International Journal of Mathematical, Engineering and Management Sciences

ISSN: 2455-7749

The *International Journal of Mathematical, Engineering and Management Sciences* (IJMEMS) is an online scientific journal which publish bi monthly (6 issues per year) technical and more informal communications directed to a large readership, as well as articles/review papers/case studies that demonstrate interaction between various disciplines such as *mathematical sciences, engineering sciences, engineering and technology, management sciences, numerical and computational sciences*. Research papers, expository and survey papers of high quality are considered from any discipline. In submitting the papers, author(s) must guarantee that they are not submitted elsewhere simultaneously.

Articles in *International Journal of Mathematical, Engineering and Management Sciences* are Open Access articles published under the Creative Commons CC BY License Creative Commons Attribution 4.0 International License http://creativecommons.org/licenses/by/4.0/ . This license permits use, distribution and reproduction in any medium, provided the original work and source is properly cited. The publisher of the journal is IJMEMS, India itself.

Acceptance Rate Around: **Below 10%**
CiteScore 2018: **0.68**
CiteScoreTracker 2019: **2.58** (updated monthly)

FAQ

Important Information: It is requested to manuscript submitter/corresponding author, kindly do not contact telephonically to the Mentor, Advisor, Editorial Board Members & EiC of IJMEMS. Most of the answers of author's queries are available on website. The corresponding author must submit his/her article(s) online through e-mail at eicijmems@ijmems.in only. There is NO other process to submit the articles to IJMEMS. Before submitting the article(s), author(s) must read the "Ethical Issues" and format the article(s) as per IJMEMS requirements at "For Authors".

Privacy Policy| Terms & Conditions

International Journal of Mathematical, Engineering and Management Sciences

ISSN: 2455-7749

- Scopus
- SJR (0.21, 2018, Q2)- SCImago
- DOAJ-Directory of Open Access Journals
- Crossref
- Publons
- CNKI
- UGC Approved
- J-Gate
- ResearchGate
- Google Scholar
- Academic Recourse Index- Research Bib
- World Catalogue of Scientific Journals

Disclaimer about Indexing of IJMEMS

The criteria for indexing is solely dependent on the respective indexing agencies. Hence, it is imperative that based on earlier indexing; we cannot predict the indexing policy as of today, since indexing belongs to a third party (i.e. indexing agencies). The indexing agencies have full right to continue or discontinue to the journal at any time without prior intimation to the journal. Therefore, please neither send any questions nor expect any answers from us on behalf of a third party i.e. indexing agencies. For authentic information, please visit to the respective indexing agency website time to time.

Privacy Policy| Terms & Conditions

International Journal of Mathematical, Engineering and Management Sciences

ISSN: 2455-7749

Editor-in-Chief

Prof. (Dr.) Mangey Ram
Department of Mathematics; Computer Science and Engineering,
Graphic Era (Deemed to be University), Dehradun, INDIA.
e-mail: eicijmems@ijmems.in
Research Areas - *Operations Research, Reliability Theory, Applied Mathematics.*

Editorial Members

Prof. (Dr.) S. B. Singh
Department of Mathematics, Statistics & Computer Science,
G. B. Pant University of Agriculture & Technology,
Pantnagar, INDIA.
Research Areas - *Operations Research, Reliability Theory, System Engineering.*

Prof. (Dr.) J. Paulo Davim
Department Mechanical Engineering,
University of Aveiro, Campus Santiago, Aveiro, PORTUGAL.
Research Areas - *Machining, Tribology, Mechanical Engineering.*

Dr. Ilia Frenkel
Centre for Reliability and Risk Management,
Sami Shamoon College of Engineering, Bialik/Basel Sts. Beer Sheva, ISRAEL.
Research Areas - *Reliability Theory, System Engineering, Multi-State Systems.*

Dr. Olga Fink
ZHAW School of Engineering,
Institute for Data Analysis and Process Design (IDP),
ZHAW Zurich University of Applied Sciences, SWITZERLAND.
Research Areas - *Fault Detection, Machine Learning, Reliability Engineering.*

Prof. (Dr.) Om Prakash Yadav

Department of Industrial and Manufacturing Engineering,
North Dakota State University, Fargo, USA.
Research Areas - *Quality & Reliability Engineering, Robust Product/Process Design, Concurrent Engineering, TQM, Lean Manufacturing, Six Sigma Methodologies, Production & Operations Management, Optimization Techniques, Supply-Chain Management, Fuzzy Logic & Neural Networks, Quantitative Analysis of Operations Management.*

Prof. (Dr.) Aliakbar Montazer Haghighi
Department of Mathematics,
Prairie View A & M University, Prairie View, USA.
Research Areas - *Probability, Statistics, Queuing Theory.*

Prof. (Dr.) Ilia Vonta
Department of Mathematics,
School of Applied Mathematical and Physical Sciences,
National Technical University of Athens, Zografou, GREECE.
Research Areas - *Survival Analysis & Semi Parametric Statistics, Medical Statistics, Model Selection, Biostatistics.*

Prof. (Dr.) Vinod Kumar
Department of Mathematics, Statistics & Computer Science,
G. B. Pant University of Agriculture & Technology, Pantnagar, INDIA.
Research Areas - *Mathematical Statics, Time Series, Reliability Theory.*

Dr. Akshay Kumar
Department of Mathematics,
Graphic Era Hill University, Dehradun Campus, INDIA.
Research Areas - *Operations Research, Reliability Theory, Fuzzy Reliability, System Engineering.*

Prof. (Dr.) Mustapha Nourelfath
Faculty of Science and Engineering,
Department of Mechanical Engineering,
Université Laval, Quebec, CANADA.
Research Areas - *Industrial Engineering & Operations Research, Design & Planning of Supply Chains, Reliability & Maintenance, Quality, Decision Making, Modeling & Simulation, Stochastic Optimization, Game Theory.*

Dr. Arpan Gupta
School of Engineering,
Indian Institute of Technology, Mandi, INDIA.
Research Areas - *Acoustics, Metamaterials, Vibrations, Bio-Mechanics, Computational Methods, Robotics.*

Dr. Rajesh Joshi
G. B. Pant Institute of Himalayan Environment and Development,
An autonomus Institue of Ministry of Environment, Forest & Climate Change, Government of India,
Kosi-Katarmal, Almora, INDIA.
Research Areas - *Soft Computing Techniques, Environmental & Hydrological Modeling, Climate Impact Studies.*

Prof. (Dr.) Alex Karagrigoriou
Department of Mathematics,
Division of Statistics & Actuarial-Financial Mathematics,
University of the Aegean, Karlovasi, GREECE.
Research Areas - *Model Selection, Information Theory & Divergence Measures, Goodness of Fit Tests, Reliability Theory, Applied Probability.*

Dr. Anuj Kumar
Department of Mathematics,
University of Petroleum & Energy Studies, Dehradun, INDIA.
Research Areas - *Reliability Theory, Wavelet Theory, Nature Inspired Optimization Algorithms.*

Prof. (Dr.) Naoto Kaio
Department of Economic Informatics, Faculty of Economic Sciences,
Hiroshima Shudo University, Hiroshima, JAPAN.
Research Areas - *Mathematical Theory of Reliability, Maintainability.*

Dr. Lokesh Kumar Joshi
Department of Mathematics,
Faculty of Engineering and Technology,
Gurukula Kangari University, Haridwar, INDIA.
Research Areas - *Fuzzy Logic Applications.*

Dr. Dinesh Bisht
Department of Mathematics,
Jaypee Institute of Information Technology, Noida, INDIA.
Research Areas - *Soft Computing, Particle Swarm Optimization.*

Dr. Navneet Joshi
Department of Mathematics,
Graphic Era Hill University, Bhimtal Campus, Nainital, INDIA.
Research Areas - *Computational Fluid Dynamics, Mathematical Modelling, Simulation, Heat & Mass Transfer.*

Dr. William Vesely

Technical Risk Assessment,
Office of Safety and Mission Assurance,
NASA Headquarters, Washington, USA.
Research Areas - *Aerospace Engineering, Fault Detection, Reliability Engineering, Risk Assessment.*

Dr. Yuriy V. Kostyuchenko
Workgroup on Disaster Study and Risk Analysis,
Scientific Centre for Aerospace Research of the Earth,
National Academy of Sciences of Ukraine, Honchar Street, Kiev, UKRAINE.
Research Areas - *Risk Assessment, Environmental Studies, Vulnerability and Reliability.*

Dr. Phuc Do
Lorraine University,
Campus Sciences BP09, 06 Vandoeuvre Cedex, FRANCE.
Research Areas - *Reliability, Maintenance, Prognostics.*

Prof. (Dr.) Bharatendra K. Rai
Department of Decision and Information Sciences,
Charlton College of Business,
University of Massachusetts – Dartmouth, USA.
Research Areas - *Multivariate Diagnosis, Pattern Recognition & Data Mining, Developing Meta-Models Using Computer Experiments, Prediction of Unexpended Warranty Costs, Field Performance Studies from Large Warranty Datasets.*

Prof. (Dr.) John Andrews
The Lloyd's Register Foundation Centre for Risk and Reliability Engineering,
Nottingham Transportation Engineering Centre, Faculty of Engineering,
University of Nottingham, Nottingham, UK.
Research Areas - *System Reliability, Maintenance Modelling, Asset Management, Fault Diagnostics, Optimisation, Risk Assessment.*

Prof. (Dr.) Tadashi Dohi
Department of Information Engineering,
Graduate School of Engineering,
Hiroshima University, Hiroshima, JAPAN.
Research Areas - *Reliability & Maintenance, Software Engineering, Dependable Computing.*

Prof. (Dr.) Madhu Jain
Department of Mathematics,
Indian Institute of Technology Roorkee, Roorkee, INDIA.
Research Area - *Stochastic Processes, Queuing Theory, Inventory Models.*

Dr. Sangeeta Pant
Department of Mathematics,
University of Petroleum & Energy Studies, Dehradun, INDIA.
Research Area - *Nature Inspired Optimization Techniques, Reliability Optimization.*

Dr. Shshank Chaube
Department of Mathematics,
University of Petroleum & Energy Studies, Dehradun, INDIA.
Research Area - *Fuzzy Reliability, Reliability Theory.*

Prof. (Dr.) Revaz Kakubava
Scientific-Research Center for System Problems,
Georgian Technical University, Kostava str., Tbilisi, GEORGIA.
Research Areas - *Reliability Theory, Queuing Theory, Mathematical Modeling & Optimization of Complex Systems.*

Prof. (Dr.) S. C. Dimri
Department of Computer Science and Engineering,
Graphic Era (Deemed to be University), Dehradun, INDIA.
Research Areas - *Advance Algorithms, Computer Graphics, Network Optimization.*

Prof. (Dr.) Priti Kumar Roy
Centre for Mathematical Biology and Ecology,
Department of Mathematics,
Jadavpur University, Kolkata, INDIA.
Research Area - *Mathematical Biology on Ecology with Special Emphasis on Transmissible Disease, Modeling on Disease Psoriasis, HIV, Enzyme Kinetics, Bio-Diesel Production.*

Prof. (Dr.) Lirong Cui
School of Management and Economics,
Beijing Institute of Technology, Beijing, CHINA.
Research Areas - *Stochastic Modeling, Quality & Reliability Engineering, Simulation & Optimization, Operations Research, Applications of Probability & Statistics.*

Prof. (Dr.) Won Young Yun
Department of Industrial Engineering,
Pusan National University, Pusan, REPUBLIC OF KOREA.
Research Areas - *System Reliability Analysis, Maintenance Optimization, Warranty Policies.*

Prof. (Dr.) Yi-Kuei Lin
Department of Industrial Engineering and Management,
Department of Electrical and Computer Engineering,
National Chiao Tung University,
1001 University Rd., Hsinchu 30010, TAIWAN.
Research Areas - *Network Analysis, System Reliability, Performance Evaluation, Service Management, Information Ethics.*

Prof. (Dr.) Gopi Chattopadhyay
Faculty of Science and Technology,
Federation University, Mount Helen VIC 0, AUSTRALIA.
Research Areas - *Asset Management, Maintenance & Reliability Engineering, Operations Management, Project Management & Quality Engineering.*

Dr. Adarsh Anand
Department of Operational Research,
University of Delhi, New Delhi, INDIA.
Research Areas - *Innovation Diffusion & Multi Generation Modeling in Marketing, Software Reliability Growth Modeling.*

Prof. (Dr.) H. M. Srivastava
Department of Mathematics and Statistics,
University of Victoria, Victoria, British Columbia VW R, CANADA.
Research Areas - *Analytic Number Theory, Inventory Modelling & Optimization, Fractional Calculus, q-Series & q-Polynomials, Real & Complex Analysis, Fractional Calculus, Integral Equations & Transforms, Higher Transcendental Functions.*

Prof. (Dr.) Patricio Franco
Department of Materials and Manufacturing Engineering,
Technical University of Cartagena, Murcia, SPAIN.
Research Areas - *Manufacturing Processes, Dimensional Metrology, Metal Forming Processes, Joining & Welding Processes, Optimization Of Manufacturing Systems, Numerical Modelling Of Manufacturing Processes, CAD/CAE/CAM/CAPP, Quality Assurance, Productivity Improvement.*

Prof. (Dr.) Suresh Kumar Sharma
Centre for Systems Biology and Bioinformatics,
Panjab University, Chandigarh, INDIA.
Research Areas - *Statistical Modelling, Biostatistics, Ranking & Selection, Statistical Inference, Distribution Theory.*

Dr. Anatoly Lisnianski
The Israel Electric Corporation Ltd.,
Planning, Development & Technology Division, Haifa, ISRAEL.
Research Areas - *Reliability, Risk Analysis, Maintenance.*

Dr. Sanjay Kumar Chaturvedi
Subir Chowdhury School of Quality and Reliability,
Indian Institute of Technology Kharagpur, Kharagpur, INDIA.
Research Areas - *System Reliability Modelling & Analysis, Reliability Data Analysis, Reliability Estimation, Maintenance Engineering.*

Dr. Geeta Arora
Department of Mathematics,
Lovely Professional University, Phagwara, INDIA.
Research Areas - *Numerical Analysis, Partial Differential Equations.*

Prof. (Dr.) J. K. Pandey
Department of Research & Development,
University of Petroleum & Energy Studies, Dehradun, INDIA.
Research Areas - *Nano-technology, Bio-polymers, Water Treatment and Composites.*

Dr. Jitendra Pal Singh
Advanced Analysis Center,
Korea Institute of Science and Technology, Seoul, SOUTH KOREA.
Research Areas - *Nanomaterials, Magnetism, Electronic Structure.*

Dr. Surya Parkash
National Institute of Disaster Management (NIDM),
Ministry of Home Affairs, Government of India, New Delhi, INDIA.
Research Areas - *Landslides, Avalanches, Earthquakes, Floods, Tsunami, GLOFs, LLOFs, Dam Bursts, Mining Disasters, Community Based Disaster Risk Management, Damage and Loss Assessment, Disaster Safe Hill Area Development, Environmental Management.*

Dr. Sachin Kumar Mangla
Plymouth Business School,
University of Plymouth, UK.
Research Areas - *Green Supply Chain Management, Sustainability, Risk Management, Decision Support System/MCDM, Operations Management.*

Dr. Rajeev Singh
Department of Computer Engineering,
G. B. Pant University of Agriculture & Technology, Pantnagar, INDIA.
Research Areas - *Data Structures, Algorithms, Computer Networks, Network Security.*

Dr. Santosh Kumar
Department of Computer Science and Engineering,
Graphic Era (Deemed to be University), Dehradun, INDIA.

Research Areas - *Ubiquitous Computing, Network Security, Internet on things, Software Engineering.*

Dr. Amit Kumar
Department of Mathematics,
Lovely Professional University, Phagwara, INDIA.
Research Areas - *Reliability Theory, System Engineering, Multi-State Systems.*

Dr. Nupur Goyal
Department of Mathematics,
Roorkee Institute of Technology, Roorkee, INDIA.
Research Areas - *Reliability Theory, System Engineering, Multi-State Systems.*

Prof. (Dr.) Ali Muhammad Ali Rushdi
Department of Electrical and Computer Engineering,
King Abdulaziz University, Jeddah 19, SAUDI ARABIA.
Research Areas - *System Reliability, Boolean Analysis and Reasoning, Analysis and Design of Algorithms, Engineering Mathematics.*

Prof. (Dr.) G. S. Ladde
Department of Mathematics and Statistics,
University of South Florida, 0 East Fowler Avenue,Tampa, Florida, USA.
Research Areas - *Hereditary Dynamic Systems; Local Lagged Generalized Method of Moments (LLGMM); Multi-agent & Multi-Market/Finance; Multivariate/Large-Scale Systems Analysis; Stability Stochastic Hybrid Dynamical; Stochastic Modeling of Dynamical Processes in Biological, Chemical, Engineering, Medical, Physical and Social Sciences; Stochastic Modeling of Network Dynamic; Stochastic System Parameter and State Estimation; Time Series Analysis and Applications.*

Dr. Harish Garg
School of Mathematics,
Thapar Institute of Engineering and Technology University, Patiala, INDIA.
Research Areas - *Multi Criteria Decision Making, Reliability, Fuzzy Optimization, Multi Objective Optimization, Aggregation Operators, Computational Intelligence, Optimization Techniques, Various Nature Inspired Algorithms (E.G. Genetic Algorithms, Swarm Optimization, Artificial Bee Colony Etc.,), Fuzzy and Intuitionistic Fuzzy Set Theory, Expert Systems. Application Areas Include Wide Range of Industrial and Structural Engineering Design Problems.*

Prof. (Dr.) Santosh Kumar
Department of Mathematics and Statistics,
University of Melbourne, Melbourne, AUSTRALIA.
Research Areas - *Operations Research, Optimization Techniques, Applied Mathematics.*

Prof. (Dr.) Ljubisa Papic

DQM Research Center, Prijevor, SERBIA.
Research Areas - *Reliability Engineering, Maintainability Engineering, Failure Analysis, Safety Analysis, Quality Management.*

Prof. (Dr.) Piotr Kulczycki
Centre of Information Technology for Data Analysis Methods,
Systems Research Institute,
Polish Academy of Sciences, Warsaw, POLAND.
Research Areas - *Data Analysis, Mathematical Statistics, Control Engineering, Mathematical Modelling, Computational Intelligence.*

Dr. Garima Chopra
Department of Mathematics,
University Institute of Engineering & Technology,
Maharshi Dayanand University, Rohtak, INDIA.
Research Areas - *Wavelet Analysis, Image Processing, Reliability Analysis.*

Prof. (Dr.) P. K. Kapur
Former Professor, University of Delhi,
Amity Center for Interdisciplinary Research,
Amity University, Noida, INDIA.
Research Areas - *Software Reliability, Modeling, Optimization, Marketing Research, Vulnerability Discovery Modeling.*

Dr. Ritu Arora
Department of Mathematics,
Gurukul Kangari University, Haridwar, INDIA.
Research Areas - *Mathematical Modeling, Fuzzy Logic, Wavelet Theory, Applied Mathematics.*

Prof. (Dr.) Chandra K. Jaggi
Department of Operational Research,
Faculty of Mathematical Sciences,
New Academic Block, University Of Delhi, Delhi, INDIA.
Research Areas - *Supply Chain Management, Inventory Management, Trade Credit, Two Warehousing Models.*

Prof. (Dr.) J. P. Singh Joorel
Department of Statistics,
University of Jammu, Jammu, INDIA.
Research Areas - *Operations Research, Reliability Theory, Sampling & amp; Statistical Inference.*

Dr. Kanchan Das
Department of Technology Systems,

College of Engineering and Technology,
East Carolina University, USA.
Research Areas - *Operations Management, Inventory Planning, Supply Chain Management, Operations Research, Management Science; Quality Assurance.*

Prof. (Dr.) Jianqiang Gao
College of Computer and Information Engineering,
Hohai University, Nanjing 1009, CHINA.
Research Areas - *Remote Sensing Image Classification, Image Processing, Information Processing System and Pattern Recognition.*

Dr. Bidyut Bikash Gogoi
Space Navigation Group,
Indian Space Research Organization, ISRO, INDIA.
Research Areas - *u1-69Heat and Mass Transfer, Mathematical Biology, Computational Fluid Dynamics, Numerical Methods for PDEs, Scientific Computing, Gas Dynamics, Lattice Boltzmann Methods, Time Synchronization, Navigation, Precision Farming, Numerical Analysis.*

Dr. Bhagawati Prasad Joshi
Department of Applied Sciences,
Seemant Institute of Technology, Pithoragarh, INDIA.
Research Areas - *Soft Computing Techniques: Theories and Applications, Reliability Theory, Decision Making under Uncertain Environment.*

Prof. (Dr.) Hoang Pham
Department of Industrial & Systems Engineering,
School of Engineering,
Rutgers State University, New Jersey, USA.
Research Areas - *Reliability Engineering, Optimization, Statistical Inference.*

Prof. (Dr.) Antonella Petrillo
Department of Engineering,
University of Napoli "Parthenope", ITALY.
Research Areas - *Operation Research, Industrial Engineering, Product Life Cycle Management, Business Model, Decision Analysis and Decision Support System, Modeling and Simulation, Safety-Critical System Performance.*

Prof. Xiao-Zhi Gao
School of Computing,
University of Eastern Finland, Kuopio, FINLAND.
Research Areas - *Artificial Intelligence, Machine Learning, Data Mining, Optimization.*

Prof. (Dr.) Elżbieta Macioszek

Faculty of Transport,
Transport Systems and Traffic Engineering Department,
Silesian University of Technology, Gliwice, POLAND.
Research Areas - *Traffic Engineering, Traffic Modelling, Driver Behaviour Analysis, Transport Statistics, Optimization in Transport.*

Prof. (Dr.) Sanjeev Kumar Singh
Department of Mathematics,
University of Petroleum & Energy Studies, Dehradun, INDIA.
Research Areas - *Differential Geometry, Mathematical Modeling, Soft Computing, Data Mining.*

Prof. (Dr.) Yuri Klochkov
Director of the Monitoring Centre for Science and Education,
Department of Economics and Management in Machine Building,
Saint Petersburg Polytechnic University, Saint Petersburg, RUSSIA.
Research Areas – *Statistical Quality Control and Quality Management, Systems Reliability.*

Dr. A. P. Singh
Department of Mathematics,
S. G. R. R. (P. G.) College, Dehradun,
H.N.B. Garhwal Central University, Srinagar (Garhwal), INDIA
Research Areas - *Operations Research, Inventory Management, Optimization.*

Prof. (Dr.) Stefan Wolfgang Pickl
Computer Science Faculty, Core Competence Center C for Operations Research,
Chair for Operations Research,
Universität der Bundeswehr München, Neubiberg, GERMANY.
Research Areas - *Optimization of Complex Systems, IT Based Decision Support Systems/ Reachback Architectures, Strategic Management.*

Prof. (Dr.) Alexander Bochkov
Sustainability Analysis in the Oil and Gas Industry Department,
Centre "Risk Analysis", LLC NIIGAZECONOMIKA, Moscow, RUSSIA.
Research Areas - *Non-Stationary Processes, Risk Assessment, Analysis and Management, System Analysis and Operation Research, Vulnerability and Survivability of Large-Scale Systems.*

Prof. (Dr.) Anu A. Gokhale
Department of Technology,
Illinois State University, Normal, USA.
Research Areas - *Analytics, Computational Algorithms, Data Mining.*

Dr. Sunil Luthra
Department of Mechanical Engineering,
State Institute of Engineering & Technology (Formerly known as Government Engineering College) Nilokheri, Haryana, INDIA
Research Areas - *Operations Management, Green/Sustainable Supply Chain Management, Production and Industrial Engineering, Renewable/Sustainable Energies.*

Prof. (Dr.) Liudong Xing
Department of Electrical and Computer Engineering,
University of Massachusetts, Dartmouth, USA.
Research Areas - *Systems Reliability Engineering.*

Prof. (Dr.) Sankar Kumar Roy
Department of Applied Mathematics with Oceanology and Computer Programming,
Vidyasagar University, Midnapore, INDIA.
Research Areas - *Operations Research- Transportation Problem, Game Theory, Inventory Management and Facility Location Problem.*

Prof. (Dr.) Paulo Peças
IDMEC, Instituto Superior Técnico,
Universidade de Lisboa, Lisboa, PORTUGAL.
Research Areas - *Life Cycle Engineering, Life Cycle Cost Modelling, Life Cycle Analysis, Eco-Efficiency, Sustainable Production, Lean Manufacturing, Lean Digitalization, Lean & Industry 4.0 (Lean 4.0), Additive Manufacturing, Natural Fibre Composites.*

Dr. Vijay Kumar
Department of Mathematics,
AIAS, Amity University, Noida, INDIA.
Research Areas - *Reliability Engineering, Optimization, Modeling, Optimal Control Theory.*

Dr. Deepti Aggrawal
University School of Management & Entrepreneurship,
Delhi Technological University, East Delhi Campus, INDIA.
Research Areas - *Operational Research, Decision Sciences, Marketing Analytics.*

Dr. Saurabh Kapoor
Department of Education in Science and Mathematics,
Regional Institute of Education, Bhubneshwar,
(National Council of Educational Research and Training, Delhi),
Ministry of HRD, Government of India, INDIA.
Research Areas - *Computational Fluid Dynamics, Hydrodynamics Stability of Flow through Porous Medium, Applied Numerical Method, FDM, FEM, SCM, B Spline FEM, ICT in Mathematics, Mathematics Education.*

Prof. (Dr.) Nita H. Shah
Department of Mathematics,
University School of Sciences, Gujarat University,
Ahmedabad – 380 009, Gujarat, INDIA.
Research Areas - *Operations Research, Dynamical Systems, Applied Mathematics.*

Prof. (Dr.) Ajit Kumar Verma
Department of Fire Safety and HSE Engineering,
Western Norway University of Applied Sciences,
Haugesund, NORWAY.
Research Areas - *Reliability in Engineering Design, Software Reliability, Design and Analysis of Fault Tolerant Systems, Physics of Failures, Power System Reliability.*

Dr. Chandra Shekhar
Department of Mathematics,
Birla Institute of Technology & Science,
Pilani, 333031, Rajasthan, INDIA.
Research Areas - *Queueing theory, Stochastic processes, Computer and communication system, Reliability and maintainability, Manufacturing and machine repair problem, Reliability engineering, Fuzzy set and Fuzzy logic, Inventory Theory, Statistical inference and analysis, Evolutionary and Nature Inspired Optimization Techniques.*

Dr. Mukesh Kumar
Department of Mathematics,
Motilal Nehru National Institute of Technology Allahabad, Prayagraj, INDIA.
Research Areas - *Nonlinear Partial Differential Equations, Similarity Transformation Method, Lie Group Theory.*

Prof. (Dr.) Yigit Kazancoglu
Department of International Logistics Management,
Yasar University, Izmir, TURKEY.
Research Areas - *Operations Management, Supply Chain Management, Multi-Criteria Decision Making, Total Quality Management.*

Dr. Shailendra Rajput
Department of Electrical and Electronic Engineering,
Ariel University, P.O.B. 3, Ariel, ISRAEL.
Research Areas - *Energy Harvesting, Dielectric Materials, Piezoelectricity and Ferroelectricity.*

Prof. (Dr.) Xufeng Zhao
Nanjing University of Aeronautics and Astronautics,

No. 29, Jiangjun Avenue, Nanjing 211106, CHINA.
Research Areas - *Quality Management, Reliability Engineering, Maintenance Modeling and Optimization, Management of Information and Industrial Systems.*

Privacy Policy| Terms & Conditions

Source details

International Journal of Mathematical, Engineering and Management Sciences

Open Access

Scopus coverage years: from 2016 to 2019

Publisher: International Journal of Mathematical, Engineering and Management Sciences (IJMEMS)

E-ISSN: 2455-7749

Subject area: Mathematics: General Mathematics | Engineering: General Engineering | Business, Management and Accounting: General Business, Management and Accounting | View all

CiteScore 2018
0.68
Add CiteScore to your site

SJR 2018
0.208

SNIP 2018
0.378

View all documents > | Save to source list | Journal Homepage

CiteScore | CiteScore rank & trend | CiteScore presets | Scopus content coverage

CiteScore 2018

Calculated using data from 30 April, 2019

CiteScore rank

$$0.68 = \frac{\text{Citation Count 2018}}{\text{Documents 2015 - 2017*}} = \frac{\text{25 Citations >}}{\text{37 Documents >}}$$

*CiteScore includes all available document types

View CiteScore methodology > CiteScore FAQ >

CiteScoreTracker 2019

Last updated on *08 January, 2020*
Updated monthly

$$2.62 = \frac{\text{Citation Count 2019}}{\text{Documents 2016 - 2018}} = \frac{\text{194 Citations to date}}{\text{74 Documents to date}}$$

Metrics displaying this icon are compiled according to Snowball Metrics ↗ , a collaboration between industry and academia.

Category	Rank	Percentile
Mathematics		
General Mathematics	#153/338	54th
Engineering		
General Engineering	#147/275	46th
Business,		

View CiteScore trends >

About Scopus

What is Scopus

Content coverage

Scopus blog

Scopus API

Privacy matters

Language

% & # ! $ "

" ! $ #

! # $ "

Русский язык

Customer Service

Help

Contact us

ELSEVIER

Terms and conditions ↗ Privacy policy ↗

Copyright © Elsevier B.V ↗. All rights reserved. Scopus® is a registered trademark of Elsevier B.V.

We use cookies to help provide and enhance our service and tailor content. By continuing, you agree to the use of cookies.

RELX

also developed by scimago: SCIMAGO INSTITUTIONS RANKINGS

SJR Scimago Journal & Country Rank Enter Journal Title, ISSN or Publisher Name

Home Journal Rankings Country Rankings Viz Tools Help About Us

International Journal of Mathematical, Engineering and Management Sciences

Country	India - SIR Ranking of India
Subject Area and Category	Business, Management and Accounting Business, Management and Accounting (miscellaneous) Computer Science Computer Science (miscellaneous) Engineering Engineering (miscellaneous) Mathematics Mathematics (miscellaneous)
Publisher	International Journal of Mathematical, Engineering and Management Sciences (IJMEMS)
Publication type	Journals

4

H Index

ISSN 24557749

Coverage 2016-ongoing

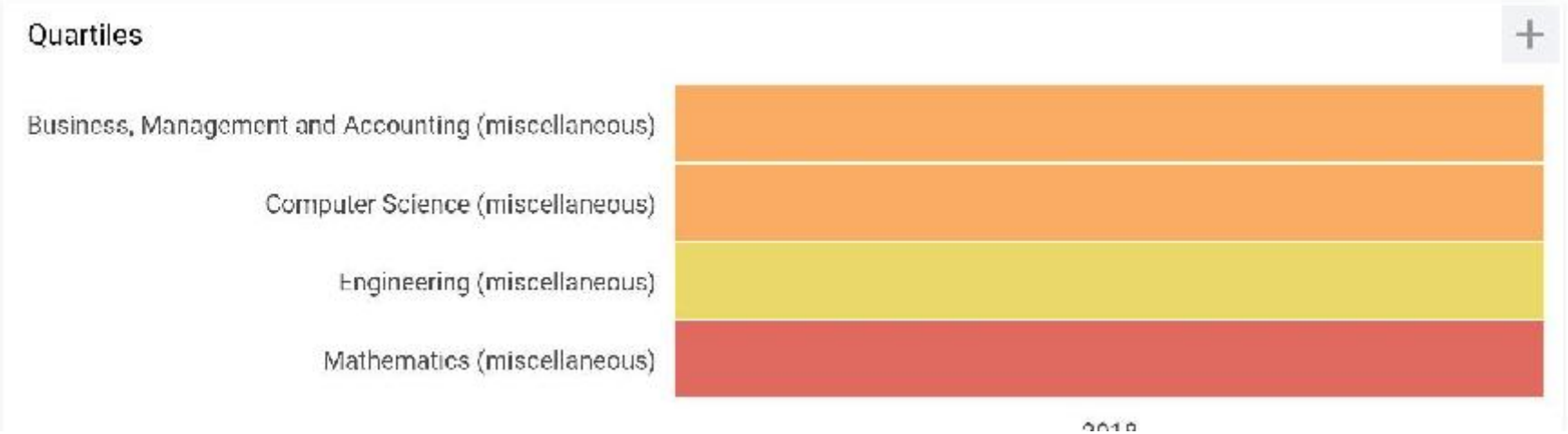
Scope IJMEMS is a peer reviewed international journal aiming on both the theoretical and practical aspects of mathematical, engineering and management sciences. The original, not-previously published, research manuscripts on topics such as the following (but not limited to) will be considered for publication: *Mathematical Sciences- applied mathematics and allied fields, operations research, mathematical statistics. *Engineering Sciences- computer science engineering, mechanical engineering, information technology engineering, civil engineering, aeronautical engineering, industrial engineering, systems engineering, reliability engineering, production engineering. *Management Sciences- engineering management, risk management, business models, supply chain management.

Homepage

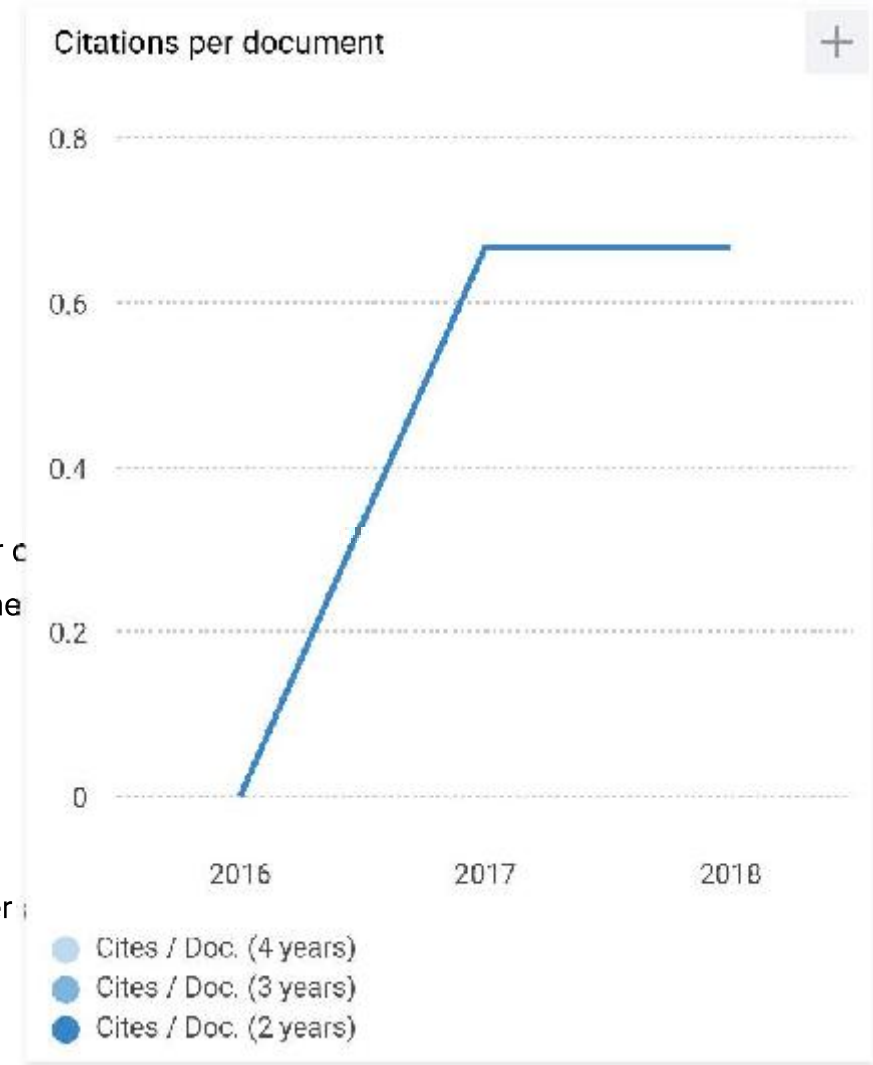
How to publish in this journal

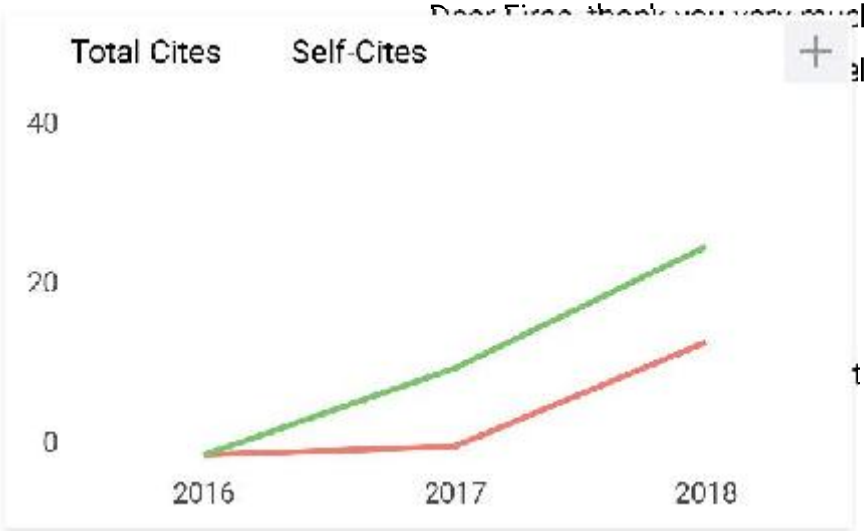
Contact

Join the conversation about this journal



SJR
NB
0.21
NB
2018


Citations per document
0.8
0.6
0.4
0.2
0
2016
2017
2018
Cites / Doc. (4 years)
Cites / Doc. (3 years)
Cites / Doc. (2 years)

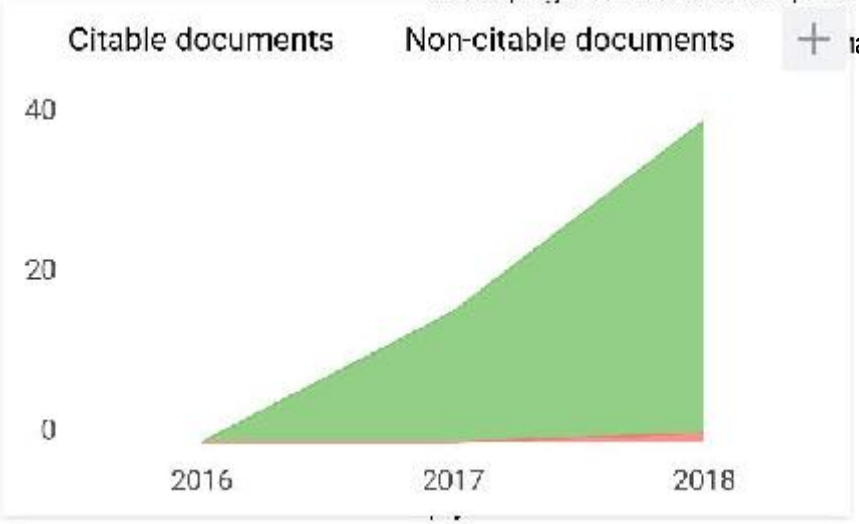

Total Cites
Self-Cites
40
20
0
2016
2017
2018

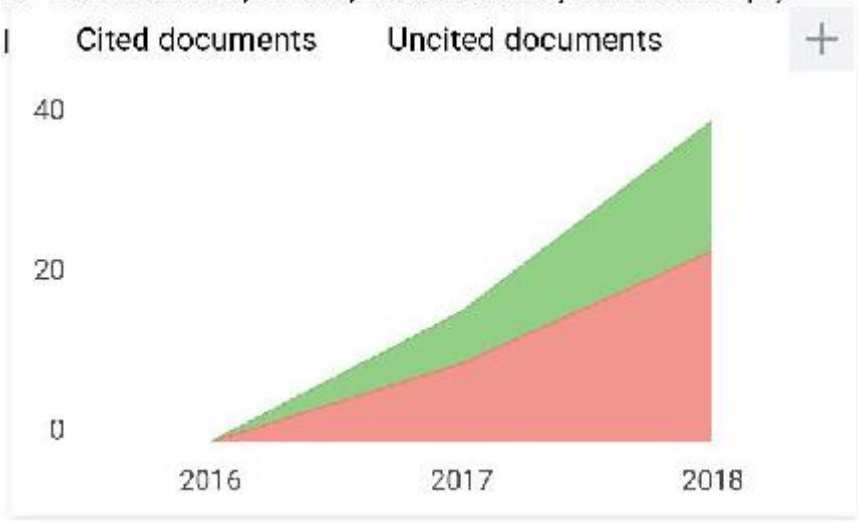
External Cites per Doc
Cites per Doc
0.8
0.4

% International Collaboration
40
20

Jc
ırr
yc

homepage or contact the journal's editorial staff , so they could inform you more deeply.





← Show this widget in your own website

Just copy the code below and paste within your html code:

```
<a href="https://www.scim
```

Submit

The users of Scimago Journal & Country Rank have the possibility to dialogue through comments linked to a specific journal. The purpose is to have a forum in which general doubts about the processes of publication in the journal, experiences and other issues derived from the publication of papers are resolved. For topics on particular articles, maintain the dialogue through the usual channels with your editor.

Developed by:

Powered by:

Follow us on @ScimagoJR

Scimago Lab, Copyright 2007-2019. Data Source: Scopus®

EST MODUS IN REBUS

Horatio (Satire 1.1.106)

International Journal of Mathematical, Engineering and Management Sciences

ISSN: 2455-7749

Custom Search

Volume 5 (2020)

- Number 2 (April)
- Number 1 (February)

Volume 4 (2019)

- Number 6 (December)
- Number 5 (October)
- Number 4 (August)
- Number 3 (June)
- Number 2 (April)
- Number 1 (February)

Volume 3 (2018)

- Number 4 (December)
- Number 3 (September)
- Number 2 (June)
- Number 1 (March)

Volume 2 (2017)

- Number 4 (December)
- Number 3 (September)
- Number 2 (June)
- Number 1 (March)

Volume 1 (2016)

Number 3 (December)

Number 2 (September)

Number 1 (June)

Privacy Policy| Terms & Conditions

International Journal of Mathematical, Engineering and Management Sciences

ISSN: 2455-7749

Volume 5, Number 2, April 2020

Determining the Economic Manufacturing Lot Size with Expedited Fabrication Rate and Product Quality Assurance
Yuan-Shyi Peter Chiu, Yu-Ru Chen, Hua-Yao Wu, Chung-Li Chou
193-207
https://doi.org/10.33889/IJMEMS.2020.5.2.016

Determining Software Time-to-Market and Testing Stop Time when Release Time is a Change-Point
Ompal Singh, Saurabh Panwar, P. K. Kapur
208-224
https://doi.org/10.33889/IJMEMS.2020.5.2.017

Effect of Carbon Emission and Human Errors on a Two-Echelon Supply Chain under Permissible Delay in Payments
Manavi Gilotra, Sarla Pareek, Mandeep Mittal, Vinti Dhaka
225-236
https://doi.org/10.33889/IJMEMS.2020.5.2.018

Bioconvection of Micropolar Fluid in an Annulus
D. Srinivasacharya, I. Sreenath
237-247
https://doi.org/10.33889/IJMEMS.2020.5.2.019

CFD Modelling of Multi-Particulate Flow through Concentric Annulus
Satish Kumar Dewangan, Vivek Deshmukh
248-259
https://doi.org/10.33889/IJMEMS.2020.5.2.020

Mathematical Modelling of Postindustrial Land Use Value in the Big Cities in Ukraine
I. Openko, Y. V. Kostyuchenko, R. Tykhenko, O. Shevchenko, O. Tsvyakh, T. Ievsiukov, M. Deineha
260-271
https://doi.org/10.33889/IJMEMS.2020.5.2.021

7th -Order Caudrey-Dodd-Gibbon Equation and Fisher-Type Equation by Homotopy Analysis Method
Ankita Sharma, Rajan Arora

272-282
https://doi.org/10.33889/IJMEMS.2020.5.2.022

Geostatistical Analysis on Spatial Variability of Soil Nutrients in Vertisols of Deccan Plateau Region of North Karnataka, India
Vinod Tamburi, Amba Shetty, S. Shrihari
283-295
https://doi.org/10.33889/IJMEMS.2020.5.2.023

Forecasting the Long-Run Behavior of the Stock Price of Some Selected Companies in the Malaysian Construction Sector: A Markov Chain Approach
Wajeeh Mustafa Sarsour, Shamsul Rijal Muhammad Sabri
296-308
https://doi.org/10.33889/IJMEMS.2020.5.2.024

A Non-Parametric Approach for Survival Analysis of Component-Based Software
Sandeep Chopra, Lata Nautiyal, Preeti Malik, Mangey Ram, Mahesh K. Sharma
309-318
https://doi.org/10.33889/IJMEMS.2020.5.2.025

Control of the System of Piezoelectric Actuator Devices for Precision Drive Systems
Stanislav Matveev, Nikolai Yakovenko, Yuri Konoplev, Andrei Gorbunov, Alexander Shirshov, Nikolay Didenko
319-327
https://doi.org/10.33889/IJMEMS.2020.5.2.026

Assessment of Software Vulnerabilities using Best-Worst Method and Two-Way Analysis
Misbah Anjum, P. K. Kapur, Vernika Agarwal, Sunil Kumar Khatri
328-342
https://doi.org/10.33889/IJMEMS.2020.5.2.027

Heat and Mass transfer in MHD Nanofluid over a Stretching Surface along with Viscous Dissipation Effect
K. Govardhan, G. Narender, G. Sreedhar Sarma
343-352
https://doi.org/10.33889/IJMEMS.2020.5.2.028

Influence of Hybrid Fibres on Bond Strength of Concrete
Srinivasa Rao Naraganti, Rama Mohan Rao Pannem, Jagadeesh Putta
353-362
https://doi.org/10.33889/IJMEMS.2020.5.2.029

Dynamics of A Re-Parametrization of A 2-Dimensional Mapping Derived from Double Discrete Sine-Gordon Mapping
La Zakaria, Johan Matheus Tuwankotta

363-377
https://doi.org/10.33889/IJMEMS.2020.5.2.030

Erratum to: Effectiveness Analysis of a Two Non-Identical Unit Standby System with Switching Device and Proviso of Rest
Darpandeep Kour, J. P. Singh Joorel, Neha Sharma
378-380
https://doi.org/10.33889/IJMEMS.2020.5.2.031

Privacy Policy| Terms & Conditions

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

Dynamics of A Re-Parametrization of A 2-Dimensional Mapping Derived from Double Discrete Sine-Gordon Mapping

La Zakaria
Department of Mathematics,
Faculty of Mathematics and Natural Sciences, Universitas Lampung, Indonesia.
Corresponding author: lazakaria.1969@fmipa.unila.ac.id

Johan Matheus Tuwankotta
Analysis and Geometry Group,
Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Indonesia.
E-mail: theo@dns.math.itb.ac.id

(Received June 24, 2019; Accepted November 14, 2019)

Abstract
We study the dynamics of a two dimensional map which is derived from another two dimensional map by re-parametrizing the parameter in the system. It is shown that some of the properties of the original map can be preserved by the choice of the re-parametrization. By means of performing stability analysis to the critical points, and also studying the level set of the integrals, we study the dynamics of the re-parametrized map. Furthermore, we present preliminary results on the existence of a set where iteration starts at a point in that set, in which it will go off to infinity after finite step.

Keywords- Re-parametrizing, 2-dimensional mapping, Generalized double discrete sine-Gordon, Integral.

1. Introduction

Arguably, one of the most important and general integrable maps is known in the literature as the Quispel-Roberts-Thompson map (QRT). It is a two-dimensional map depending on 18 parameters. The QRT map is closely related to so called soliton equations (Quispel et al., 1988; Quispel et al., 1989). More recent studies have focused on generalizations of QRT maps. One of them was proposed by Joshi and Kassotakis (2019). Main result of their paper is a new connection between two major theories that generalize QRT maps. They provide a new formulation of QRT involutions in terms of Hirota derivatives and discover conditions under which each involution can be factorized into two further involutions.

The sine-Gordon equation is a partial differential equation, which is known to have soliton solutions; hence, it is a soliton equation (Quispel et al., 1991). Discretizations of the sine-Gordon equation have been done in various ways (Quispel et al., 1988; Quispel et al., 1989; Quispel et al., 1991). The reduction of the sine-Gordon equation to a two dimensional ordinary difference equations using a standard staircase (see Van der Kamp and Quispel, 2010 for the method) is known as being a special case of the celebrated QRT map. Recently, Celledoni et al. (2019) have studied a new systematic approach for calculating the preserved measures and integrals of a rational map in which a two-dimensional sine-Gordon (standard) map was chosen as an example (see Celledoni et al., 2019 §4.3 Example 4).

Generating a new family of mapping from a known one is an interesting topic to study. A piece-wise linear map from a known integrable map by using the ultra discretization technique is

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

generated by Tuwankotta et al. (2004). The number of independent integrals are preserved by the transformation which implies that the integrability is preserved. In our article (Zakaria and Tuwankotta, 2016) a straight-forward generalization by adding parameters in the Lax pair of the ordinary discrete sine-Gordon partial difference equation has done. By using the standard staircase method, the resulted equation is then reduced to system of ordinary difference equations (see Van der Kamp et al., 2007 for the method). Note that this generalized sine-Gordon system is also analyzed in (Duistermaat, 2010).

Another novel method to introduce a new class of discrete systems from an integrable discrete system, is by introducing the concept of dual (Quispel et al., 2005). This works beautifully for a single discrete equation, although the resulting equation may not be new nor integrable. This idea of dual is extended to system of discrete equations in (Tuwankotta et al., 2019). The latter is interesting in the sense that the method proposed there produces in general more than one system.

Roberts et al. (2002) constructed a new family of mapping by interchanging the parameter and the integral. For example, consider a discrete dynamical system in $\mathbb{R}^n$ which is denoted by: $\mathbf{x} = \mathbf{f}(\mathbf{x})$, where the prime denotes the upshift, is a parameter in the system, and $\mathbf{f}: \mathbb{R}^n \quad \mathbb{R}^n$. We assume that the exists a smooth function $G: \mathbb{R}^n \quad \mathbb{R} \quad \mathbb{R}^n$ such that: $G(\mathbf{x}, \quad) \quad G(\mathbf{x}, \quad)$, which is called an integral for the system. Suppose that we can solve the equation $G(\mathbf{x}, \quad) = 0$ for $(\mathbf{x})$. Then by substituting this solution to $f(\mathbf{x})$ (and call it $g(\mathbf{x})$), we derive a new discrete system: $\mathbf{x} = g(\mathbf{x})$, with integral: $(\mathbf{x})$.

We will follow this technique and apply it to a generalized sine-Gordon equation. The aim of this paper is to show a number of properties of the new mapping (after re-parameterization) and to compare them with the original mapping. Furthermore, the qualitative behavior of the new mapping is studied by means of obtaining fixed points and their stability, and also the base points.

The outline of this paper is the following. In Section 2, a system of first order difference equations derived from generalized double discrete sine-Gordon (sine-Gordon) equations is formulated by restricting to traveling wave solution. In Section 3 a new integrable mapping derived from sine-Gordon mapping by interchanging the role of the integral and the parameter in the original system is presented. The new system is then analyzed by means of describing its symmetry and measure preservation. In Section 4 finding fixed points and periodic points (and their linear stability) and also computing the base points are discussed. These are done in the remaining subsections in Section 4. We have divided Section 4 into five subsections. This paper ends with some concluding remarks in Section 5.

2. Formulation of the Problem

Consider a three parameters family of partial difference equation on two dimensional lattice:

$$_1 V_{l,m} {}_1V_{l\ 1,m} \quad V_{l\ 1,m} {}_1V_{l,m} \qquad {}_2V_{l\ 1,m} {}_1V_{l,m} {}_1V_{l\ 1,m} V_{l,m} = \ _3. \tag{1}$$

This equation is derived from the compatibility condition of the generalized Lax pair of the classical discrete sine-Gordon equation (see Zakaria and Tuwankotta, 2016). The travelling wave solutions of (1) can be obtained by considering the following form:

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

$V_{l,m} = V_n, n = z_1 l \quad z_2 m,$

where z_1 and z_2 are relatively prime integers. Substituting this to (1), we have an infinite hierarchy of mapping labeled by z_1 and z_2, i.e.

$$_1 V_{n\ z_1\ z_2} V_n \quad V_{n\ z_1} V_{n\ z_2} \qquad _2 V_{n\ z_1\ z_2} V_{n\ z_2} V_{n\ z_1} V_n \qquad _3 \quad 0. \tag{2}$$

If z_1 and z_2 are fixed, a mapping from $\mathbb{R}^{z_1\ z_2} \quad \mathbb{R}^{z_1\ z_2}$ can be obtained from (2). It can be noted that by setting $_2 \quad _3 \quad 1$ and $_1 = pq$ in (2), we can derive the two dimensional mappings in Quispel et al., (1991).

Let $z_1 = z_2 = 1, \quad _1 = \quad _2$, and $\quad _3 = \quad _2$. From (2), we derive:

$$V_{n\ 2} \quad \frac{V_{n\ 1}^2}{V_n \ V_{n\ 1}^2},$$

which is a second order difference equation. Then, by writing

$x_n = V_{n\ 1}$, and $y_n = V_n$.

We derive a system of first order difference equations:

$$\begin{aligned} x_{n\ 1} \quad &= \quad \frac{x_n^2}{y_n \ x_n^2}, \\ y_{n\ 1} \quad &= \quad x_n. \end{aligned} \tag{3}$$

Let us define: $f : \mathbb{R}^2 \quad \mathbb{R}^2$, by

$$f(x, y) \quad \frac{x^2}{y \ x^2}, x \ .$$

Using this, we can write (3) as:

$$(x\ , y\ ) = f(x, y) \tag{4}$$

where the prime denotes the upshift. The mapping (4) has an integral, i.e.

$$F(x, y) = \quad \frac{x}{y} \ \frac{y}{x} \qquad xy \quad \frac{1}{xy} \ . \tag{5}$$

Thus, $F(x_{n\ 1}, y_{n\ 1}) = F(x_n, y_n)$ for all $n \quad \mathbb{N}$.

3. Reparametrized Mapping and Its Properties

Consider the mapping in (4). For $\quad = 1$, we solve the equation $F(x, y) = 0$ for $\quad$ to derive:

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

$$(x, y) = \frac{1 \quad x^2 y^2}{x^2 \quad y^2}. \tag{6}$$

Substituting (6) into (4), we derive a new mapping:

$$(x\,, y\,) = \hat{f}(x, y) = \quad y, x\ , \tag{7}$$

with integral

$$\hat{F}(x, y) = \frac{1 \quad x^2 y^2}{x^2 \quad y^2}. \tag{8}$$

Note that the integral (8) is nothing but the function (x, y) in (6). The new map, $\hat{f}(x, y)$, has the following properties:

The orbits of $\hat{f}(x, y)$ is 4-periodic. This is simple to show that the linear map in (7) is trivial, its second iterate leads to $x\,, y \;=\; x,\; y$ thus all the results are immediate (the fourth iterate is the identity function).

$\hat{f}(x, y)$ is area preserving. A two-dimensional map is area preserving (also called conservative) when its Jacobian determinant $|J\ \ x, y\ |$ is equal to 1 in all the points x, y of the plane, which for the linear map in equation (7) is immediate (the Jacobian determinant is constant and equal to 1), as well as the so-called "reverse symmetry", since $\hat{f}^{\ 1}(u, v) = \hat{f}(u, v)$. Alternatively, we can use different procedure to show that an integrable map is measure preserving (Roberts et al., 2002). For our map, $\hat{f}(x, y)$, the procedure can be followed as shown below.

The mapping $\hat{f}(x, y)$ is measure preserving (area preserving) because there is the density $\hat{}(x, y)$ such that

$$\left|D\hat{f}(x, y)\right| = \frac{\hat{}(x, y)}{\hat{}(x\,, y\,)} = 1$$

when the density $\hat{}$ is given by

$$\hat{}(x, y) = \frac{1}{xy} \quad \frac{F(x, y)}{} \quad ^{1} = \frac{1}{x^2 \quad y^2}.$$

Consider $G_1(x, y) = \quad y, x$. Note that $G_1(G_1(x, y)) = G_1(y, x) = (x, y)$. This implies that: $G_1^{\ 1} = G_1$. Since $G_1 \circ \tilde{f} \circ G_1^{\ 1} = \tilde{f}^{\ 1}$ then G_1 is a reversing symmetry for $\hat{f}$.

There exists a symmetry $S_1(x, y) = \quad x, \quad y$ such that $S_1 \circ \hat{f} \circ S_1^{\ 1} = \hat{f}$.

The dynamics of the mappings in (7) for $\quad = 1$ on every level set $\hat{F} = c$ is basically identical to the dynamics of the mappings in (4) on the level set $F = 0$ for $\quad = c$. Furthermore, this provides

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

us with the existence of a 4-periodic points of the mappings: (7) for $=1$ for every . The locations of these points are in the level set $F=0$.

Still fixing the value of $=1$, let us now, reparametrize the parameter in (4) by a_0 a_1 . It follows immediately that the map is given by

$$\tilde{f}(x,y)=\frac{1 \quad a_0 \quad a_1 \quad x^2}{y \quad x^2 \quad a_0 \quad a_1}, x \quad . \tag{9}$$

Consequently, we transform the integral (5) to

$$\tilde{F}(x,y)= a_0 \quad a_1 \quad \frac{x}{y} \quad \frac{y}{x} \quad xy \quad \frac{1}{xy} \quad . \tag{10}$$

For the case where: $a_1 \quad 0$, let us add a constant in (10) which takes a special form: $b_0 \quad b_1$, i.e.

$$\tilde{\tilde{F}}(x,y)= a_0 \quad a_1 \quad \frac{x}{y} \quad \frac{y}{x} \quad xy \quad \frac{1}{xy} \quad b_0 \quad b_1 \quad , \tag{11}$$

Note that $\tilde{\tilde{F}}(x,y)$ is linear in . Furthermore:

$$\tilde{\tilde{F}} \circ \tilde{f} = \tilde{\tilde{F}} \quad \frac{1 \quad a_0 \quad a_1 \quad x^2}{y \quad x^2 \quad a_0 \quad a_1}, x \quad \tilde{\tilde{F}}(x,y).$$

This implies that we can look at the zero level curve of $\tilde{\tilde{F}}$ and solve it for :

$$\tilde{}(x,y)=\frac{1 \quad x^2y^2 \quad x^2a_0 \quad y^2a_0 \quad xyb_0}{x^2a_1 \quad y^2a_1 \quad xyb_1}. \tag{12}$$

Substituting this expression of $\tilde{}(x,y)$ into $\tilde{f}$ in (9) gives:

$$\hat{\tilde{f}}(x,y)=\frac{a_1 \quad y \quad x^4y \quad x^3b_0 \quad x \quad 1 \quad x^2a_0 \quad b_1}{a_1 \quad 1 \quad x^4 \quad xyb_0 \quad xy \quad x^2 \quad a_0 \quad b_1}, x \quad .$$

It is interesting to note that $\tilde{} \quad \hat{\tilde{f}}(x,y) = \tilde{}(x,y)$ which implies that $\tilde{}(x,y)$ is an integral for the system:

$$(x\,,y\,) = \hat{\tilde{f}}(x,y). \tag{13}$$

The mapping (13) has some properties:

$\hat{\tilde{f}}$ is measure preserving and orientation-reversing (or anti-measure preserving), which means (Roberts et al., 2002)

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

$$|D\hat{\hat{f}}| \quad \frac{\hat{}(x,y)}{\hat{} \dfrac{a_1\ y\ x^4y\ x^3b_0\ x\ 1\ x^2a_0\ b_1}{a_1\ 1\ x^4\ xy\ b_0\ xy\ x^2\ a_0\ b_1}, x}.$$

Note that from the right-hand side, the determinant of the Jacobian of $\hat{\hat{f}}$ is

$$\frac{b_1^2x^2\ a_0^2x^2\ a_0\ x^4\ 1\ x^2\ a_1b_0\ b_1x^2\ 2a_0x^2\ x^4\ 1\ a_1^2\ b_0^2\ 2\ x^4\ x^8\ 1}{b_1xy\ x^2\ a_0\ a_1\ b_0xy\ x^4\ 1}^2.$$

Meanwhile, from the right-hand side, we have

$$\frac{b_1^2\ x^2\ a_0\ x^2\ a_0x^2\ 1}{a_1\ 1\ x^4\ xyb_0\ xy\ x^2\ a_0\ b_1}^2 \quad \frac{a_1b_0\ b_1x^2\ 2a_0x^2\ x^4\ 1}{a_1\ 1\ x^4\ xyb_0\ xy\ x^2\ a_0\ b_1}^2$$

$$\frac{a_1^2\ b_0^2\ 2\ x^4\ x^8\ 1}{a_1\ 1\ x^4\ xyb_0\ xy\ x^2\ a_0\ b_1}^2$$

where the so-called density $\hat{}$ is given by

$$\hat{}(x,y) \quad \frac{1}{x\,y} \quad \tilde{\tilde{F}}^{\ 1} \quad \frac{1}{a_1(x^2\ \ y^2)\ \ b_1x\,y}.$$

The function $\hat{G}_1(x,y) =\ \ y,\ \ x$, is a reversing symmetry for $\hat{\hat{f}}$.

4. Critical Point and Base Point of the Integral

There are two important elements in analyzing the dynamics of system (13), i.e. Fixed Point (FP) and Base Point (BP). FP can be obtained by finding the critical point of the integral function, while BP is defined as the point where the integral function is singular. At the BP, level sets for various values of the integral function intersect each other.

4.1 The Critical Point

The critical points of the integral function (12) are solutions of

$$\frac{\hat{}}{x} = \frac{a_1\ 2x\ y^4\ 1\ \ y\ x^2\ \ y^2\ b_0\ \ y\ x^2y^2\ 1\ \ \ x^2\ \ y^2\ a_0\ b_1}{x^2\ \ y^2\ a_1\ \ xyb_1}^2\ \ 0,$$

$$\frac{\hat{}}{y} = \frac{a_1\ 2y\ x^4\ 1\ \ \ \ x^3\ \ xy^2\ b_0\ \ x\ x^2y^2\ 1\ \ x^2\ \ y^2\ a_0\ b_1}{x^2\ \ y^2\ a_1\ \ xyb_1}^2\ \ 0.$$

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

To obtain the solutions, we can do as follow:

$$y\frac{\hat{}}{x} \quad x\frac{\hat{}}{y} = 0 \qquad \frac{x^2 \quad y^2 \quad x^2 \quad y^2 \quad a_1 \quad 2xy \quad b_0 \qquad x^2y^2 \qquad x^2 \quad y^2 \quad a_0 \quad b_1}{x^2 \quad y^2 \quad a_1 \quad xyb_1} \quad 0$$

From the left-hand side in the last equation, we have two lines, $y = x$ and $y = \quad x$ as solutions. By substituting these into $\frac{\hat{}}{x} \quad 0$ or $\frac{\hat{}}{y} \quad 0$ and then solve it, we have

$$x, y \;=\; 1, \;\; 1 \quad \text{and} \quad 1, \mp 1 \tag{14}$$

It is easy to verify that: $(1,1)$ and $(\;\; 1, \;\; 1)$ are two fixed points while $(1, \;\; 1)$ and $(\;\; 1,1)$ are two 2-periodic points.

4.2 The Base Point

Apart from the critical points, the so-called base points also play a crucial role in the dynamics of (13). Note that the invariant (12) can be written as a rational function. A point x_0, y_0 is a base point if it is a common zero of the numerator and denominator of $\tilde{} \; x, y$ equal to zero. In our case, we will discuss two conditions, $a_1 \quad 0$ and $a_1 \quad 0$.

For $a_1 \quad 0$, the base points are

$$(x_0, y_0) \qquad 0, \sqrt{\frac{1}{a_0}} \;\; , \;\; 0, \;\; \sqrt{\frac{1}{a_0}} \;\; , \;\; \sqrt{\frac{1}{a_0}}, 0 \;\; , \;\; \sqrt{\frac{1}{a_0}}, 0 \tag{15}$$

Note that the points in eq. (15) are base points of the system (13) when the parameter $a_1 \quad 0$. Two base points, $\sqrt{\frac{1}{a_0}}, 0$, are directly mapped to infinity by the mapping $\hat{\tilde{f}}$, in the sense that one of both of the component of $\hat{\tilde{f}}$ blows up at these points. And then the other points, $0, \;\; \sqrt{\frac{1}{a_0}}$, are base points in which they are mapped to other base points after one iterate of $\hat{\tilde{f}}$.

For $a_1 \quad 0$, the base points can be obtained by solving $1 \quad x^2y^2 \quad x^2a_0 \quad y^2a_0 \quad xyb_0 \quad 0$ and $x^2a_1 \quad y^2a_1 \quad xyb_1 \quad 0$. Based on our computations, the base points are

$$x_0, y_0 \qquad x_{10}, \;\; y_{10} \;\; , \;\; \mp x_{10}, \;\; y_{10} \;\; , \;\; x_{20}, \;\; y_{20} \;\; , \;\; \mp x_{20}, \;\; y_{20}$$

where

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

$$x_{10} \quad \frac{\sqrt{\quad d^{\ 2}\ b_0 \quad a_0\, d^{\ 2} \quad 8 \quad d \quad d^2 \quad 2 \quad a_0 \quad d \quad a_0 d^2 \quad b_0 \quad b_0\, d}}{2 \quad d \quad d^2 \quad 2};$$

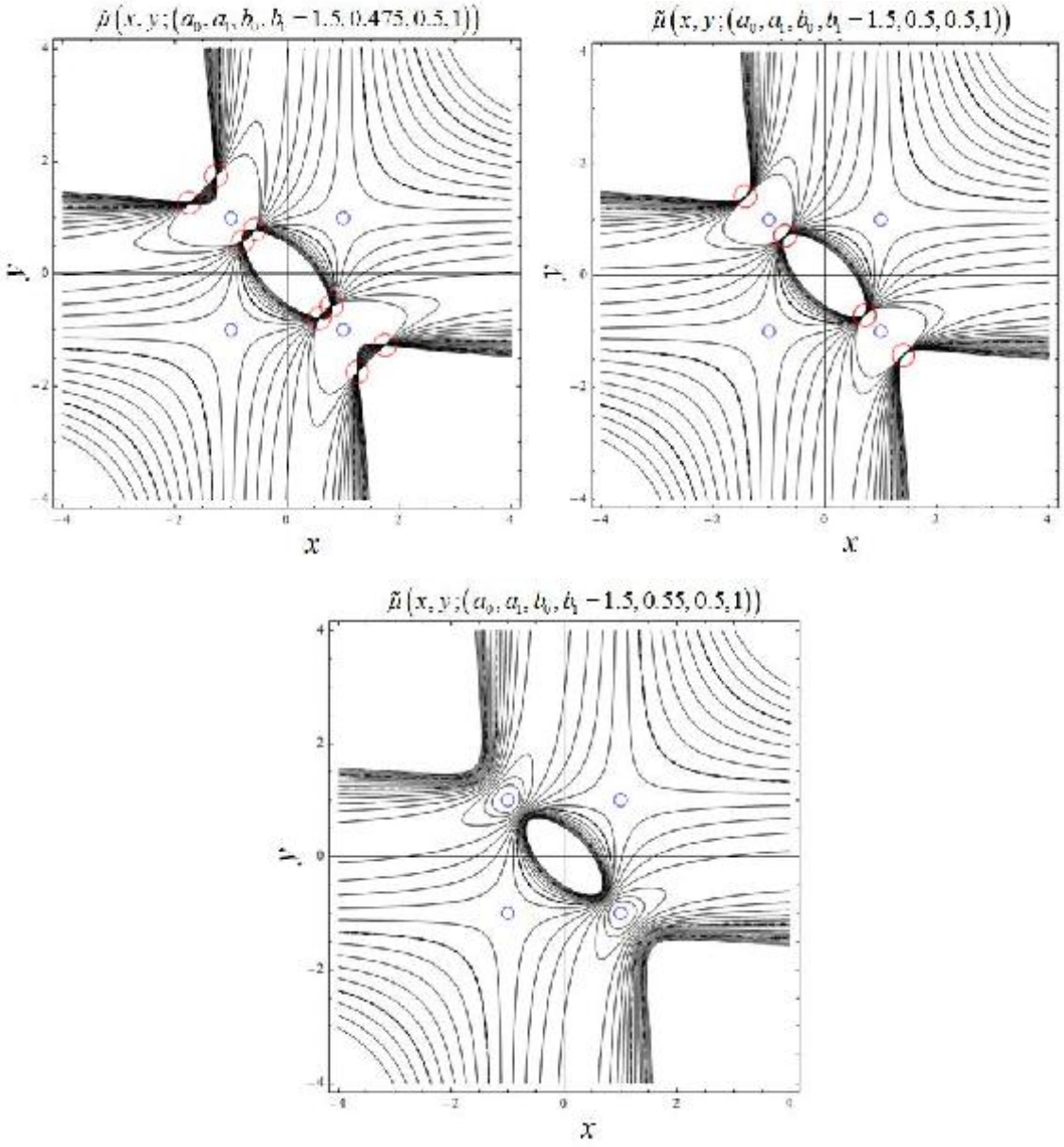


Figure 1. The level sets, the fixed points, the 2-periodic points, and the real base points of the integral (12) for parameter values $a_0, a_1, b_0, b_1 = 1.5, 0.475, 0.5, 1$ (the diagram in the first row-left), $a_0, a_1, b_0, b_1 = 1.5, 0.5, 0.5, 1$ (the diagram in the first row-right), and $a_0, a_1, b_0, b_1 = 1.5, 0.55, 0.5, 1$ (the diagram in the second row).

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

y_{10} $\frac{x_{10}}{2}$ d ;

x_{20} $\sqrt{\frac{\sqrt{d \quad ^2 \; b_0 \quad a_0 d \; ^2 \quad 8 \quad d \quad 8d^2 \quad 16} \quad a_0 \quad d \quad a_0 d^2 \quad b_0 \quad b_0 d}{2 \quad d \quad d^2 \quad 2}}$;

y_{20} $\frac{x_{20}}{2}$ d ; d $\frac{b_1}{a_1}, a_1$ 0; $\sqrt{d^2 \quad 4}$.

All base points depend on the parameters $a_0, a_1, b_0,$ and b_1 . To obtain the real base points for x_0, y_0 $x_{10}, \; y_{10}$, $\mp x_{10}, \; y_{10}$, the parameters should satisfy the following conditions

$b_0 \quad \mathbb{R} \quad a_0 \quad \frac{b_0}{d} \quad 2\sqrt{\frac{2 \quad d \quad 2d^2 \quad 4}{d^2 \quad d \; ^2}} \quad 0 \quad d \quad 2 \quad 0 \quad d \quad \frac{2}{d} \quad d \quad 2$.

And then to obtain the real base points for x_0, y_0 $x_{20}, \; y_{20}$, $\mp x_{20}, \; y_{20}$, the parameters should satisfy the following conditions

$b_0 \quad \mathbb{R} \quad d \quad 2 \quad 0 \quad a_0 \quad \frac{b_0}{d} \quad 2\sqrt{\frac{2 \quad d \quad 2d^2 \quad 4}{d^2 \quad d \; ^2}}$

$d \quad 2 \quad 0 \quad \frac{d^2 \quad 2}{d} \quad a_0 \quad \frac{b_0}{d} \quad 2\sqrt{\frac{2 \quad d \quad 2d^2 \quad 4}{d^2 \quad d \; ^2}}$

Figure 1 shows the base points (red-circle) together with fixed points and 2-periodic points (blue-circle). Three diagrams in Figure 1 are some level sets of integral (12) together with the fixed points, the 2-periodic points, and the base points for $a_0 = 1.5.$ The diagram has fixed parameter values $(a_1, b_0, b_1) = (0.475, 0.5, 1.0)$ (up-left diagram), $(a_1, b_0, b_1) = (0.5, 0.5, 1.0)$ (up-right diagram) and $(a_1, b_0, b_1) = (0.55, 0.5, 1.0)$ (down diagram).

4.3 Preimages of the Base Points

A base point is mapped to infinity by the mapping $\hat{\hat{f}}$. Furthermore, there are two base points which do not refer to a base point. But after one iteration they are mapped to base points. This means that if we start at those points, the iteration of $\hat{\hat{f}}$ will be sent to infinity after two iterations. Let us name the set of points which are mapped by $\hat{\hat{f}}$ to a base point by P_1. We can then look at the set of points which are mapped by $\hat{\hat{f}}$ into P_1, or the preimage of P_1 under $\hat{\hat{f}}$; and name the

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

set P_2. Continuing in a similar way, we constructed P_3, P_4 and so on. Thus, the dynamical system (13) is well defined if we exclude the points in $\bigcup_1 P_k$.

Let us present a few explicit computations of $P_k, k = 1, 2, 3$ as examples.

Consider the situation where $(x, y) = \frac{1}{\sqrt{a_0}}, 0$ for $a_0 > 0$ and denote the system (13) by

$$\hat{\hat{f}} \quad : \quad \mathbb{R}^2 \quad \mathbb{R}^2$$

$$(x, y) \mapsto \frac{a_1 \; y \quad x^4 \quad 1 \quad x^3 b_0 \quad x \; 1 \quad x^2 a_0 \; b_1}{a_1 \; x^4 \quad 1 \quad xyb_0 \quad xy \; x^2 \quad a_0 \; b_1}, x \; . \tag{16}$$

The preimage of $(x, y) = \frac{1}{\sqrt{a_0}}, 0$ by (16) is the solution of the following system

$$x \quad = \quad 0$$

$$\frac{a_1 \; y \quad x^4 \quad 1 \quad x^3 b_0 \quad x \; 1 \quad x^2 a_0 \; b_1}{a_1 \; x^4 \quad 1 \quad xyb_0 \quad xy \; x^2 \quad a_0 \; b_1} \quad = \quad \frac{1}{\sqrt{a_0}}.$$

The solution is

$$P_1 \quad \hat{\hat{f}}^{\,1} \qquad \frac{1}{\sqrt{a_0}}, 0 \qquad 0, \frac{1}{\sqrt{a_0}} \quad | a_0 > 0 \; . \tag{17}$$

For the preimages P_2 and P_3, we have

$$P_2 \quad \hat{\hat{f}}^{\,1} \quad 0, \frac{1}{\sqrt{a_0}} \qquad , \frac{^3 b_0}{^4 \; 1} \quad | b_0 \quad \mathbb{R} \; ,$$

$$P_3 \quad \hat{\hat{f}}^{\,1} \quad , \frac{^3 b_0}{^4 \; 1} \qquad \frac{^3 b_0}{^4 \; 1}, \; \frac{^4 \; 1^{\,2} \qquad ^4 b_0^2}{^4 \; 1^{\,2} \qquad ^8 b_0^2} \; \Bigg| \; > 0, b_0 \quad \mathbb{R} \; ,$$

where $= \frac{1}{\sqrt{a_0}} > 0$.

The graphs of P_2 to P_6 for $a_0 = 4$ are presented in Figure 2. The graph of P_2, P_3, P_4, P_5, and P_6 are plotted using green, red, blue, orange, and black, respectively. It is interesting to note that if

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

we fix to one of the level sets of the integral, an preliminary observation shows an indication that there are only finitely many intersection points between $\bigcup_1^6 P_k$. Whether it is true when $\bigcup_1 P_k$ is a subject of future investigation.

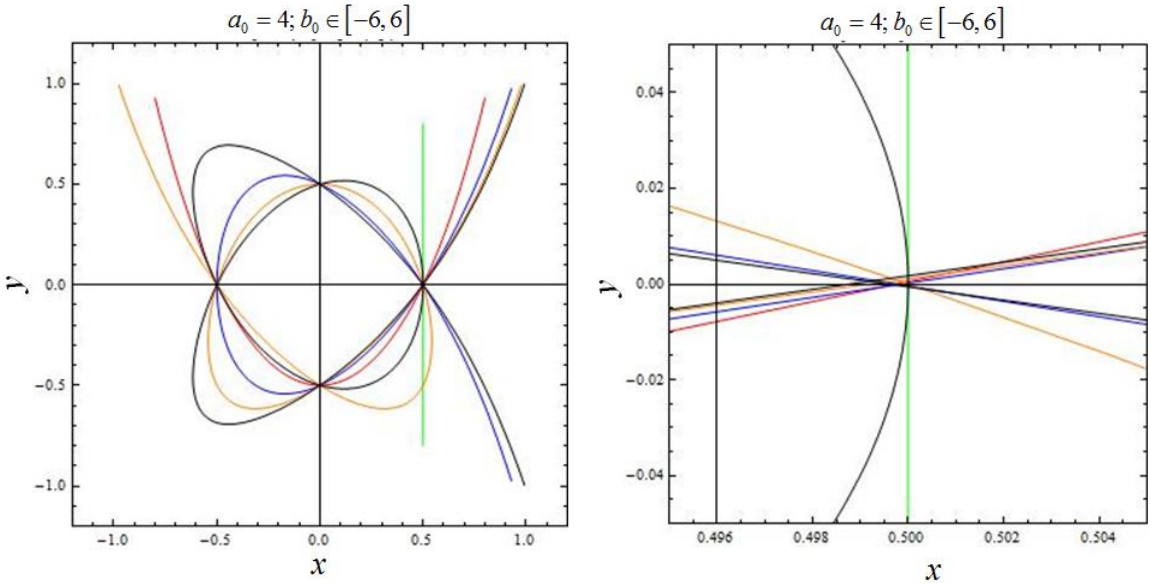


Figure 2. Plotting of the curves $P_2 ... P_6$ for $a_0 = 4$. We see that all curves pass through all base points (left). Enlarged view of the left image around a point $(1/2, 0)$ to see the finer structure of the graph (right).

4.4 Generic Situation

To study the dynamics of the system (13), we have plotted some of the level sets of the integral (see Figure 3). These level sets contain solution of the system (13). Furthermore, by studying how the level sets deformed as we vary the value parameter, the bifurcations in the system can be studied. This is however beyond the scope of this paper.

The two diagrams in the upper part of Figure 3 are some the level sets of integral (12) for $_0 = 4$. The diagram on the left-hand side has fixed $(_1, _0, _1) = (0.7, 0.5, 1.0)$, while for the diagram on the right-hand side is $(_1, _0, _1) = (-0.875, 0.5, 1.0)$. In the Figure 3 (down), we plot $_0 = 4$, $(_0, _1) = (0.5, 1.0)$, and $_1$ $(-3, 0)$ around a fixed point $(\ ,\ ) = (-1, 1)$ (left); and we plot $_0 = 4$, $(_0, _1) = (0.5, 1.0)$, and $_1$ $(-1, 2)$ around the fixed point $(\ ,\ ) = (1, 1)$ (right).

By setting $_0 = 4$ and $_1 = 0.7$, we have all of the curves around the fixed points, (1,1) and (-1,-1), are of hyperbolic type. On the other hand we have all of the curves around the 2-periodic points, (-1,1) and (1,-1), are of elliptic type. But for $_0 = 4$ and $_1 = -0.875$, the situation is reverse, i.e. all of the curves around the fixed points are of elliptic type while all of the curves around the 2-periodic points are of hyperbolic type.

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

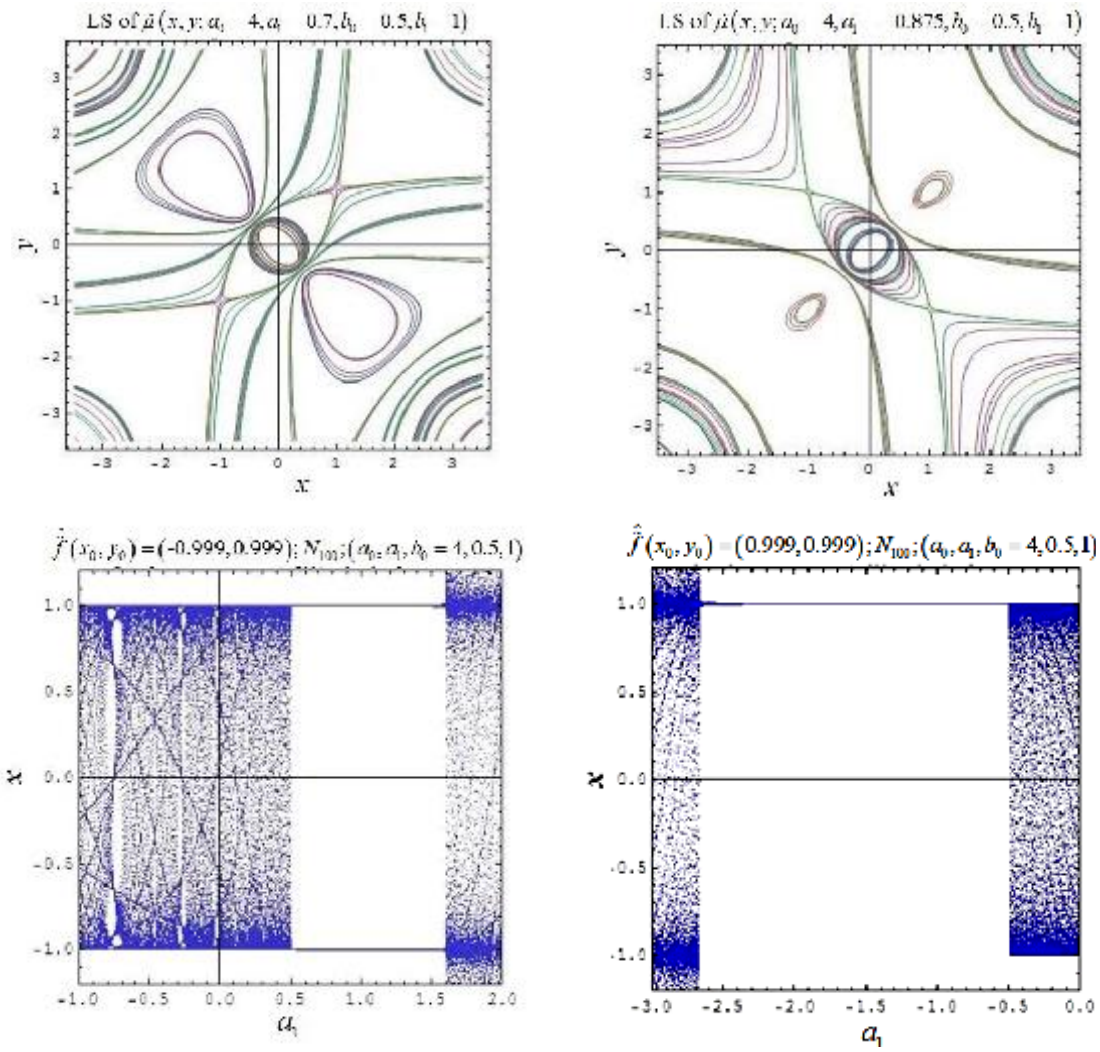


Figure 3. The level set forms of the integral (12) for the parameter value $_0 = 4$ (up). Bifurcation situation corresponds with its (down).

4.5 Stability

The stability of the system in the vicinity of the fixed points can be extracted from the integral function (see Kulenovic and Merino, 2002). In this case, we use the concept of Lyapunov stability to obtain the information of the stability of system (13). Note that the Hessian matrix evaluated at the fixed point $x^*, y^* = 1, 1$ is

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

$$H = \begin{matrix} A & B \\ B & A \end{matrix}$$

where

$$A = \frac{2\ a_1 b_0 \quad a_0 \quad 1\ b_1}{2a_1 \quad b_1^{\ 2}};$$

$$B = \frac{2 \quad a_1 \quad 4 \quad b_0 \quad 1 \quad a_0 \ b_1}{2a_1 \quad b_1^{\ 2}}.$$

The determinant of the Hessian matrix H is

$$\text{Det } H \ = \ \frac{4\ a_1\ 8 \quad 4b_0 \quad 4a_0 b_1}{2a_1 \quad b_1^{\ 3}}$$

If the determinant of Hessian matrix H is positive then the integral (12) attains a minimum at $x^*, y^* = 1, \ 1$. In Table 1, we have listed the condition for the value of the parameters of the system, so that the determinant of H is positive. Consequently, the fixed points of (13), $x^*, y^* = 1, \ 1$, are the centre points (stable).

Table 1. The conditions for the value of the parameters so that the determinant of H is positive

No.	a_0	b_1	b_0	a_1
1.	$a_0 > 0$	$b_1 > 0$	$2 \quad 2a_0 < b_0 < 2$	$\frac{a_0 b_1}{2 \quad b_0} < a_1 < \frac{b_1}{2}$
2.	$a_0 > 0$	$b_1 > 0$	$b_0 < 2 \quad 2a_0$	$\frac{b_1}{2} < a_1 < \frac{a_0 b_1}{2 \quad b_0}$
3.	$a_0 > 0$	$b_1 > 0$	$b_0 > 2$	$a_1 < \frac{b_1}{2} \qquad a_1 > \frac{a_0 b_1}{2 \quad b_0}$
4.	$a_0 < 0$	$b_1 > 0$	$2 < b_0 \quad 2 \quad 2a_0$	$a_1 > \frac{b_1}{2} \qquad a_1 < \frac{a_0 b_1}{2 \quad b_0}$
5.	$a_0 < 0$	$b_1 > 0$	$b_0 > 2 \quad 2a_0$	$a_1 > \frac{a_0 b_1}{2 \quad b_0} \quad a_1 < \frac{b_1}{2}$
6.	$a_0 < 0$	$b_1 > 0$	$b_0 < 2$	$\frac{b_1}{2} < a_1 < \frac{a_0 b_1}{2 \quad b_0}$
7.	$a_0 > 0$	$b_1 < 0$	$2 \quad 2a_0 < b_0 < 2$	$\frac{b_1}{2} < a_1 < \frac{a_0 b_1}{2 \quad b_0}$

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

Table 1 continued…

8.	$a_0 > 0$	$b_1 < 0$	$b_0 > 2$	$a_1 > \frac{b_1}{2}$
9.	$a_0 > 0$	$b_1 < 0$	$b_0 < 2 \quad 2a_0$	$\frac{a_0 b_1}{2 \quad b_0} < a_1 < \frac{b_1}{2}$
10.	$a_0 < 0$	$b_1 < 0$	$2 < b_0 \quad 2 \quad 2a_0$	$a_1 < \frac{b_1}{2} \qquad a_1 > \frac{a_0 b_1}{2 \quad b_0}$
11.	$a_0 < 0$	$b_1 < 0$	$b_0 > 2 \quad 2a_0$	$a_1 > \frac{b_1}{2} \qquad a_1 < \frac{a_0 b_1}{2 \quad b_0}$
12.	$a_0 < 0$	$b_1 < 0$	$b_0 < 2$	$\frac{a_0 b_1}{2 \quad b_0} < a_1 < \frac{b_1}{2}$
13.	$a_0 = 0$	$b_1 < 0$	$b_0 < 2$	$0 < a_1 < \frac{b_1}{2}$
14.	$a_0 = 0$	$b_1 < 0$	$b_0 > 2$	$a_1 > \frac{b_1}{2} \qquad a_1 < 0$
15.	$a_0 = 0$	$b_1 > 0$	$b_0 < 2$	$\frac{b_1}{2} < a_1 < 0$
16.	$a_0 = 0$	$b_1 > 0$	$b_0 > 2$	$a_1 > 0 \qquad a_1 < \frac{b_1}{2}$
17.	$a_0 \quad \mathbb{R}$	$b_1 = 0$	$b_0 \quad 2$	$a_1 > 0$
18.	$a_0 < 0$	$b_1 \quad 0$	$b_0 \quad \mathbb{R}$	$a_1 = 0$

5. Conclusion

After re-parametrizing the parameter in (9) and in (10) by $a_0 \quad a_1$ and $\tilde{F} \quad \tilde{F} \quad (b_0 \quad b_1)$, we have a new mapping $\hat{\hat{f}}$. The properties of this mapping are integrable, measure preserving, and reversible. Furthermore, it has two fixed points and two 2-periodic points which are of an elliptic type and a hyperbolic type. It is interesting to underline the fact that we have the set of P_k consisting of points in $\mathbb{R}^2$ which is mapped to infinity after k-iterates of the map. Then we can consider the set of $\mathcal{P} = \bigcup_1 P_k$. For an arbitrary level set of the integral (12), the question should be whether the intersection between the set of $\mathcal{P}$ with the level set is finite or infinite (could it be dense on the level set). This is a subject to future investigation.

Conflict of Interest
The authors confirm that there is no conflict of interest for this publication.

Acknowledgements

International Journal of Mathematical, Engineering and Management Sciences
Vol. 5, No. 2, 363-377, 2020
https://doi.org/10.33889/IJMEMS.2020.5.2.030

LZ acknowledges supports from a The Research and Community Services of Lampung University through Lampung University DIPA BLU Research Grants 2017. JMT research is supported by: Riset KK ITB 2017. The authors would like to thank you very much for the reviewer's comments/suggestions that improved the contents of the article.

References

Celledoni, E., Evripidou, C., McLaren, D.I., Owren, B., Quispel, G.R.W., & Tapley, B.K. (2019). Discrete Darboux polynomials and the search for preserved measures and integrals of rational maps. *Arxiv Preprint Arxiv:* 1902.04685v1.

Duistermaat, J.J. (2010). *Discrete integrable systems: QRT maps and elliptic surfaces*. Springer Monographs in Mathematics, Springer-Verlag, New York.

Joshi, N., & Kassotakis, P. (2019). Re-factorising a QRT map. *Arxiv Preprint Arxiv:*1906.00501v1[nlin.SI]

Kulenovic, M.R.S., & Merino, O. (2002). *Discrete dynamical systems and difference equations with mathematica*. Chapman and Hall/CRC, Boca Raton, Florida. USA.

Quispel, G.R.W., Capel, H.W, & Roberts, J.A.G. (2005). Duality for discrete integrable systems.*Journal of Physics A: Mathematical and General*, *38*(18), 3965.

Quispel, G.R.W., Capel, H.W, Papageorgiou V.G, & Nijhoff, F.W (1991). Integrable mappings derived from soliton equations. *Physica A: Statistical Mechanics and its Applications*, *173*(1-2), 243–266.

Quispel, G.R.W., Roberts, J.A.G., & Thompson, C.J. (1988). Integrable mappings and soliton equations. *Physics Letters A*, *126*(7), 419-421.

Quispel, G.R.W., Roberts, J.A.G., & Thompson, C.J. (1989). Integrable mappings and soliton equations II. *Physica D*, *34*(1-2), 183-192.

Roberts, J.A.G., Iatrou A., & Quispel, G.R.W. (2002). Interchanging parameters and integrals in dynamical systems: the mapping case, *Journal of Physics A: Mathematical and General*, *35*(9), 2309-2325.

Tuwankotta, J.M., Van der Kamp, P., Quispel, G.R.W., & Saputra, K.V.I. (2019). Generating a chain of maps which preserve the same integral as a given map. *arXiv Preprint arXiv*:1902.05206.

Tuwankotta, J.M., Quispel G.R.W., & Tamizhmani, K.M. (2004). Dynamics and bifurcations of a three-dimensional piecewise-linear integrable map. *Journal of Physics A: Mathematical and General*, *37*(50), 12041.

Van der Kamp, P.H., & Quispel, G.R.W. (2010). The staircase method: integrals for periodic reductions of integrable lattice equations. *Journal of Physics A: Mathematical and Theoretical*, *43*(46) , 465207.

Van der Kamp, P.H., Rojas, O., & Quispel, G.R.W. (2007). Closed-form expressions for integrals of mKdV and sine-Gordon maps. *Journal of Physics A: Mathematical and Theoretical*, *40*(42), 12789.

Zakaria, L., & Tuwankotta, J.M. (2016). Dynamics and bifurcations in a two-dimensional maps derived from a generalized ΔΔsine-Gordon equation. *Far East Journal of Dynamical Systems*, *28*(3), 165-194.

Original content of this work is copyright © International Journal of Mathematical, Engineering and Management Sciences. Uses under the Creative Commons Attribution 4.0 International (CC BY 4.0) license at https://creativecommons.org/licenses/by/4.0/