



# SEMINAR NASIONAL SAINS MIPA DAN APLIKASINYA

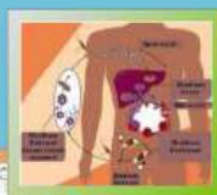
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# **Prosiding** Seminar Nasional Sains MIPA dan Aplikasinya Tahun 2010 FMIPA Universitas Lampung

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Tim Penyunting



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# COMPUTATIONAL ASPECT OF WADR1 AND WADR2 ALGORITHMS FOR THE MULTI PERIOD DEGREE CONSTRAINED MINIMUM SPANNING TREE PROBLEM

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## ABSTRACT

The Multi Period Degree Constrained Minimum Spanning Tree Problem (MPDCMST) is a problem of determining the total minimum installation cost in a certain network, where the installation process is divided into some periods due to the fund limitation. In addition, the network itself must maintain its reliability by restricting the number of links at every node (can be computer, city, road, and so on). In this paper we will discuss the computational aspects of two algorithms, WADR1 and WADR2 for the MPDCMST and compare the solutions of those two algorithms.

*Keywords: multi period, degree constrained, computational aspects*

## INTRODUCTION

Multi Period Degree Constrained Minimum Spanning Tree (MPDCMST) Problem is one of network problems that frequently occur in real life situation, especially in networks installation. For example, the installation of power supply network system, the installation of telecommunication network, computer network, water supply network, and so on. However, in reality, the installation process needs to be done in some periods, mostly because of the fund limitation. When the installation process only needs one period, the problem is called as Degree Constrained Minimum Spanning Tree (DCMST) Problem.

DCMST can be applied in some cases where  $n$  vertices (represents terminals, servers, depot, etc) must be connected in such a way so that the minimum distance/cost/time (represented by edges/arcs) is minimum whilst also restrict the interconnection in every vertex. The restriction needed to ensure reliability of the network (Wamiliana, 2002). If we disregard the restriction in every vertex, the problem reduced to a well known problem in network design, which is The Minimum Spanning Tree (MST) problem.

Minimum spanning trees or spanning trees in general, are used in many network optimization problems as the key structure. Since G.R. Kirchoff designed electrical circuits in the 19<sup>th</sup> century, spanning trees have been considered as one of the most used subgraphs in many network design applications. Deo and Kumar (1997) reported that spanning trees can be computed in linear time for a given connected weighted graph. When the spanning tree is required to have minimum weight (minimum spanning tree), the computational time only slightly increases. Rouvray (1995), reported that Flaviitsky, in 1871 was the one who was the first to use combinatorial techniques to determine isomer counts. Flaviitsky studied the homologues series of the alcohols and enumerated the first ten members. His investigations arrived at a conclusion that this problem is the equivalent to enumerating rooted tree graphs all of whose vertices are of degree one or



four. Based on this investigation, then Cayley in 1874 introduced generating functions into isomer enumeration.

In this paper we will discuss two algorithms for solving the MPDCMST problem, and we will organize our paper as follow: In Section 1 we give a brief information about MPDCMST, its derivates and applications, in Section 2 we give information about some of the methods already investigated, in Section 3 we will discuss our proposed algorithm, In Section 4 we give the implementation and results, followed by conclusion.

### METHODS AVAILABLE IN LITERATURE

The Minimum Spanning Tree (MST) problem is one of the classical problems arising in many network design applications. To find a minimum-spanning tree, there are two well-known algorithms: Kruskal's (1956) and Prim's (1957). However, the earliest algorithm for finding a minimum spanning tree according to Graham and Hell (1982) was suggested by Boruvka (1926a,b) who developed an algorithm for finding the most economical layout for a power-line network. Minimum spanning tree algorithms have been studied extensively and a variety of fast algorithms have been developed. Gabow et al. (1986) developed an efficient and fast minimum spanning tree algorithm that requires computational time nearly linear in the number of edges.

The DCMST problem is MST with additional constrained. The objective is to construct a minimum cost network that satisfies the prescribed graph parameters, which is the degree. The DCMST can be formulated as a Mixed Integer Linear Programming as follow (Wamiliana, 2002):

$$\text{Minimise } \sum_i^n \sum_j^n c_{ij} x_{ij} \quad (1)$$

Subject to:

$$\sum_{i,j} x_{ij} = n - 1 \quad (2)$$

$$\sum_{i,j \in V'} x_{ij} \leq |V'| - 1, \quad \forall \emptyset \neq V' \subset V \quad (3)$$

$$1 \leq \sum_{j=1, j \neq i} x_{ij} \leq b_i, \quad i = 1, 2, \dots, n \quad (4)$$

$$x_{ij} = 0 \text{ or } 1, \quad 1 \leq i \neq j \leq n. \quad (5)$$

$c_{ij}$  is the weight (or distance or cost) of the edge  $(i,j)$ ,  $b_i$  is the degree bound on vertex  $i$  and  $n$  is the number of vertices. Constraint (2.2) ensures that  $(n-1)$  edges are selected. Constraint (2.3) is the usual subtour elimination constraints. Constraint (2.4) specifies the degree restriction on the vertices. The last constraint (2.5), is just the variable constraint, which restricts the variables to the value of 0 or 1.  $x_{ij}$  is 1 if the edge  $x_{ij}$  is selected or included in the tree  $T$  and 0, otherwise.

Many authors have proposed solution methods for the DCMST problem, which include both exact and heuristic methods. Since this problem is NP complete, heuristic methods have dominated. Some of the heuristics that have been investigated include: a number of basic MST algorithms of Prim and Kruskal (see Narula and Ho ,1980); the genetic Algorithm by Zhou and Gen (1997); Simulated Annealing by Krishnamoorthy et al. (2001); and Iterative Refinement by Boldon et al. (1996) and Deo and Kumar (1997), Modified Penalty by Wamiliana (2002); Wamiliana and Caccetta (2006), and Tabu Search by Caccetta and Wamiliana (2001,2004), and Wamiliana and Caccetta (2004).

Multi Period Degree Constrained Minimum Spanning Tree (MPDCMST) is a problem derived from the Degree Constrained Minimum Spanning Tree Problem (DCMST). The difference of the two problems lies on the period in which restrict the network considered to be installed in some periods or stages. Usually, in real-life implementation, the installation must be done in some periods due to fund limitation. In DCMST, all the vertices (can represent computers, cities, and so on) can be installed in one period and there is no center vertex specified. Either DCMST or MPDCMST problems appear in some real-life problems such as installation of pipe for: water supply, liquid natural gas, electricity, telephone (cable) and so on.

Kawatra (2002) solved MPDCMST using *hybrid* methods between Lagrangean Relaxation and branch exchange. This method implemented using vertices varying from 40 to 100 vertices. They used a 10-year planning horizon and the time period for activating each terminal is uniformly distributed from 1 to 6, and vertex 1 as central vertex. Wamiliana et al (2005) investigated other types of MPDCMST where they used a one year planning horizon and divided the installations into three periods (4 months each), and four periods (three months each), and set vertex 1 as central vertex. They did the modifications in order to mimic the real situations occurs in Indonesia where the funding for each project usually divided into three or four periods. Wamiliana et al (2005) had investigated the greedy method to solve the MPDCMST problem and get feasible solution for all data tested. However, the solutions are still not guarantee to be optimal because they were not using benchmark problems and there is no exact method yet investigated to be compared. Junaidi et al (2008) improved the method proposed by Wamiliana et al (2005) and tested using some problems taken from TSPLIB. Next, we will propose the new algorithm for MPDCMST which are WADR1 and WADR2.

### WADR1 and WADR2 Algorithms

Both WADR1 and WADR2 algorithms in general use Modified Kruskal's algorithms to find the MPDCMST, but in the sorting process we adopt the DFS techniques until two vertices and two edges ahead to be connected in the tree ( or maybe forest ). Below is pseudocode of the algorithms:

**Initialization :** Set  $V = \{1\}$ ,  $T = \emptyset$ ,  $k = 2$ , and set vertex 1 as central of the network, set  $HVT_i$ ,  $MAXVT_i$

**begin**

Determine vertices in  $HVT_i$

Sort edges connected with vertices in  $HVT_i$  whose path length  $k$  in ascending order

Set  $i = 1$

**if**  $|HVT_i| \geq |MAXVT_i|$ , Stop

**else do**

**while** {the number of edges in  $T$  is  $< n - 1$ }

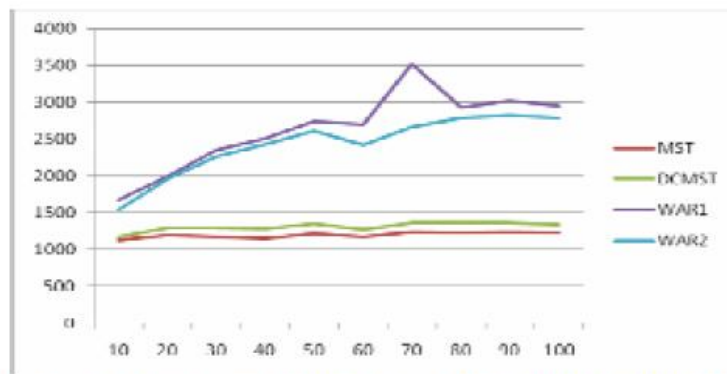
**do**





**Table 1.** Comparative Analysis of MST, DCMST, WADR1, and WADR2

n	Average (Solution)			
	MST	DCMST	WADR1	WADR2
10	1129.43	1178.80	1672.80	1544.37
20	1196.10	1299.60	1999.30	1966.17
30	1177.43	1304.03	2351.77	2263.13
40	1151.23	1287.97	2515.40	2439.57
50	1223.43	1357.07	2741.03	2609.77
60	1175.57	1282.30	2699.20	2427.80
70	1242.10	1367.53	3527.00	2672.97
80	1236.83	1367.70	2939.53	2794.93
90	1248.00	1367.93	3027.90	2823.50
100	1234.10	1340.23	2962.20	2786.60
<b>Average</b>	<b>1201.42</b>	<b>1315.32</b>	<b>2643.61</b>	<b>2432.88</b>



**Figure 1.** The Graph of average solution among MST, DCMST, WADR1 and WADR2

### CONCLUSION

From the graphs we can see that the solution found using WADR2 are better than WADR1. The reason is in WADR1 algorithm we must enter other vertices if the one must be installed in a certain period, already installed before, while in WADR2, if the vertices must be installed in certain period already installed before then the algorithm just continue to find the next available minimum edge. But, these two algorithms give alternative to the decision maker, which problem they are facing for. If they are not really care about what next vertex (city /terminal or other) is supposed to be in the network, then choose WADR2, the cheaper; or use WADR1 if they are concern about what city must be installed soon in a certain period.

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