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The Locating-Chromatic Number for Certain Operation of Generalized Petersen Graphs \( sP(4, 2) \)

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**Abstract.** The locating-chromatic number of a graph combined two graph concept, coloring vertices and partition dimension of a graph. The locating-chromatic number, denoted by \( \chi_L(G) \), is the smallest \( k \) such that \( G \) has a locating \( k \)-coloring. In this paper, we discuss the locating-chromatic number for certain operation of generalized Petersen graphs \( sP(4,2) \).

1. **Introduction**

Chartrand et al. [1] in 2002 introduced the locating-chromatic number of a graph, with derived two graph concept, coloring vertices and partition dimension of a graph. Let \( G = (V, E) \) be a connected graph and \( c \) be a proper \( k \)-coloring of \( G \) with color 1, 2, ..., \( k \). Let \( \Pi = \{C_1, C_2, ..., C_k\} \) be a partition of \( V(G) \) which is induced by coloring \( c \). The color code \( c_{\Pi}(v) \) of \( v \) is the ordered \( k \)-tuple \( (d(v, C_1), d(v, C_2), ..., d(v, C_k)) \) where \( d(v, C_i) = \min \{d(v, x) | x \in C_i\} \) for any \( i \). If all distinct vertices of \( G \) have distinct color codes, then \( c \) is called a locating coloring of \( G \). The locating-chromatic number, denoted by \( \chi_L(G) \), is the smallest \( k \) such that \( G \) has a locating \( k \)-coloring.


Specially for non-homogenous tree graph in 2014, Asmiati [9] determined the locating-chromatic number of non-homogeneous amalgamation of stars, then Asmiati [10] for caterpillar graphs and non-homogenous firecracker graphs. In 2017, Asmiati et al. [11] determined some generalized Petersen graphs \( P(n, 1) \) having locating-chromatic number 4 for odd \( n \geq 3 \) or 5 for even \( n \geq 4 \).
The generalized Petersen graph $P(n, k)$, $n \geq 3$ and $1 \leq k \leq [(n - 1)/2]$, consists of an outer $n$-cycle $u_1, u_2, ..., u_n$, a set of $n$ spokes $u_i, v_i, 1 \leq i \leq n$, and $n$ edges $v_i, v_{i+k}, 1 \leq i \leq n$, with indices taken modulo $n$. The generalized Petersen graph was introduced by Watkins in [12].

To define the generalized Petersen graph $sP(4,2)$, suppose there are $s$ generalized Petersen graph $P(4,2)$. Some vertices on the outer cycle $u_i, i = 1, 2, 3, 4$ for the generalized Petersen graph $t^e, t = 1, 2, ..., s, s \geq 1$ denoted by $u_i^t$, while some vertices on the inner cycle $v_i, i = 1, 2, 3, 4$ for the generalized Petersen graph $t^e, t = 1, 2, ..., s, s \geq 1$ denoted by $v_i^t$. Generalized Petersen graph $sP(4,2)$ obtained from $s \geq 1$ graph $P(4,2)$, which every vertices on the outer cycle $u_i^t, i \in [1,4], t \in [1, s]$ connected by a path $(u_i^t u_i^{t+1}) t = 1, 2, ..., s - 1, s \geq 2$.

Some researchers have determined the locating-chromatic number for certain operation. Behtoei and Omoomi [13] obtained locating-chromatic number from the grid, cartesian multiplication for trajectories and complete graphs, and cartesian multiplication of two complete graphs. Furthermore Behtoei and Omoomi [14] determined the locating-chromatic number of the fan graph, wheel and friendship graph for join multiplication of two graphs. Asmiati [15] found locating-chromatic number for certain operation of tree. In this paper, we discuss the locating-chromatic number for certain operation of generalized Petersen graphs $sP(4,2)$.

The following theorems basic to determine the locating chromatic number of a graph. The set of neighbours of a vertex $s$ in $G$, denoted by $N(s)$.

**Theorem 1.1.** Chartrand et al.[1] Let $c$ be a locating coloring in a connected graph $G$. If $r$ and $s$ are distinct vertices of $G$ such that $d(r,w)=d(s,w)$ for all $w \in V(G) \setminus \{r, s\}$, then $c(r) \neq c(s)$. In particular, if $x$ and $y$ are non-adjacent vertices of $G$ such that $N(x) \neq N(y)$, then $c(x) \neq c(y)$.

**Theorem 1.2.** Chartrand et al.[1] The locating chromatic number of a cycle $C_n$, is 3 for odd $n$ and 4 for otherwise.

2. **Results and Discussion**

In this section we will discuss the locating chromatic number of $sP(4,2)$.

**Theorem 2.1.** The locating chromatic number of generalized Petersen graph $sP(4,2)$ is 5 for $s \geq 2$.

**Proof:** First, we determine lower bound of $\chi_L(sP(4,2))$ for $s \geq 2$. Because generalized Petersen graph $P(4,2)$, for $s \geq 2$, contains some even cycles. Then by Theorem 2, $\chi_L(sP(4,2)) \geq 4$. Next, we will show that $\chi_L(sP(4,2)) \geq 5$, for $s \geq 2$. For a contradiction, suppose that $c$ is 4-locating coloring on $sP_{4,1}$ for $s \geq 2$. Consider $c(u_i^j) = i, i = 1, 2, 3, 4$ and $c(v_j^k) = j, j = 1, 2, 3, 4$ such that $c(u_i^j) \neq c(v_j^k)$ for $c(u_i^j)$ adjacent to $c(v_j^k)$. Observe that if we assign color 4 for any vertices in $u_i^2$ or $v_j^2$, then we have two vertices which have color codes. Therefore, $c$ is not locating 4-coloring on $sP(4,2)$. As the result $\chi_L(sP(4,2)) \geq 5$ for $s \geq 2$.

Next, we determine the upper bound of $\chi_L(sP(4,2))$ for $s \geq 2$. Let $c$ be a coloring of generalized Petersen graph $sP(4,2)$ for $s \geq 2$. We make the partition of the vertices of $V(sP(4,2))$:

- $C_1 = \{u_1^1, v_1^1\}$ for odd $s \cup \{u_2^1, v_2^1\}$ for even $s$
- $C_2 = \{u_2^1, u_4^1\}$ for odd $s \cup \{u_3^1, v_3^1\}$ for even $s$
- $C_3 = \{u_3^1, v_1^1, v_2^1\}$ for odd $s \cup \{u_4^1, v_4^1, v_3^1\}$ for even $s$
- $C_4 = \{v_3^1\}$ for odd $s \cup \{u_1^1\}$ for even $s$
- $C_5 = \{v_4^1\}$

Therefore the color codes of all the vertices of $G$ are:

(a) $C_1 = \{u_1^1\}$ for odd $s \cup \{u_2^1, v_2^1\}$ for even $s$

For odd $s$, the color codes of $sP(4,2)$ are:

$$c_{01}(u_i^j) = \begin{cases} 0, & \text{for } 1^{st} \text{ component} \\ 1, & \text{for } 2^{nd}, 3^{rd} \text{ and } 4^{th} \text{ component} \\ s + 1, & \text{for } 5^{th} \text{ component} \end{cases}$$
For even $s$, the color codes of $sP(4,2)$ are:

$$
c_{II}(u_2^s) = \begin{cases} 
0 & \text{for 1 st component} \\
1 & \text{for 2 nd, 3 rd and 4 th component} \\
s + 1 & \text{for 5 th component}
\end{cases}
$$

$$
c_{II}(v_4^s) = \begin{cases} 
2 & \text{for 2 nd and 4 th component} \\
1 & \text{for 3 rd component} \\
s + 1 & \text{for 5 th component}
\end{cases}
$$

(b) $C_2 = \{u_2^s, u_4^s\}$ for odd $s$ \cup $\{u_3^s, v_1^s\}$ for even $s$

For odd $s$, the color codes of $sP(4,2)$ are:

$$
c_{II}(u_2^s) = \begin{cases} 
1 & \text{for 1 st and 3 rd component} \\
0 & \text{for 2 nd component} \\
4 & \text{for 4 th component} \\
s + 1 & \text{for 5 th component}
\end{cases}
$$

$$
c_{II}(u_4^s) = \begin{cases} 
1 & \text{for 1 st, 3 rd and 5 th component} \\
0 & \text{for 2 nd component} \\
2 & \text{for 4 th component}
\end{cases}
$$

For odd $s \geq 3$, the color codes of $sP(4,2)$ are:

$$
c_{II}(u_4^s) = \begin{cases} 
1 & \text{for 1 st, 3 rd and 4 th component} \\
0 & \text{for 2 nd component} \\
s & \text{for 5 th component}
\end{cases}
$$

For even $s$, the color codes of $sP(4,2)$ are:

$$
c_{II}(u_2^s) = \begin{cases} 
1 & \text{for 1 st and 3 rd component} \\
0 & \text{for 2 nd component} \\
2 & \text{for 4 th component} \\
s + 1 & \text{for 5 th component}
\end{cases}
$$

$$
c_{II}(v_1^s) = \begin{cases} 
1 & \text{for 1 st and 4 th component} \\
0 & \text{for 3 rd component} \\
2 & \text{for 2 nd component} \\
s + 2 & \text{for 5 th component}
\end{cases}
$$

(c) $C_3 = \{u_3^s, v_1^s, v_2^s\}$ for odd $s$ \cup $\{u_4^s, v_1^s, v_3^s\}$ for even $s$.

For odd $s$, the color codes of $sP(4,2)$ are:

$$
c_{II}(u_2^s) = \begin{cases} 
2 & \text{for 1 st component} \\
1 & \text{for 2 nd and 4 th component} \\
0 & \text{for 3 rd component} \\
s + 1 & \text{for 5 th component}
\end{cases}
$$

$$
c_{II}(v_4^s) = \begin{cases} 
1 & \text{for 1 st and 4 th component} \\
0 & \text{for 3 rd component} \\
2 & \text{for 2 nd component} \\
s + 2 & \text{for 5 th component}
\end{cases}
$$
\[
c_{\Pi}(v_2^1) = \begin{cases} 
2, & \text{for 1}\text{st component} \\
1, & \text{for 2}\text{nd and 5}\text{th component} \\
0, & \text{for 3}\text{rd component} \\
3, & \text{for 4}\text{th component} 
\end{cases}
\]

For odd \(s \geq 3\) the color codes of \(sP(4,2)\) are:
\[
c_{\Pi}(v_2^1) = \begin{cases} 
2, & \text{for 1}\text{st and 4}\text{th component} \\
1, & \text{for 2}\text{nd component} \\
0, & \text{for 3}\text{rd component} \\
s + 2, & \text{for 5}\text{th component} 
\end{cases}
\]

For even \(s\) the color codes of \(sP(4,2)\) are:
\[
c_{\Pi}(u_1^2) = \begin{cases} 
1, & \text{for 1}\text{st, 2}\text{nd and 4}\text{th component} \\
0, & \text{for 3}\text{rd component} \\
s, & \text{for 5}\text{th component} \\
1, & \text{for 1}\text{st component} 
\end{cases}
\]
\[
c_{\Pi}(v_2^2) = \begin{cases} 
2, & \text{for 2}\text{nd and 4}\text{th component} \\
0, & \text{for 3}\text{rd component} \\
s + 2, & \text{for 5}\text{th component} 
\end{cases}
\]
\[
c_{\Pi}(v_3^2) = \begin{cases} 
2, & \text{for 1}\text{st and 4}\text{th component} \\
1, & \text{for 2}\text{nd component} \\
0, & \text{for 4}\text{th component} \\
s + 2, & \text{for 5}\text{th component} 
\end{cases}
\]
\[
c_{\Pi}(v_4^2) = \begin{cases} 
1, & \text{for 2}\text{nd component} \\
0, & \text{for 3}\text{rd component} \\
s + 2, & \text{for 5}\text{th component} 
\end{cases}
\]

(d) \(C_4 = \{v_2^2\} | \text{for odd } s \} \cup \{u_1^2\} | \text{for even } s \} \cup \{v_3^2\} | \text{for odd } s \geq 3 \} \)

For odd \(s\) the color codes of \(sP(4,2)\) are:
\[
c_{\Pi}(v_3^2) = \begin{cases} 
2, & \text{for 1}\text{st and 2}\text{nd component} \\
1, & \text{for 3}\text{rd component} \\
0, & \text{for 4}\text{th component} \\
s + 2, & \text{for 5}\text{th component} 
\end{cases}
\]

For odd \(s \geq 3\) the color codes of \(sP(4,2)\) are:
\[
c_{\Pi}(v_4^2) = \begin{cases} 
2, & \text{for 1}\text{st component} \\
1, & \text{for 2}\text{nd and 3}\text{rd component} \\
0, & \text{for 4}\text{th component} \\
s + 1, & \text{for 5}\text{th component} 
\end{cases}
\]

For even \(s\) the color codes of \(sP(4,2)\) are:
\[
c_{\Pi}(v_4^2) = \begin{cases} 
1, & \text{for 1}\text{st, 2}\text{nd and 3}\text{rd component} \\
0, & \text{for 4}\text{th component} \\
s + 1, & \text{for 5}\text{th component} 
\end{cases}
\]
(e) $C_5 = \{v_4^1\}$

$$c_{fl}(v_4^1) = \begin{cases} 
2 & \text{for 1}^{st} \text{ component} \\
1 & \text{for 2}^{nd} \text{ and 3}^{rd} \text{ component} \\
3 & \text{for 4}^{th} \text{ component} \\
0 & \text{for 5}^{th} \text{ component}
\end{cases}$$

Since all the vertices have different color codes, $c$ is a locating coloring of generalized Petersen graphs $sP(4,2)$, so $\chi_L(sP(4,2)) = 5$, for even $s \geq 2$.

In figure 1 is illustrated a locating coloring of generalized Petersen graphs $4P(4,2)$ with the locating chromatic number 5.

![Figure 1](image)

**Figure 1.** A minimum locating coloring of $4P(4,2)$

3. **Conclusion**

Based on the results, locating chromatic number of generalized Petersen graph $sP(4,2)$ is 5 for $s \geq 2$.

**References**


