the system are more considered. While the Kalman-Bucy filter is employed to provide the full states estimation. According to the Nyquist plot, the proposed control system is stable in any working conditions. This presented results are limitedin simulation study, a further real-time application with real measured tractor parameters might need to be done in order to verify the proposed method.

6. References

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