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PREFACE

The Fifth ICNOOR V, International Conference on Numerical Optimization and Operations Researchcontinues the tradition of providing an open forum for the presentation, discussion, and development of innovative application of mathematics, statistics, operations research and other scientific methods in defense and tactical operations.

The Journal of KALAM, an internationally refereed journal has collaborated with Universiti Malaysia Trengganu (UMT) and UniversitasAbulyatama (UNAYA) to organize the 5th International Conference on Numerical Optimization and Operations Research (NOOR-V) 2013 in the campus of UNAYA, Aceh, Indonesia.

This year's keynote and invited speakers will highlight new trends in non-linear programming problems, algorithm development, discuss new trends and challenges for data mining from an industrial perspective, expose logic and its application in the methods of proof in mathematics, discuss implicit finite difference, and also the applications of fuzzy soft sets. Then, application with mathematical approach, optimzations and trends in research methods as well as its applications within a qualitative reseachis briefly discussed by some researchers.

We would like to thank the entire organizing committee for the tremendous job that have ben done in putting together such a strong technical program and diligently overseeing the entire conference paper selection process, for effectively publicizing the conference at countless venues. We are also very grateful to all of our sponsors, Abulyatama Aceh foundation, AlihTeknologi Bandar Lampung foundation, Green Paradise Cottage and Bank SyariahMandiri(BSM) UleeKareng Branch as sponsors for the conference, without their support this conference would not have been possible.

It is my hope that by providing a forum in which these papers can appear, the energy and vitality experienced by the participants at the 5th ICNOOR – 2013 can be enjoyed by many more members of the education community.

Steering Committee Chair: Agung Efriyo Hadi
Abulyatama University, Aceh Besar, 26 – 27 June 2013

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COMPUTATIONAL ASPECTS OF MODIFIED KRUSKAL ALGORITHM FOR DIAMETER RESTRICTED MINIMUM SPANNING TREE PROBLEM

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Abstract

The Diameter Restricted Minimum Spanning Tree Problem (DRMST) is a problem of finding a minimum weight spanning tree (where every edge has associated weight c_{ij}, and a positive integer d), having diameter at most d. Typically this problem arises when we are seeking the minimum length (for instance: cable or distance needed to connect between to terminals or cities, etc). In this paper we will discuss the modification of Kruskal' algorithm applied to DRMST and show the results.

Keywords: diameter restricted, minimum spanning tree, Kruskal's algorithm.

1. Introduction

One of classical problem which arises in many in many network design applications is The Minimum Spanning Tree (MST). To find a minimum-spanning tree, there are two well-known algorithms: Kruskal's and Prim's. However, the earliest algorithm for finding a minimum spanning tree according to Graham and Hell (1982) was suggested by Boruvka (Wamiliana, 2002) who developed an algorithm for finding the most economical layout for a power-line network.

The Diameter Restricted Minimum Spanning Tree Problem (DRMST) is a problem where given a weighted graph G=(V,E), where every edge has associated weight c_{ij}, and a positive integer d, the Bounded Diameter Restricted Minimum Spanning Tree Problem is a problem of finding a minimum weight spanning tree having diameter at most d. In application the DRMST could represent the maximum distance allowed for delivery, and in a telecommunication network a diameter restriction could represent the maximum number of links allowed for communication between two terminals.

The Minimum spanning tree algorithms have been studied extensively and a variety of fast algorithms have been developed. Gabow et al. (1986) developed an efficient and fast minimum spanning tree algorithm that requires computational time nearly linear in the number of edges. All algorithms for finding a minimum spanning tree exploit, in one or other way, the following fact:

Theorem. (Papadimitriou and Steiglitz, 1982)

Let $\{(U_1,T_1), (U_2,T_2),...,(U_k,T_k)\}$ be a forest spanning V, where $U_1, U_2, ... U_k \subseteq V$, and let (a,b) be the shortest of all edges with only one point in U_1. Then among all spanning trees containing all edges in $T = \bigcup_{j=1}^{k} T_j$, there is an optimal one containing (a,b).

The minimum spanning tree algorithm is a polynomial time algorithm, which means that any instance of the minimum spanning tree problem can be solved in polynomial time (polynomial solvable). However, an additional constraint often makes the problem hard, for example finding a minimum spanning tree with the additional constraint such as degree or diameter is computationally difficult, in fact (apart from some trivial cases) it is NP- complete (Wamiliana, 2002)

In this paper we will discuss the Modified Kruskal's algorithms for solving one of the various problems having MST as the backbone, which is the Diameter Restricted d Minimum Spanning Tree Problem, and we organize this paper as follow: in Section 2 we give introduction about the DRMST and discuss papers that available in literature, especially related to the problem discussed; in Section 3 we give the formulations of the problems and discuss the algorithms developed and in section 4 we show the computational results, concluding remark and possible future works.

2. The Diameter Restricted Minimum Spanning Tree (DRMST) Problem

The Diameter Restricted Minimum Spanning Tree (DRMST) problem typically arises in the design of telecommunication, transportation and energy networks. It is concerned with finding a minimum-weight (distance or cost) spanning tree that satisfies the diameter restriction. The diameter constrained restricts the number of path between to vertices in the networks. The applications of the Diameter Restricted Minimum Spanning Tree problems that may arise in real-life include: the design of telecommunication, transportation, and energy networks.

The Methods Available in literature

The Diamater Restriction Minimum Spanning Tree DRMST) Problem had been investigated by some researchers including Noronha et al (2007), Achutan et al (1992, 1994), Gouveia and Magnanti (2000, 2003), Santos et al (2004), and Trick (2003). Noronha et al (2007) developed a Constrained Programming (CP) which motivated by the weak bounds of MIP programming while in the other side needs huge computer memory due to large numbers of variables and constraints. Achutan et al (1992) gave a DRMST model and formulations using single commodity flow, and in 1992, Achutan et al ((1992) improved the formulations by valid inequalities. Santos et al (2004) proposed the way of solving the DRMST by using lifting procedure. Gouveia and Magnanti (2003) proposed the new formulation using the multicommodity flow formulations with tighter linear programming relaxation, and Tricks (2003) presented a successful application of DRMST.

3. Formulations for the Diameter Restricted Minimum Spanning Tree

The Diameter Restricted Minimum Spanning Tree (DRMST) Problem can be formulated as a Mixed Integer Linear Programming problem as follow:`

$$(P_0)\ : Minimise\ \sum_{i}^{n}\sum_{j}^{n} c_{ij}x_{ij} \qquad (1)$$

subject to

$$\sum_{i,j} x_{ij} = n-1 \qquad (2)$$

$$\sum_{i,j\in V'} x_{ij} \le |V'|-1, \qquad \forall \varnothing \ne V' \subseteq V \qquad (3)$$

$$\sum_{j=1,i\ne j} x_{ij} \le d \qquad i = 1,2,\ldots, n \qquad (4)$$

$$x_{ij} = 0 \text{ or } 1, \qquad 1 \le i \ne j \le n. \qquad (5)$$

In the method developed we apply one of the most well known algorithms for solving the MST problem which is Kruskal Algorithm. But, to accommodate the diameter restrictions, we do make some modifications on the algorithms. The modifications made mostly when the connection of the edges should be made, because this connection processes is supposed not to be done if it not satisfies the diameter restrictions. We propose the algorithm in the following pseudo code:

Input : T = ∅, d = max diameter, n = the number of vertices

begin

Sort edges from the smallest to the highest weight.

Choose the smallest weight edge and put in T

Remove the chosen edge from the sorting.

while

$|T| < n-1$,

do

repeat

Consider the next smallest edge

if if the diameter of the network is ≤ d,

Put the edge in T

Remove the edge from the sorting

end

until $|T| = n-1$

end

end

end

The example (for vertex order 10) below illustrates how the algorithm works for $D \leq 5$.

Edge	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
From	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	4
To	2	3	4	5	6	7	8	9	10	3	4	5	6	7	8	9	10	4	5	6	7	8	9	10	5
Weight	10	57	44	43	35	41	64	92	47	10	38	32	27	77	79	49	24	33	92	21	44	78	2	73	86

Edge	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
From	4	4	4	4	4	5	5	5	5	5	6	6	6	6	7	7	7	8	8	9
To	6	7	8	9	10	6	7	8	9	10	7	8	9	10	8	9	10	9	10	10
Weight	22	23	75	44	44	59	71	9	52	44	32	77	5	13	96	91	64	96	6	48

The DRMST for this problem is

Edge	e_{39}	e_{69}	$e_{8,10}$	e_{58}	e_{23}	e_{46}	e_{67}	e_{16}	$e_{9,10}$
Weight	2	5	6	9	10	22	32	35	48

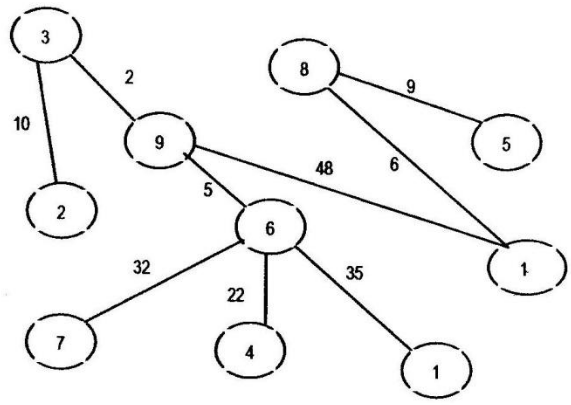

Figure 1. DRMST with d ≤ 5

4. Results and Concluding Remark

We use complete graph K_n with vertex order n to represent the problem. The data generated assigned for edge weight are uniformly distributed with the weight vary between 1 - 1000. For every order of the graph we generate 30 problems and for every order we restrict our diameter d as follow: $D_1 = 3$, $D_2 = \lfloor n/5 \rfloor$, $D_3 = \lfloor n/4 \rfloor$, $D_4 = \lfloor n/3 \rfloor$, dan $D_5 = \lfloor n/2 \rfloor$. The following chart shows the results.

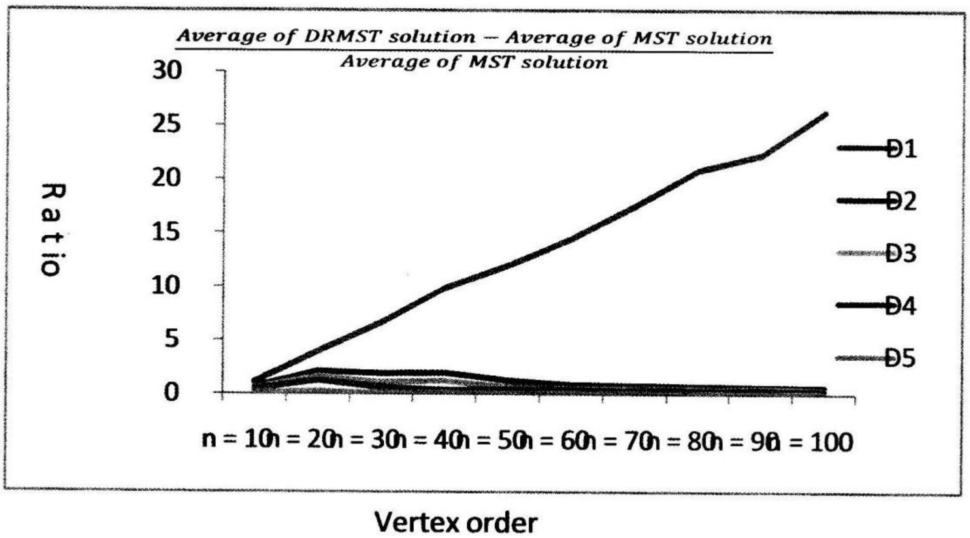

Figure 2. Analysis comparative for DRMST and MST with different d values

He chart shows the ratio of the solution of DRMST and MST as:

$$\frac{Average\ of\ DRMST\ solution - Average\ of\ MST\ solution}{Average\ of\ MST\ solution}$$

We can see that the longer the diameter, the solution for the DRMST is closer to that of MST . For D1 for example, the ratio for vertex order 100 is more than 26 times of that of MST. However, for D5, the ratio is only 0.04 which means only 4% and it is very close to the solution of the MST.

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